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For Feedback Control

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A TIME SERIES CONTROLLER ALGORITHM FOR FEEDBACK CONTROL

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ABSTRACT

Statisticians and control engineers, often taking different perspectives, have long practised the ‘art’ of process control. In recent years, the focus of attention has been on bringing the statistical process control (SPC) and automatic process control (APC) methodologies closer together. Some of the issues of automatic process control, in particular, of continuous processes, in the existence of dead time (time delay), such as stability of the closed loop, (feedback control path), are discussed in this paper. Derivations for the general formulae for making an input adjustment that would yield the smallest possible mean square error (variance) of the output controlled variable are available in the literature. However, a method to derive a time series (stochastic) feedback control algorithm for making an adjustment in the input manipulated variable that would yield minimum variance at the output is described by considering a concrete example of a the critically damped second-order dynamic system with delay (dead time). One of the main aims of this paper is to show that such a stochastic feedback control algorithm has the benefits of both integral control and dead-time compensation while providing minimum variance control.

1 INTRODUCTION

The manufacture of products of a desired quality are realised by monitoring and controlling production and by adjusting a process variable or variables in order to maintain a performance criterion in some desirable neighbourhood of a given target value, (called ‘set points’). This control action is possible by an appropriate *feedback* or *feedforward* control procedure which indicates when and by how much to ‘adjust’ the process automatically, by either using past output deviations from target (or ‘error’, that is, the difference between the output and the target values), or by cancelling these deviations by using knowledge of the value of some fluctuating measured input variable respectively or by a combination of both the feedforward and feedback control

procedures. Various forms of feedback and feedforward control regulation schemes are used in automatic process control (APC) for process adjustment (control action) to make the required change in the input in order to compensate for the output deviation. An approach to control is to *forecast* the output deviation from target which would occur if no action were taken and then act so as to cancel out this deviation. If no compensatory adjustments are made by taking proper control action, the process drifts off target and the output follows the course of the 'disturbance' (noise).

The term 'controlled process' is often used to mean a 'process state' that is (narrowly) interpreted as *stationary* having *iid variation about a target value*. An alternative has a 'state of control' as *a process state in which future behaviour can be predicted within probability limits determined by a common-cause system* (Box and Kramer [1992]).

If a state of statistical control is identified by a process generating independent and identically distributed (iid) random variables, control of such random processes by automatic means invariably leads to an undesirable increase in process variability. So, APC must be properly applied to obtain successful results.

APC, also referred to as 'engineering feedback control', is an approach to minimise the (unexpected) (output) variation (due to the presence of non-random causes), by transferring the predictable component of the output variation to the input manipulated variable [MacGregor] (Box and Kramer [1992]). The appropriate engineering control strategy depends upon (i) the *characteristics of the stochastic (statistical) component of the process modelled by a suitable time series* and (ii) the *costs associated with making dynamic adjustments*.

The general purpose of automatic control is to get satisfactory process operation by adjustment of a controller (control mechanism). By using a logical method for selecting controller adjustments and by suitable tuning, there is the potential for efficient process operation. It is possible to formulate a control mechanism by suitably modelling a process that is non-stationary but probabilistically predictable (Box and Jenkins [1970]).

2 STOCHASTIC MODELS

A disturbance causes a process to drift off target, and so it is necessary to compensate this by taking proper control action. Disturbances, entering at various points

of a process, can be envisaged as the result of a sequence of independent random shocks, and represented by (linear) differential equations. Similarly, linear difference equations represent discrete processes in which the sampling intervals are short enough to account for the dynamic or inertial properties of a process.

The class of non-stationary stochastic time series models, called Autoregressive Integrated Moving Average (ARIMA) models, are used to describe and model certain disturbances that regularly occur in industrial processes and the process dynamics (inertia). The models are used also to design feedforward and feedback control schemes.

3 CONTROLLERS

In automatic controllers, the input control actions (adjustments) help bring the process into a state of control by means of discrete data obtained through measurements. With the process data available, it is possible to control the mean square error, (sum of the squared deviations between an output and target value), about the target by (proportional -integral) feedback control schemes.

PID, (three-term) controllers, are continuous time controllers. It is believed that these controllers (i) are not capable of providing (tight) control over processes in which the effect of an adjustment is delayed until the following sample due to time taken to deliver material from the point of adjustment to the sample point (called, the 'dead time'), (ii) tend to perform poorly unless 'detuned' in the face of dead times in order to take necessary action at each sampling instant (Harris, MacGregor & Wright [1982]), and (iii) are not (ideally) suited for direct-digital (discrete) control. In certain situations, the capability of the PID controllers may be limited to produce control actions that might be called for by a minimum variance feedback controller. Time series controllers employ stochastic characteristics to regulate production processes. Time series controllers provide tight control of processes with dead time and minimum variance at the same time.

