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Robust H_∞ Synchronization of a hyper-chaotic system with disturbance input

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Abstract

This paper concerns the robust control problems on the synchronization of a hyper-chaotic system with disturbance input. Using an appropriate Lyapunov function, we design the multi-dimensional and the single-dimensional robust H_∞ synchronization controllers in terms of linear matrix inequalities for the application in practical engineering. Corresponding theoretical derivations are given subsequently. Finally, some numerical simulations are provided to demonstrate the effectiveness of the proposed techniques.

***Keywords**:* hyper-chaotic; synchronization; H_∞ performance, Lyapunov function.

1. **Introduction**

Chaos synchronization has powerful application in chemical reactions, power converters, biological systems, information processing, secure communications, etc. During the last decade, many techniques for handling chaos synchronization have been studied [1-11]. In [4], through separation between linear and nonlinear terms of the chaotic system, an adaptive synchronous controller for a general class of non-autonomous chaotic systems is proposed. In [5], based on delayed feedback control and intermittent linear state delayed feedback control, the synchronization problem on non-autonomous chaotic systems is discussed. In [6], an adaptive controller with parameter update laws is designed to realize the projective synchronization of two different chaotic systems. In [7], based on the time-domain approach, the tracking synchronization control method is proposed for the uncertain Genesio-Tesi chaotic systems with dead zone nonlinearity. In [8], the projective synchronization problem for different fractional order chaotic systems is investigated. In [9], some adaptive control schemes are developed to anti-synchronize two chaotic complex systems. However, in practical control systems,

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disturbance inputs exist widely, which make the real control problem much more complicated. H_∞ theory is an efficient approach to handle such problem and has received increasing attentions in recent years, corresponding research outcomes can be seen in [12-31] and references therein. In this paper, an H_∞ method will be introduced to solve the robust synchronization control problem for a hyper-chaotic system with disturbance input.

Hyper-chaotic systems, characterized as a chaotic attractor with more than one positive Lyapunov exponents, can generate much more complicated dynamics. Therefore, the relative research, especially for new one, is an interesting and challenging issue. In [32], a new hyper-chaotic system is introduced, and the dynamical equation is described by

$$\begin{aligned}
\dot{z}_1 &= r_1(z_2 - z_1 - g(z_1)) \\
\dot{z}_2 &= z_1 - z_2 + z_3 \\
\dot{z}_3 &= -r_2[z_2 - z_4] \\
\dot{z}_4 &= -r_3[z_3 + z_5] \\
\dot{z}_5 &= r_4[z_4 - r_5 z_5]
\end{aligned} \tag{1}$$

with

$$g(z_1) = bz_1 + \frac{(a-b)[|z_1+1| - |z_1-1|]}{2}$$

where $z_i,\ i = 1, 2, \cdots, 5$ is the system state variable, and $r_i > 0,\ i = 1, 2, \cdots, 5$ is the system parameter.

Such chaotic system has two positive Lyapunov exponents when $a = -1.27, b = -0.68,$ $r_1 = 10, r_2 = 14, r_3 = 0.2, r_4 = 20, r_5 = 0.03$, and will produce the complicated dynamics. Unfortunately, to our knowledge, such system is not paid much attention and corresponding investigations are seldom found so far. This motivates our research.

In this paper, we will study the robust synchronization control problem on the hyper-chaotic system (1) with disturbance input, corresponding theoretical derivations will be presented to support the results obtained. Finally some simulation examples will be included to validate the effectiveness of our synchronization methods.

Notations used in this paper are fairly standard. Let R^n be the *n*-dimensional Euclidean space, $R^{n\times m}$ denotes the set of $n \times m$ real matrix, the symbol * represents the symmetric part in a matrix, *I* represents an identity matrix with appropriate dimensions, the superscript *T* stands for matrix transposition, and diag{...} represents a block diagonal matrix. $\|\cdot\|_2$ refers to the Euclidean vector norm or the induced matrix 2-norm. By $A > 0$ we mean that *A* is a real symmetric positive definitive matrix.