4 TIME SERIES CONTROLLERS-CHARACTERISTICS AND FEATURES

Time series controllers are used in the process industries for regulating quality variables measured at discrete time intervals. Their 'stochastic feedback control algorithms' are used to calculate a series of adjustments which compensate the

disturbances. ARIMA models forecast the drifting behaviour of the disturbances and the stochastic feedback control algorithm or equation derived from the ARIMA models calculates a series of adjustments that compensate for the disturbances by making an adjustment at every sample point.

5 MINIMUM VARIANCE CONTROL

A feedback control strategy to minimise the mean square of the output deviation (error) from target is by minimum mean square error or variance control. It is possible that minimum variance control can be achieved in processes described by linear functions with disturbances which are added together and treated as a single disturbance for purposes of mathematical analysis and convenience. Though its implementation may demand aggressive control much in excess of what is (normally) required, minimum variance control provides a convenient bound on achievable performance against which the performance of other controllers can be compared. Such a basis is important in the context of deciding corrective control actions.

Time series controllers are capable of giving minimum control (error) variance even when there are dynamics (inertia) and dead time (delay) in a process. It may be possible to restrict sampling a process until an acceptable minimum of control error variance can be achieved by making use of the time series controller's minimum variance control property and to minimise monitoring and adjustment costs.

6 FEEDBACK CONTROL DIFFERENCE EQUATION

Notation

The 'stochastic feedback control (linear) difference equation' for a first-order single-input, single-output (SISO) dynamic (process) (control) system with b units of delay can be (approximately) represented by

$$Y_t(1-\delta B) = g(1-\delta)X_{t-b} = g(1-\delta)B^b X_t, \quad 0 \leq \delta < 1 \quad (1)$$

where

X_{t+} is the setting of the input manipulative variable (Linear function of e_t and of integral over time of past errors, e_t),

e_t is the forecast error,

a_t is the random shocks NID $(0, \sigma_a^2)$,

Z_t is the disturbance,

Y_t is the output or controlled variable and

$$e_t = Z_t + Y_t,$$

g is the (steady-state) gain denoting the ratio of change in the steady-state process output to the change in the input which caused it (Deshpande and Ash [1981]), (Shinsky [1988]),

δ represents the (dynamics) inertial capacity of a process to recover back to its (equilibrium) conditions after an adjustment is made to the process and due to which the adjustments do not have an immediate effect on the process and

$$B^b X_t = X_{t-b}.$$

The block diagram for the feedback control model is shown in Figure 1.

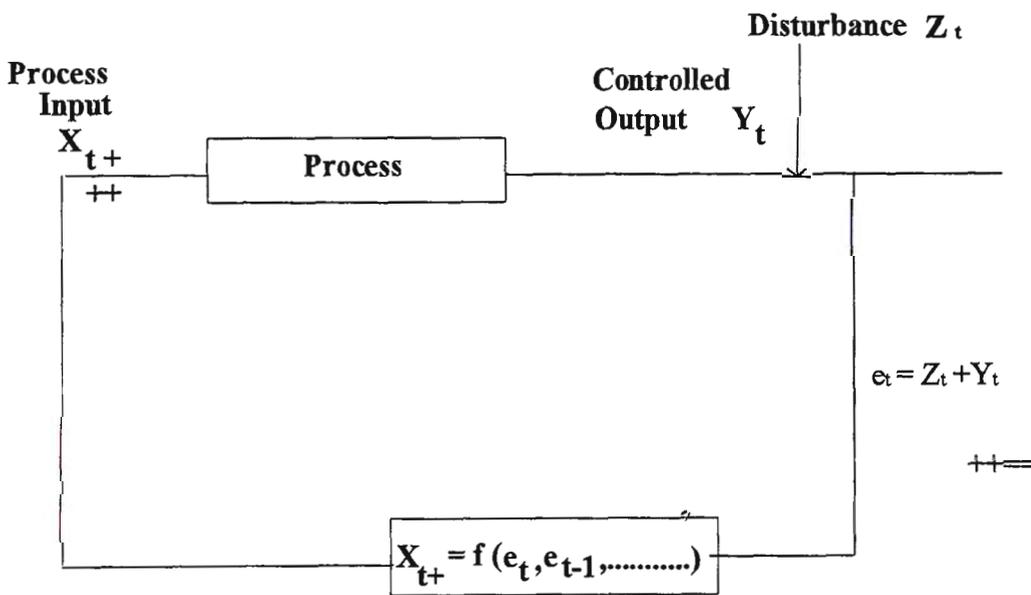


Figure 1 Block Diagram for the Feedback Control Model

In the feedback control model shown in Figure 1, the process is regulated by manipulating the input variable X_{t+} which in turn affects the controlled output Y_t . The plus sign on the subscript of X_{t+} implies that the adjustment is made during the interval between t and $t+1$. The definite deterministic relationship existing between the process input X_t and its output Y_t described by Equation (1) does not exhibit stochastic characteristics. Z_t , a non-stationary disturbance, is the output of the (linear) system, when subjected to a sequence of uncorrelated random shocks $\{a_t\}$ where $a_t \sim N(0, \sigma_a^2)$.

For this feedback control first-order difference equation for the dynamic model that can approximate the behaviour of a number of processes, the output change asymptotically approaches 'g' for a unit change in the input.

7 JUSTIFICATION FOR CONSIDERING SECOND-ORDER MODELS

For feedback control (closed-loop) stability, the dynamic (inertial) parameter δ must satisfy the condition that $0 < \delta < 1$ for the discrete dynamic model of the process and the gain should be less than or equal to 1.0 (Shinskey [1988]).

The first-order dynamic model characterised by the linear difference equation (1) can be written as

$$Y_t = \delta Y_{t-1} + g(1-\delta)X_{t-b} \quad 0 \leq \delta < 1 \quad (2)$$

The MMSE (minimum mean square error) or minimum variance control schemes based on the first-order dynamic model and the ARIMA (0,1,1) disturbance model produce the minimum mean square error (MMSE) at the output requiring excessive control action in the following situations in which (i) the values of δ are not fairly small; and in (ii) as δ becomes larger and in particular, as it approaches unity (Box and Kramer [1992]). As δ becomes larger, the minimum variance control exhibits large 'alternating' character in the required adjustments to give minimum output variance (Box and Kramer [1992]).

For larger values of δ , the general recourse is to go in for constrained variance control schemes. In such control schemes, reduced control action is achieved at a cost of small increases in the mean squared error at the output by placing a constraint on the input manipulated variable.

Processes found in practice are often complex because of their dynamic characteristics which change with time. Approximating such processes by first-order dynamic models does not always seem to be satisfactory. It can be shown from the simulation study results of the time series controller for a first-order (plus dead time) dynamic model and the ARIMA (0,1,1) model that for drifting processes, for values of δ from 0 to 1, the required adjustments are of alternating character and sometimes with huge increases in control error standard deviation and its adjustment variance. It is likely

that some processes may have more than one dynamic element and the exact mathematical model relating the output and the input could be greater than the first-order. Many complicated dynamic systems can be fairly closely approximated by second-order systems with delay (dead time). So, it is conjectured that the dynamic system will be better described by a second-order system that is represented by a dynamic model of the order (2,1), ('a discretely coincident' continuous system, Box and Jenkins [1976]).

Some commonly used identification techniques are maximum likelihood or prediction error methods and the recursive least squares method. The recursive least squares method and its extensions require that the value of the dead time (time delay) of the process must be known in order to account for the delay. Palmor and Shinnar [1979], though were of the view that the first-order model with dead time is sufficient for representing some of the most commonly occurring simple processes, cautioned that complex (process) model that helps to identify processes is (also) not (very) accurate. They contended that there had been several claims in the process industries to second-order models coupled with delays as sufficient for most purposes. In particular, the second-order dynamic model (transfer function) of the form with polynomials for $\omega(B)$ and $\delta(B)$ described by

$$Y_t = [(\omega_0 - \omega_1 B) / (1 - \delta_1 B - \delta_2 B^2)] B^{b+1} X_t$$

is probably the highest order that can be justified.

It can be shown that for a given stable (transfer function) dynamic model, the higher terms in the polynomials decrease exponentially with increasing sampling interval (T). The transfer function, $[(\omega_0 - \omega_1 B) / (1 - \delta_1 B - \delta_2 B^2)]$ depends on the sampling interval (T) and also on the tuning of the process controller and therefore it can be adjusted and can approximate any (linear) response at fixed time intervals. However, the polynomial in

the numerator can be extended further if there is a large variation in the assumed 'a priori' dead time value.

It is possible to approximate the behaviour of high-order processes by a system having one or two time constants and a dead time. When one or two time constants dominate (i.e. are much larger than the other time constants in the system), all the smaller time constants ('work together') add up to produce a lag that almost resembles (pure) dead time. It is possible also to approximate the actual input-output mathematical model of a high-order, complex dynamic process with a simplified model consisting of a second-order process combined with a dead-time element. The second-order model will reduce to the first-order model if one of the two time constants of the former model is smaller than the other (Deshpande and Ash [1981]).

Box and Jenkins [1963] suggested that many dynamic models could be adequately represented by, at most, two exponential stages with variable gains and delay governed by parameters δ_1 , δ_2 , g and dead time b . Box and Jenkins [1976] mentioned also in their monograph that complex processes with dead time (delay) can reputedly be closely approximated by second-order systems having more than one time constant, (usually, two). This view is shared also by MacGregor [1988]. A second-order process with dead time is a useful model for some complex processes which are fairly common (Shinsky [1988]).