2. **System description and preliminaries**

First, based on the hyper-chaotic system (1), we construct the following master-slave systems

$$\begin{aligned}
\dot{x}_1(t) &= r_1(x_2(t) - x_1(t) - g(x_1(t))) \\
\dot{x}_2(t) &= x_1(t) - x_2(t) + x_3(t) \\
\dot{x}_3(t) &= -r_2[x_2(t) - x_4(t)] \\
\dot{x}_4(t) &= -r_3[x_3(t) + x_5(t)] \\
\dot{x}_5(t) &= r_5[x_4(t) - r_4 x_5(t)]
\end{aligned} \tag{2}$$

and

$$\begin{aligned}
\dot{y}_1(t) &= r_1(y_2(t) - y_1(t) - g(y_1(t))) + w_1(t) + u_1(t) \\
\dot{y}_2(t) &= y_1(t) - y_2(t) + y_3(t) + w_2(t) + u_2(t) \\
\dot{y}_3(t) &= -r_2[y_2(t) - y_4(t)] + w_3(t) + u_3(t) \\
\dot{y}_4(t) &= -r_3[y_3(t) + y_5(t)] + w_4(t) + u_4(t) \\
\dot{y}_5(t) &= r_5[y_4(t) - r_4 y_5(t)] + w_5(t) + u_5(t)
\end{aligned} \tag{3}$$

where $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t), x_5(t))^T$ and $y(t) = (y_1(t), y_2(t), y_3(t), y_4(t), y_5(t))^T$ are the state vectors of the master and slave systems, respectively, $u(t) = (u_1(t), u_2(t), u_3(t), u_4(t), u_5(t))^T$ is the multiple-dimensional synchronization controller, $W(t) = (w_1(t), w_2(t), w_3(t), w_4(t), w_5(t))^T$ is the disturbance input.

Define the synchronization error vector of the master-slave systems (2) and (3) as

$$E(t) = y(t) - x(t) \tag{4}$$

where $E(t) = (e_1(t), e_2(t), e_3(t), e_4(t), e_5(t))^T$.

Then the error dynamics can be expressed by

$$\begin{aligned}
\dot{e}_1(t) &= r_1(e_2(t) - e_1(t) - M(t)e_1(t)) + w_1(t) + u_1(t) \\
\dot{e}_2(t) &= e_1(t) - e_2(t) + e_3(t) + w_2(t) + u_2(t) \\
\dot{e}_3(t) &= -r_2[e_2(t) - e_4(t)] + w_3(t) + u_3(t) \\
\dot{e}_4(t) &= -r_3[e_3(t) + e_5(t)] + w_4(t) + u_4(t) \\
\dot{e}_5(t) &= r_5[e_4(t) - r_4 e_5(t)] + w_5(t) + u_5(t)
\end{aligned} \tag{5}$$

where $a \le M(t) \le b$.

In this paper, the following lemmas and definition are concerned

Lemma 1 [33]. If $f(t) \in L_\infty \cap L_2$ and $\dot{f}(t) \in L_\infty$, we have

$$\lim_{t \to +\infty} f(t) = 0 \tag{6}$$

Lemma 2 [34]**.** Given any real vectors D and E with appropriate dimensions, and any positive scalar $k > 0$, the following inequality holds

$$DE + E^T D^T \le kDD^T + \frac{1}{k} E^T E \tag{7}$$

Definition 1: Under the assumption of zero initial condition, the systems (3) can synchronize to system (2) with an H_∞ norm bound γ, if there exists a scalar $\gamma > 0$ such that

$$\|E(t)\|_2 \le \gamma \|w(t)\|_2 \tag{8}$$

for any nonzero $w(t) \in L_2[t_0, \infty]$

3. Main Results

In this section, based on Lyapunov method and LMI technique, the following theoretical results are proposed.

Theorem 1. If there exist scalars $\gamma > 0$, $K_i > 0, i = 1, 2, \cdots, 5$, we design an H_∞ synchronization law with the following control regulation

$$u(t) = -K^T E(t) \tag{9}$$

with

$$\begin{aligned} K &= diag\{k_1, k_2, k_3, k_4, k_5\} \\ k_1 &= K_1 - r_1(1+a) + 4, \\ k_2 &= K_2 + \frac{(r_1+1)^2}{16} + 3, \\ k_3 &= K_3 + \frac{(1-r_2)^2}{16} + 4, \\ k_4 &= K_4 + \frac{(r_2 - r_3)^2}{16} + 4, \\ k_5 &= K_5 + \frac{(r_5 - r_3)^2}{16} - r_4 r_5 \end{aligned} \tag{10}$$

$$\begin{bmatrix} I - K & I \\ * & -\gamma^2 I \end{bmatrix} < 0 \tag{11}$$

then, system (3) with any initial conditions can synchronize to system (2) with H_∞ norm bound γ.

Proof. Choose the following Lyapunov function candidate

$$V(t) = \frac{1}{2}(e_1^2(t) + e_2^2(t) + e_3^2(t) + e_4^2(t) + e_5^2(t)) \tag{12}$$

The time derivative of $V(t)$ along trajectories of error model (5) is

$$\begin{aligned}
\dot{V}(t) &= e_1(t)\dot{e}_1(t) + e_2(t)\dot{e}_2(t) + e_3(t)\dot{e}_3(t) + e_4(t)\dot{e}_4(t) + e_5(t)\dot{e}_5(t) \\
&= e_1(t)[r_1(e_2(t) - e_1(t) - Me_1(t)) - k_1 e_1(t) + w_1(t)] \\
&\quad + e_2(t)[e_1(t) - e_2(t) + e_3(t) - k_2 e_2(t) + w_2(t)] \\
&\quad + e_3(t)[-r_2(e_2(t) - e_4(t)) - k_3 e_3(t) + w_3(t)] \\
&\quad + e_4(t)[-r_3(e_3(t) + e_5(t)) - k_4 e_4(t) + w_4(t)] \\
&\quad + e_5(t)[r_5(e_4(t) - r_4 e_5(t)) - k_5 e_5(t) + w_5(t)] \\
&= -r_1(1+M)e_1^2(t) + (r_1+1)e_1(t)e_2(t) - (1+k_2)e_2^2(t) + (1-r_2)e_2(t)e_3(t) - k_3 e_3^2(t) \\
&\quad + (r_2 - r_3)e_3(t)e_4(t) - k_4 e_4^2(t) + (r_5 - r_3)e_4(t)e_5(t) - (k_5 + r_4 r_5)e_5^2(t) + W^T(t)E(t) \\
&\le -[r_1(1+M) - 4 + k_1]e_1^2(t) - [2e_1(t) - \frac{(r_1+1)e_2(t)}{4}]^2 - [-\frac{(r_1+1)^2}{16} - 3 + k_2]e_2^2(t) \\
&\quad - [2e_1(t) - \frac{(1-r_2)e_3(t)}{4}]^2 - [-\frac{(1-r_2)^2}{16} - 4 + k_3]e_3^2(t) - [2e_3(t) - \frac{(r_2-r_3)e_4(t)}{4}]^2 \\
&\quad - [-\frac{(r_2-r_3)^2}{16} - 4 + k_4]e_4^2(t) - [2e_4(t) - \frac{(r_5-r_3)e_5(t)}{4}]^2 \\
&\quad - [-\frac{(r_5-r_3)^2}{16} + r_4 r_5 + k_5]e_5^2(t) + W^T(t)E(t)
\end{aligned} \tag{13}$$