Some methodologies were suggested by MacGregor [1988], Box and Kramer [1992] to use statistical process control charts to monitor the performance of closed-loop controlled systems. Such methodologies, though taking care of on-line process control and monitoring, do not always seem to deal with stability problems of the feedback control loop in an explicit and simple manner. Consider a concrete example of

controlling a second-order dynamic system by a second-order dynamic model of the form:-

$$Y_t(1 - \delta_1 B - \delta_2 B^2) = (\omega_0 - \omega_1 B) B^{b+1} X_t \quad (3)$$

For stability reasons, interest is focused on the 'critically damped' behaviour of the second-order dynamic system (for which the time constants τ_1 and τ_2 are real and equal) and not on the behaviour of the system when it is either 'overdamped' or 'underdamped' for which τ_1 and τ_2 may be complex.

The second-order dynamic system can then be thought of as equivalent to two discrete first-order systems arranged in series.

The second-order model will be critically damped, when the roots of the second-order model described by Equation (3) are real and equal, that is, when

$$\delta_1^2 + 4\delta_2 = 0.$$

Stability is achieved when the point (δ_1, δ_2) lies in a triangular region defined by the conditions $\delta_2 - \delta_1 = 1$, $\delta_1 + \delta_2 = 1$ and $\delta_2 < 1$. This is shown in Figure 2.

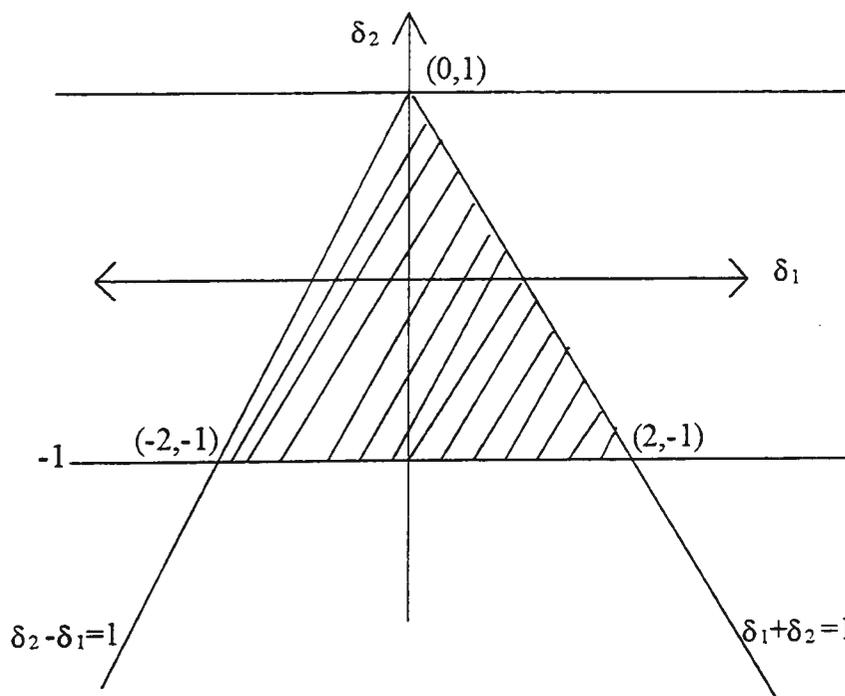


Figure 2 Triangular region defined by the inequality conditions for achieving stability.

Equation (3) is rewritten as

$$(1 - \delta_1 B - \delta_2 B^2) Y_t = \omega B^{b+1} X_t \quad (4)$$

where $(\omega_0 - \omega_1) \approx \omega$, for mathematical convenience in dealing with a single term, being, the magnitude of the process response to a unit step change in the first period following the dead time carrying over into additional sample periods. This is possible since only whole periods of dead time (delay) are being only considered.

Moreover, equation (4) reduces to that of Baxley's [1991] first-order dynamic model, namely,

$$Y_t = \delta Y_{t-1} + \omega B^{b+1} X_t$$

that describes the first-order system with dead time (delay) when $\delta_2 = 0$ and $\delta_1 = \delta$.

The steady-state gain, 'g', of such a second-order discrete dynamic model is given by

$$g = (\omega_0 - \omega_1)/(1 - \delta_1 - \delta_2). \quad (\text{Box and Jenkins [1970]})$$

It can be shown for a critically damped system that

$$\omega = \omega_0 - \omega_1 = PG[1 - \delta_1 - \delta_2]$$

where PG represents the process gain, realised by the total effect in output caused by a unit change in the input variable after the completion of the dynamic response (Baxley [1991]).

Therefore, the steady-state or system gain

$$g = (\omega_0 - \omega_1)/(1 - \delta_1 - \delta_2) = PG[1 - \delta_1 - \delta_2]/[1 - \delta_1 - \delta_2] = PG, \text{ the process gain.}$$

Baxley [1991] used $PG = 1/1 - \delta$ and made $PG = 1.0$ by setting $\delta = 0$, meaning that there are no carry-over effects (inertia) and seems to have tackled the problem of feedback control stability in a convincing manner in his simulation studies for drifting processes. Kramer [1990], derived expressions for the disturbance and the output effect of control actions as functions of random shocks, *independent* of the *control scheme*. Moreover, Kramer [1990] considered approaches for reducing adjustment variability. Since, interest here is in minimising control error standard deviation, (product variability) at the output by using the algorithm derived in the next Section, it may be worthwhile to consider the critically damped behaviour of the second-order dynamic system for which the time constants are real and equal thus ensuring closed-loop stability. Furthermore, it is shown that the steady state gain of such critically damped second-order systems is PG, the process gain itself.