From the conditions (10), we have

$$\dot{V}(t) = -E^T(t)KE\ (t) + W^T(t)E(t) \tag{14}$$

Consider the following H_∞ performance index

$$\begin{aligned}
J &= \int_{t_0}^{t_T} [E^T(t)E(t) - \gamma^2 W^T(t)W(t)]dt \\
&= \int_{t_0}^{t_T} [E^T(t)E(t) - \gamma^2 W^T(t)W(t) + \dot{V}(t)]dt + V(t_0) - V(t_T)
\end{aligned} \tag{15}$$

For $V(t_0) = 0$ and $V(t_T) \ge 0$, we have

$$\begin{aligned}
J &\le \int_{t_0}^{t_T} [E^T(t)E(t) - \gamma^2 W^T(t)W(t) + \dot{V}(t)]dt \\
&= \int_{t_0}^{t_T} \eta^T(t)\Omega\eta(t)dt
\end{aligned} \tag{16}$$

where

$$\eta(t) = [E^T(t), W^T(t)]^T$$

$$\Omega = \begin{bmatrix} I - K & I \\ * & -\gamma^2 I \end{bmatrix}$$

Consider LMI (11), we have $J \le 0$ for any nonzero $W(t) \in L_2[t_0, \infty]$. According to Definition 1, the proof of Theorem 1 is thus completed. □

Next, we design a single-dimension H_∞ synchronization controller. Constructing the following master-slave systems

$$\begin{aligned}
\dot{x}_1(t) &= r_1(x_2(t) - x_1(t) - g(x_1(t))) \\
\dot{x}_2(t) &= x_1(t) - x_2(t) + x_3(t) \\
\dot{x}_3(t) &= -r_2[x_2(t) - x_4(t)] \\
\dot{x}_4(t) &= -r_3[x_3(t) + x_5(t)] \\
\dot{x}_5(t) &= r_4[x_4(t) - r_5 x_5(t)]
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
\dot{y}_1(t) &= r_1(y_2(t) - y_1(t) - g(y_1(t))) + w(t) + u(t) \\
\dot{y}_2(t) &= y_1(t) - y_2(t) + y_3(t) \\
\dot{y}_3(t) &= -r_2[y_2(t) - y_4(t)] \\
\dot{y}_4(t) &= -r_3[y_3(t) + y_5(t)] \\
\dot{y}_5(t) &= r_4[y_4(t) - r_5 y_5(t)]
\end{aligned} \tag{18}$$

where $x_i(t)$ and $y_i(t)$, $i = 1, 2, \cdots, 5$ are the state variables of the master and slave systems, respectively, $u(t)$ is the single-dimension synchronization controller, and $w(t)$ is the disturbance input.

Then we can get the following error dynamical system model

$$\begin{aligned}
\dot{e}_1(t) &= r_1(e_2(t) - e_1(t) - M(t)e_1(t)) + w(t) + u(t) \\
\dot{e}_2(t) &= e_1(t) - e_2(t) + e_3(t) \\
\dot{e}_3(t) &= -r_2[e_2(t) - e_4(t)] \\
\dot{e}_4(t) &= -r_3[e_3(t) + e_5(t)] \\
\dot{e}_5(t) &= r_4[e_4(t) - r_5 e_5(t)]
\end{aligned} \tag{19}$$

where $a \le M(t) \le b$.

Based on Lyapunov method and LMI technique, the following theoretical result can be concluded.

Theorem 2. If there exist scalars $\gamma > 0$, $n > 0$, and $\varepsilon > 0$, we design an H_∞ synchronization law with the following control regulation

$$u(t) = -L \cdot e_1(t) \tag{20}$$

with

$$L = r_1\left(\frac{1}{n} + \varepsilon - 1 - a\right) \tag{21}$$

$$\begin{bmatrix} 1-\varepsilon & 0 & 0 & \frac{1}{2r_1} \\ 0 & -1+n & 0 & 0 \\ 0 & 0 & -r_5 & 0 \\ \frac{1}{2r_1} & 0 & 0 & -\gamma^2 \end{bmatrix} < 0 \tag{22}$$

then, system (18) with any initial conditions can synchronize to system (17) with H_∞ norm bound γ.

Proof. Choose the following Lyapunov function candidate

$$V(t) = \frac{1}{2r_1} e_1^{\,2}(t) + \frac{1}{2} e_2^{\,2}(t) + \frac{1}{2r_2} e_3^{\,2}(t) + \frac{1}{2r_3} e_4^{\,2}(t) + \frac{1}{2r_4} e_5^{\,2}(t) \tag{23}$$

The time derivative of $V(t)$ along trajectories of error model (19) is given by

$$\begin{aligned} \dot V(t) &= \frac{1}{r_1} e_1(t)\dot e_1(t) + e_2(t)\dot e_2(t) + \frac{1}{r_2} e_3(t)\dot e_3(t) + \frac{1}{r_3} e_4(t)\dot e_4(t) + \frac{1}{r_4} e_5(t)\dot e_5(t) \\ &= e_1(t)[e_2(t) - e_1(t) - M(t)e_1(t) - \frac{L}{r_1} e_1(t) + \frac{1}{r_1} w(t)] \\ &\quad + e_2(t)[e_1(t) - e_2(t) + e_3(t)] \\ &\quad - e_3(t)[e_2(t) - e_4(t)] \\ &\quad - e_4(t)[e_3(t) + e_5(t)] \\ &\quad + e_5(t)[e_4(t) - r_5 e_5(t)] \\ &= -e_1^{\,2}(t)[1 + M(t) + \frac{L}{r_1}] - e_2^{\,2}(t) + 2e_1(t)e_2(t) - r_5 e_5^{\,2}(t) + \frac{1}{r_1} w(t)e_1(t) \\ &\le -e_1^{\,2}(t)[1 + M(t) + \frac{L}{r_1} - \frac{1}{n}] - (1-n)e_2^{\,2}(t) - r_5 e_5^{\,2}(t) + \frac{1}{r_1} w(t)e_1(t) \end{aligned} \tag{24}$$

Consider condition (21), we have

$$\dot V(t) \le -\bar E^T(t)\bar K \bar E\ (t) + \frac{1}{r_1} w(t)e_1(t) \tag{25}$$

where

$$\bar K = diag\{\varepsilon, 1-n, r_5\} \ge 0$$

$$\bar E = [e_1(t), e_2(t), e_5(t)]^T$$

Consider the following H_∞ performance index

$$J = \int_{t_0}^{t_T} [e_1^2(t) - \gamma^2 w^T(t) w(t)] dt$$
$$= \int_{t_0}^{t_T} [e_1^2(t) - \gamma^2 w^T(t) w(t) + \dot{V}(t)] dt + V(t_0) - V(t_T) \quad (26)$$

For $V(t_0) = 0$ and $V(t_T) \geq 0$, we have

$$J \leq \int_{t_0}^{t_T} [e_1^2(t) - \gamma^2 w^T(t) w(t) + \dot{V}(t)] dt$$
$$= \int_{t_0}^{t_T} \eta^T(t) \Omega \eta(t) dt \quad (27)$$

where

$$\eta(t) = [\bar{E}^T(t), w^T(t)]^T$$

$$\Omega = \begin{bmatrix} 1-\varepsilon & 0 & 0 & \frac{1}{2r_1} \\ 0 & -1+n & 0 & 0 \\ 0 & 0 & -r_5 & 0 \\ \frac{1}{2r_1} & 0 & 0 & -\gamma^2 \end{bmatrix}$$

Consider LMI (22), we have $J \leq 0$ for any nonzero $W(t) \in L_2[t_0, \infty]$. According to Definition 1, the proof of theorem 2 is thus completed. □

Remark 1. $r_i, i = 1, 2, \cdots, 5$ are the system parameters of the hyper-chaotic system. From the proof of Theorem 2 we can see, through constructing Lyapunov function (23) with r_i based on the characteristic of the hyper-chaotic system, chaos synchronization is achieved via a single-dimensional controller $u(t) = -L \cdot e_1(t)$, which is only suitable for this system.