An additional term in the parameter δ , (δ_2) of the second-order dynamic model makes it possible to account for more of the process dynamics for both small and large values of δ and to better represent the dynamic nature of the process. The additional term Y_{t-2} defines the input-output relationship in a better manner than the first-order dynamic model.

For stability of the second-order dynamic model, the parameters δ_1 and δ_2 must satisfy the following inequality conditions given by

$$\delta_1 + \delta_2 < 1$$

$$\delta_2 - \delta_1 < 1$$

$$-1 < \delta_2 < 1$$

The roots of the characteristic equation $(1 - \delta_1 B - \delta_2 B^2) = 0$ determine the stability of the second-order dynamic system. When these roots are real and positive, the step response, which is the sum of two exponential terms, approaches its asymptote g , the steady-state gain, without crossing it. It can be seen from Figure 3, (Reproduced from figure 10.5, page 344, Box and Jenkins [1970]), that the system has no overshoot when the characteristic equation has real positive roots. This explains the focus on the critically damped second-order discrete dynamic model which ensures closed-loop (feedback control) stability.

As shown in Figures 2 and 3, the values of δ_1 and δ_2 should satisfy

$$-2 < \delta_1 < 2$$

$$-1 < \delta_2 < 1.$$

8 EXPRESSION FOR THE CONTROL ADJUSTMENT IN THE INPUT VARIABLE OF A TIME SERIES CONTROLLER

An expression is derived for the *feedback control adjustment* required in the input manipulated variable of a time series controller for a dynamic process with dead time (delay). Figure 4 shows the feedback control scheme to compensate a disturbance Z_t by means of a time series controller. Baxley [1991] considered the dead time equal to one period when deriving the feedback control equation. In this Section, the feedback control (adjustment) algorithm is derived considering b periods of dead time.

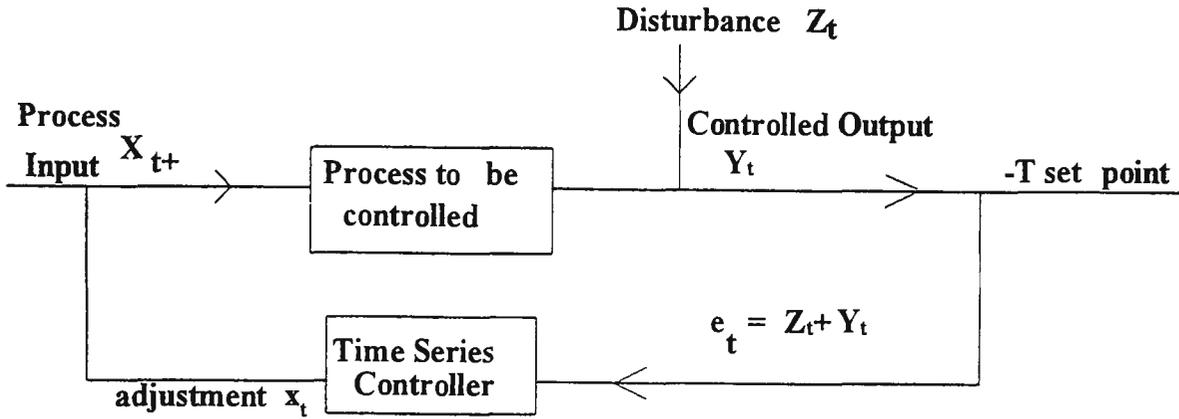


Figure 4 Feedback Control Scheme to compensate disturbance Z_t by a Time Series Controller in the existence of dynamics and dead time

From Equation (4),

$$(1 - \delta_1 B - \delta_2 B^2) Y_t = \omega B^{b+1} X_t.$$

Changes are made in the input X at times $t, t-1, t-2, \dots$, immediately after observing the disturbances $z_t, z_{t-1}, z_{t-2}, \dots$. Because of this, a pulsed input results and the level of X in the interval t to $t+1$ is denoted by X_{t+} .

For this pulsed input, assume that the dynamic model which connects the input manipulated variable X_{t+} and the controlled output Y_t is

$$Y_t = L_1^{-1}(B) L_2(B) B^{b+1} X_{t+},$$

where,

$L_1(B)$ is a polynomial in B of degree r ,

$L_2(B)$ is a polynomial in B of degree s and

b is the number of complete intervals of pure delay before an adjustment in the input X_{t+} begins to affect the output Y_t .

The non-stationary disturbance is represented by the ARIMA (0,1,1) model

$$\nabla Z_t = (1 - \Theta B) a_t.$$

Z_t measures the effect at the output of an unobserved disturbance, that is, an uncompensated non-stationary disturbance that reaches the output before it is possible for the compensating control action to become effective, this causes the process to wander off target. It is defined as the deviation from the target that would occur if no control action was taken.