4. Example and simulation

In this section, we include some examples to validate the effectiveness of our chaotic synchronization method. The numerical simulation is with step size 0.001 second and the following initial parameters

$$x_0 = [0.7557 \quad 0.0957 \quad -0.2801 \quad 0.0164 \quad 0.1498]^T$$

$$y_0 = [1.1306 \quad -0.0411 \quad -0.1034 \quad -0.0354 \quad -0.4042]^T$$

$$\gamma = 0.2$$

$$w_i(t) = \begin{cases} 2\cos(2t)\sin(\frac{e^t}{t+1}) & \text{t} \geq \text{20 s} \\ 0 & \text{else} \end{cases}$$

First, we consider the multi-dimension synchronization law in (9). Based on Theorem 1, we get $K = \mathrm{diag}\{8.7028, 12.5653, 16.5653, 17.9053, 25.9053\}$. The numerical simulation result can be seen in Fig 3.

Next, we consider the single-dimension synchronization law in (20). Based on Theorem 2, we get $L = 14.7076$. The numerical simulation result can be seen in Fig. 4.

Remark 2. Fig. 1 depicts the time response of system disturbance input. Fig. 2 depicts the attractor of the hyper-chaotic system. It can be seen the hyper-chaotic system (1) possesses complicated dynamics. Figs. 3 and 4 depict the time response of synchronization error variables of master-slave systems based on our H_∞ synchronization laws. At the first stage there is no disturbance input, we can see the slave system achieves following the master system during 10 seconds. Later the disturbance is added at the 20*th* second, we can see the synchronization error variables jitter in a small range, which satisfy the given H_∞ performance index. Compared to multi-dimensional H_∞ synchronization controller, the single-dimensional H_∞ synchronization controller has the simpler structure due to the requirement only for one system state variable. In numerical simulation, we notice that the multi-dimensional H_∞ synchronization controller has the better control performance at the cost of complicated structure with much more system state information, which may cause the generating of the new disturbances and will make a difference to the synchronization control. Therefore, both synchronization methods are meaningful and can be utilized in accordance with the practical requirements.

5. Conclusion

In this paper, we have studied the robust control problems for the synchronization of hyper-chaotic systems with disturbance input. Based on Lyapunov method, we have designed the multi-dimensional and the single-dimensional H_∞ robust synchronization controllers in terms of linear matrix inequalities for the application in practical engineering. Corresponding theoretical derivations were also presented. Finally, some numerical simulations were carried out to illustrate the effectiveness of the proposed techniques.

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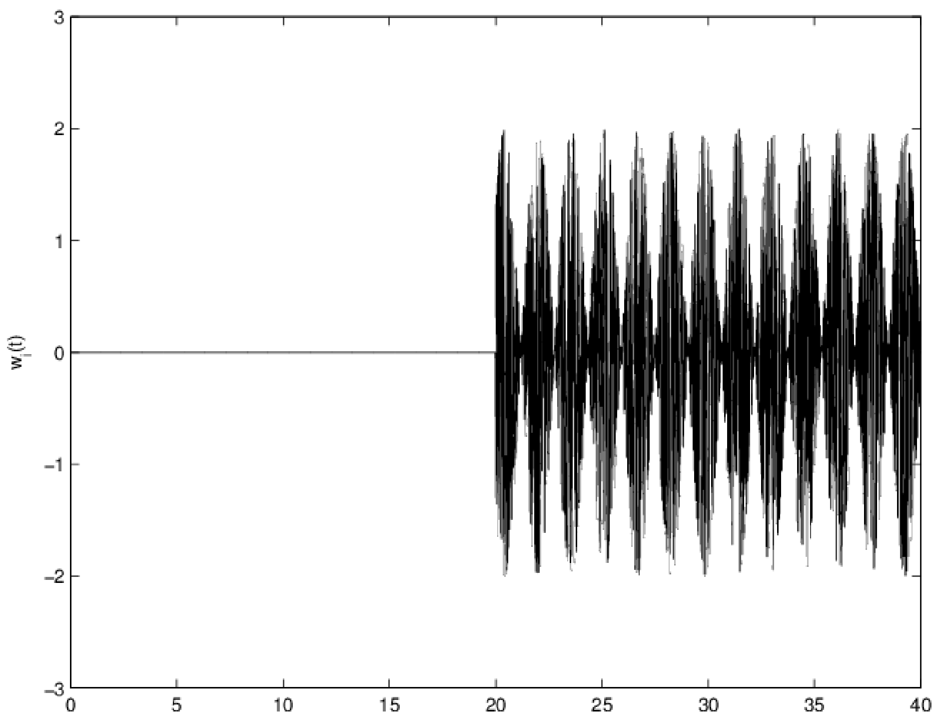