It can be shown that for a time series controller, when the disturbance is described by the ARIMA (0,1,1) model and there are definite carry over (inertial) effects, the adjustment (x_t) in the input manipulated variable required to make the control and forecast error variances equal, is given by

$$X_{t+} = -\{L_1(B)L_3(B)/(L_2(B)L_4(B))\}\varepsilon_t. \quad (\text{Box and Jenkins [1976]})$$

where the error observed at time t,

$$\varepsilon_t = e_{t-b-1}^{(b+1)}$$

and $L_3(B)$ and $L_4(B)$ are operators in B.

The expressions for $L_1(B)$, $L_2(B)$, $L_3(B)$ and $L_4(B)$ can be evaluated as

$$L_1(B) = (1 - \delta_1 B - \delta_2 B^2),$$

$$L_2(B) = PG(1 - \delta_1 - \delta_2)$$

$$L_3(B) = (1 - \Theta)/(1 - B) \text{ and}$$

$$L_4(B) = 1 + (1 - \Theta)B.$$

The control action in terms of the adjustment $x_t = x_{t+} - x_{t-1+}$ to be made at time t is

$$x_t = \frac{-L_1(B)L_3(B)(1-B)}{L_2(B)L_4(B)}\varepsilon_t. \quad (\text{page 435, Box and Jenkins [1976]})$$

This 'feedback control equation defines the adjustment to be made to the process at time t which would produce the feedback control action yielding the smallest possible mean square error since it exactly compensates the predicted deviation from target' (Box and Jenkins [1968]).

The above equation, on substituting the expressions for $L_1(B)$, $L_2(B)$, $L_3(B)$ and $L_4(B)$, results in

$$x_t = -\frac{(1 - \delta_1 B - \delta_2 B^2)(1 - \Theta)}{PG(1 - \delta_1 - \delta_2)(1 + (1 - \Theta)B)}\varepsilon_t. \quad (5)$$

where Θ is the *moving average* (operator) parameter.

The control errors which turn out to be the one-step ahead forecast errors are measured in practice.

It is known that the error ε_t at the output at time t is the forecast error at lead time b+1 for the Z_t disturbance.

So,

$$\varepsilon_t = e_{t-b-1}^{(b+1)} = \psi_0 a_t + \psi_1 a_{t-1}. \quad (6)$$

For the ARIMA (0, 1, 1) model, the weights are $\psi_0 = 1$ and $\psi_1 = 1 - \Theta$, so

$$\begin{aligned}\varepsilon_t &= a_t + (1 - \Theta) a_{t-1} \\ &= (1 + (1 - \Theta)B) a_t\end{aligned}$$

and further,

$$x_t = -\frac{(1 - \delta_1 B - \delta_2 B^2)(1 - \Theta)}{PG(1 - \delta_1 - \delta_2)(1 + (1 - \Theta)B)} (1 + (1 - \Theta)B) a_t.$$

Since $(1 - \Theta) \times 100$ per cent of the control error will affect the future process behaviour as per the disturbance model, for a dead time b ,

$$\begin{aligned}e_t &= a_t + (1 - \Theta)a_{t-b} \\ &= a_t[1 + (1 - \Theta)B^b]\end{aligned}$$

and so

$$a_t = e_t / [1 + (1 - \Theta)B^b]. \quad (7)$$

Therefore, the control adjustment equation for b periods of dead time is

$$x_t = -\frac{(1 - \delta_1 B - \delta_2 B^2)(1 - \Theta)}{PG(1 - \delta_1 - \delta_2)} \times \frac{e_t}{(1 + (1 - \Theta)B^b)}.$$

That is,

$$x_t + (1 - \Theta)x_{t-b} = -\frac{(1 - \delta_1 B - \delta_2 B^2)(1 - \Theta)}{PG(1 - \delta_1 - \delta_2)} e_t$$

giving

$$x_t = -\frac{(e_t - \delta_1 e_{t-1} - \delta_2 e_{t-2})(1 - \Theta)}{PG(1 - \delta_1 - \delta_2)} - (1 - \Theta)x_{t-b}. \quad (8)$$

The control adjustment action given by (8) minimises the variance of the output controlled variable.

Equation (8) is in conformance with the feedback control action adjustment equation of Kramer [1990] for achieving minimum variance or mean square control when $b = 0$. The control adjustment action is made up of the current deviation (e_t) and the past adjustment action x_{t-b} (Kramer [1990]). It is observed also that this is similar to the feedback control action adjustment equation for one period of dead time derived by Baxley [1991] on taking a value 1 for b , the dead time and when there are no carry-over effects for a time series controller. On comparison with the equation of Baxley [1991], it is found that the first term in Equation 8 gives the integral action and the second term, the dead-time compensator, developed by Smith [1959] (Baxley [1991]).