Fig. 1 Time response of system disturbance input

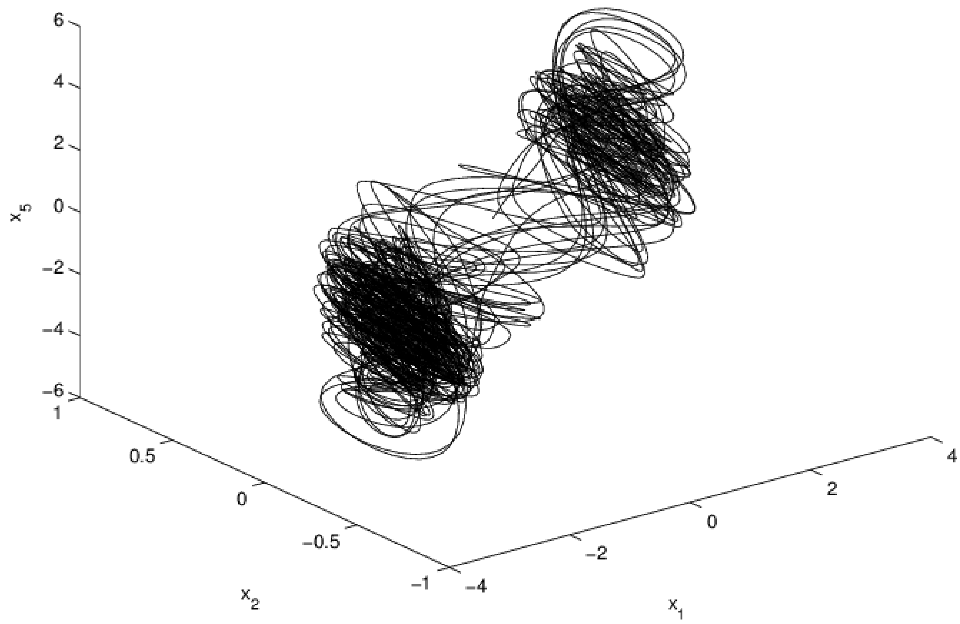

Fig. 2 Attractor of the hyper-chaotic system (1)