Some simulation results of Equation 8 obtained when $b = 1$, given below, match closely with that of Baxley's [1991] values for the time series controller.

Comparison of Time Series Controller Performance with One Dead time Period

<u>theta</u>	<u>SE (Equation 8)</u>	<u>SE (Baxley)</u>
0.25	1.260	1.250
0.50	1.112	1.118
0.75	1.010	1.031

9 BENEFITS OF THE INTEGRAL AND DEAD-TIME COMPENSATION TERMS IN EQUATION (8)

It has been pointed out that Equation (8) contains integral action. The control obtained through the first term in Equation (8) is the discrete analog of 'integral control', a form of adjustment employed in engineering or automatic process control. So, a minimum variance controller built on this algorithm will be able to maintain the desired output variable at or near a given set point (Palmor and Shinnor [1979]), thereby fulfilling some of the criteria specified as the requirements of a sampled-data (discrete) controller algorithm by Palmor and Shinnar [1979] in their paper.

The controller algorithm, Equation (8) meets with some of the requirements of a (discrete) sampled-data digital controller algorithm such as

- (i) *It is able to maintain the desired output variable at a given set point,*
- (ii) *The algorithm leads to stable overall control and converge fast on the desired steady state, thus ensuring asymptotic stability and provides satisfactory performance for different types of disturbances that could arise,*
- (iii) *A time series (integral) controller can be designed on the basis of Equation (8) with a minimum of information with respect to the nature of the inputs and the structure of the system.*
- (iv) *Such an (integral) controller will be reasonably insensitive to changes in system limits and will be stable and perform well over a reasonable range of system parameters.*

(Palmor and Shinnar [1979])

The integrating control action is successful in eliminating 'offset' (deviation obtained with proportional control) at the expense of reduced speed of response and increasing the period of the feedback control loop with its phase lag. When integral time is too long, the feedback loop is overdamped, leading to unstable conditions. Overshoot can be avoided by setting the integral time higher than that required for (load) regulation and can also be minimized by limiting the rate of set-point changes (Shinskey [1988]).

During a step change of the input process variable, the closed-loop output is serially independent when pure one-step minimum mean square control is used which is often the practice in process control operations.

A process variable that has a uniform 'cycle' and sustained oscillations does not threaten the stability of the feedback control system. Under integral control, the (feedback) closed-loop oscillates with uniform amplitude. The feedback loop tends to oscillate at the period where the system gain is unity, the integral (also known as reset) time, that is, the time constant (I) of the controller, then, affects only the period of oscillation which increases with damping for a controller with integral action in a dead-time loop. It can be shown that the 'integral time' (I_v) for zero damping (which requires a loop gain of 1.0), is $0.64PG T_d$ where PG is the process gain and T_d , the dead time (Shinskey [1988]). A process with a dead time of 1 minute would cycle with a period of 4 minutes under integral control with oscillations sustained by an integral time constant of about $0.64PG$ minutes. Control engineers endeavour constantly to have this integral time as nearly equal to the process dead time as possible so that the process variable can take the same path as the dead time. It is possible to achieve damping by reducing the closed-loop (feedback) gain and by increasing the integral time as will be shown next.

The integral action term in the control adjustment equation (8), determines the damping of the feedback control loop. The period and the integral time required for damping are established by the process characteristics. Minimizing the integral error ensures damping of the feedback control loop. The minimum integral (absolute) error, (IAE), for (integral) control of dead time is achieved by setting the integral time constant of the feedback control loop, $I = 1.6PGT_d$, where T_d is the time delay (dead time) (Shinskey [1988]). Under integral (floating) control, the feedback control loop tends to oscillate with uniform amplitude and at the period where, the (pure) steady-state gain is unity. For an (integral) time constant equal to the dead time, the period of oscillation

changes to about half of the original period. Increases in the (integral) time constant contribute more (phase) lag to the feedback control loop and extends the period of oscillation of the feedback control loop. There must be a rise in integral time constant so that the controller can then contribute less (phase) lag.

Palmor and Shinnar [1979] observed that 'the Smith predictor is a direct result of minimal variance strategy and that minimal variance control for processes having dead times includes this type of dead-time compensation'. At this stage, an intuitive conjecture, using this principle, is made that the inclusion of the dead-time compensation term of either the Smith predictor type and/or the Dahlin's (on-line) tuning parameter, (whose values range from 0 to 1), in a feedback control algorithm will also result in a minimum variance strategy for processes with dead time. This draws on the comparison made by Harris, MacGregor and Wright [1982] to the minimum variance controller they derived for the process with dead time (for which the number of whole periods of delay was equal to 2) and the Dahlin controller given in their paper. The authors showed that the two controllers were identical upon setting the value of Dahlin's parameter, (the discrete time constant of the closed-loop process), equal to Θ , the IMA parameter in the stochastic disturbance model. They reconciled the different approaches by noting that the 'IMA parameter Θ provides information about the magnitude of the closed-loop time constant'. Equation (8) is identical to Baxley's [1991] algorithm and includes both integral action and dead-time compensation terms and the IMA parameter Θ , (whose values range from 0 to 1), is set to take care of the drift ($r = 1 - \Theta$) as well as Dahlin's parameter to compensate for the dead time. So, it follows that *the variance of the output product variable achieved by using Equation (8) with integral action and dead-time compensation terms is a minimum. The dead-time compensation term (seemingly) removes the delay from stability considerations and definitely provides a stabilising effect on the feedback control system. These principles are used for designing (formulating) the discrete (sampled data) time series (integral) controller.* Such a controller will maintain the mean of the process quality variable at or near target and will allow for a (rapid) response to process disturbances without much overcompensation or overcorrection.