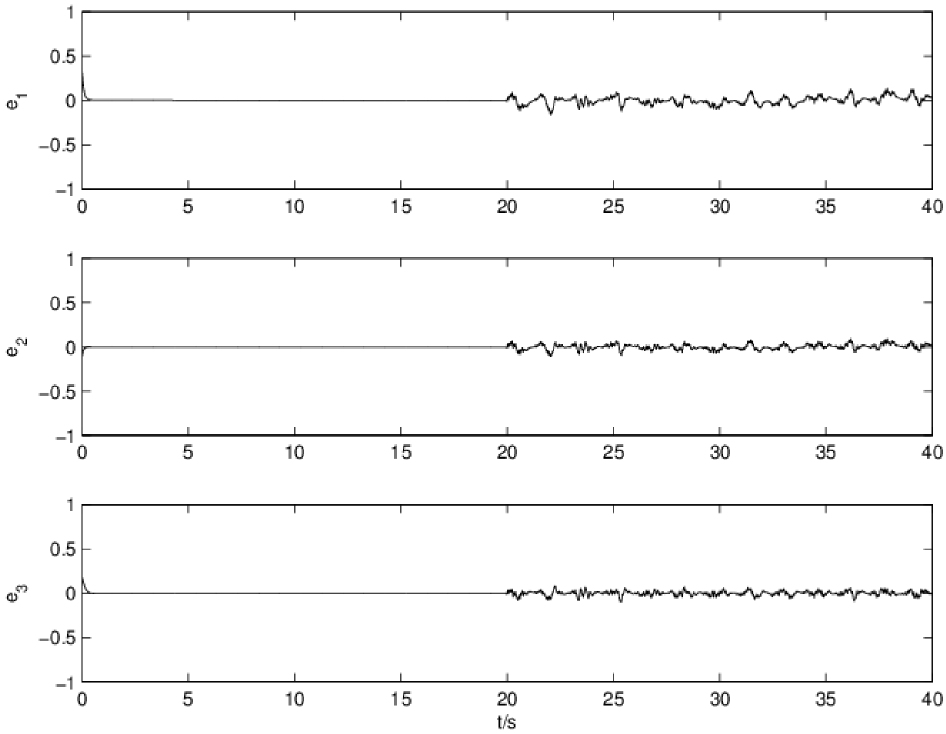

Fig. 3 Time response of synchronization error variables with multi-dimensional H_∞ controller

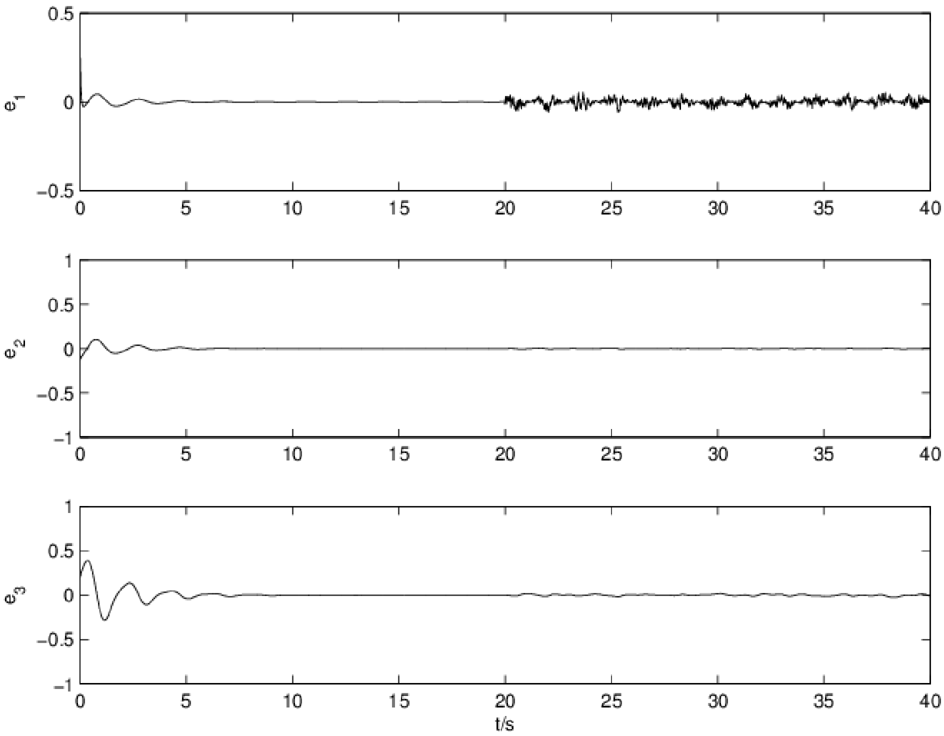

Fig. 4 Time response of synchronization error variables with single-dimensional H_∞ controller