The performance of the computer-controlled algorithm depends not only on the tuning constants but also on the sampling period (adjustment interval). It is reported in

control engineering literature that although a second-order (conventional) control system is stable for all values of the (controller) gain, it is likely that computer control of the same system can give unstable response for some specific combinations of the gain and the sampling period. This is one of the main reasons to consider the critically damped behaviour of the second-order model.

10 CONCLUSION

In this paper, a brief discussion of minimum variance control and time series controllers has been provided. A general feedback control equation has been derived and a statistical control algorithm developed for the critically damped second-order dynamic system. The discrete stochastic feedback (integral) controller can be used to reduce the control error sigma (product variability) at the output. The justification for considering a second-order dynamic model and its critically damped behaviour have also been presented in this paper.

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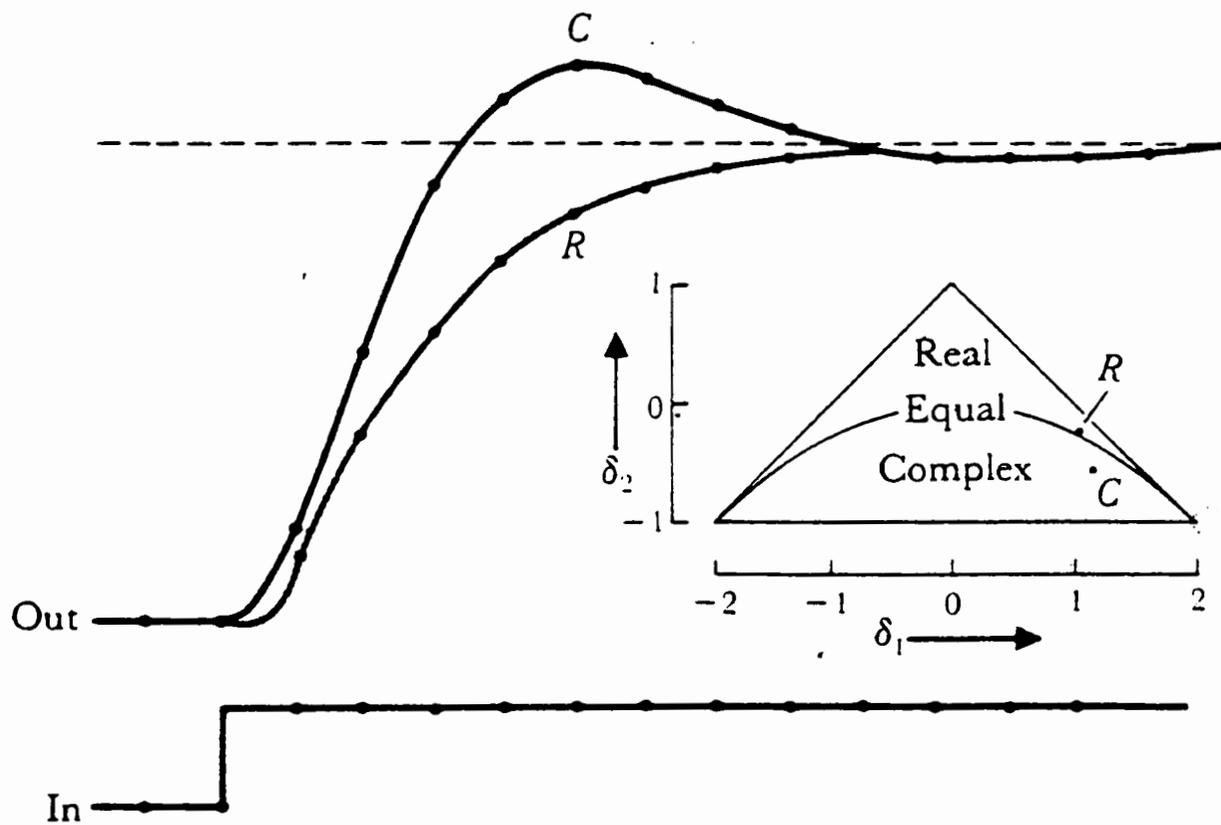


FIG. 3 Step responses of coincident, discrete and continuous second-order systems having characteristic equations with real roots (Curve R) and complex roots (Curve C)

