Markets For Non-Storable Commodities

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by

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ABSTRACT

Published empirical studies of simultaneous rational expectations models of spot and futures markets for non-storable commodities, such as finished live cattle, are extremely rare. Indeed, only two countries, the US and Australia, have produced data sets for the study of such markets. This paper develops, and presents estimates of a simultaneous rational expectations model of the live cattle market in Australia, the world’s leading beef exporting country. The model contains functional relationships for short hedgers and speculators combined (there is no disaggregation of hedgers’ and speculators’ commitments in Australian data), long hedgers and speculators, and consumers, and is completed with a spot price equation and market clearing identity. Augmented Dickey-Fuller and Phillips-Perron tests for unit roots yield ambiguous results, but the Phillips-Perron tests, taken as definitive, indicate that several variables are I(1), including spot and futures prices, real income and consumption of beef, the others being stationary. Johansen cointegration tests indicate that where two or more I(1) variables appear in a structural equation, they are cointegrated. Structural equations with rational expectations are estimated by the instrumental variables method of McCallum in the absence of serial correlation, and by non-linear least squares when a correction for autocorrelation is required. The estimates of all 15 structural parameters have the expected sign, and all are significant at the five per cent level. In a 34 month post-sample period, the model forecasts consumption with a per cent RMSE of 2.3% and spot and futures prices with per cent RMSE’s of 4.6% and 2.8% respectively. In forecasting the spot price, the model outperforms, but not significantly, conventional benchmarks such as a random walk, an ARIMA model, and a lagged futures price. The outcome of this last comparison implies that the efficient markets hypothesis cannot be rejected.

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MARKETS FOR NON-STORABLE COMMODITIES

I

INTRODUCTION

The objective of this paper is to develop and present empirical results for a simultaneous, rational expectations model of the Australian finished live cattle market. Finished live cattle are non-storable, because they can be kept in their finished condition for a period of six to eight weeks only. This model employs information from both spot and futures markets. Only two countries, the United States and Australia, have introduced futures contracts for live cattle, and hence only these two countries have produced data sets for the estimation of such models. While studies of the US live cattle market have been made, comprising simultaneous rational expectations models, none of these studies has been published, as far as the present authors are aware, and no such study has been made of the Australian market, to the best of the present authors' knowledge.

Simultaneous theoretical models of the determination of spot and futures prices have been developed by Peston and Yamey (1960), Stein (1961, 1964), Dewbre (1981) and Kawai (1983), the last of these being specifically for non-storable commodities. Empirical, simultaneous models of (non-storable) livestock markets, without rational expectations, have been developed and estimated by Leuthold and Hartmann (1979) and Leuthold and Garcia (1992) and others, while empirical, simultaneous models, with rational expectations, for storable commodities have been developed by Giles et al. (1985), Goss et al. (1992) and others. This paper extends the work of Peston and Yamey (1960), Giles et al. (1985) and Goss et al. (1992) to develop a simultaneous model, with rational expectations, of the Australian live cattle market.
A major difference between storable and non-storable commodities, for which both spot and futures markets exist, is that in the case of storable commodities, the extent of the forward premium or contango (i.e. where the futures price exceeds the spot price) is limited by the marginal net cost of storage, whereas in the case of non-storables, no such restriction applies. (On the other hand, this restriction does not apply to the spot premium or backwardation (i.e. where the spot price exceeds the futures price), even in the case of storable commodities, as Keynes (1930, pp. 142-44) pointed out.)

International live cattle markets have attracted considerable attention from researchers, particularly markets in the USA. Leuthold (1972) was reluctant to reject the random walk hypothesis with US live cattle price data, even though some filter rules yielded profits net of transaction costs. Leuthold (1974) did not reject the unbiasedness hypothesis, for US live cattle futures prices as predictors of delivery date spot prices, with lags up to three months from maturity, but did reject that hypothesis with longer lags. Giles and Goss (1980) obtained a very similar result with Australian data. Just and Rausser (1981) compared the predictive performance of various commercial econometric price forecasts with that of the futures price, for a range of commodities. In the case of live cattle they found that only one commercial forecast out of five surpassed the futures price, with a three month lag to maturity, whereas with longer lags several commercial forecasts surpassed the futures price. While Leuthold and Hartmann (1979) refined the model prediction approach to market efficiency (for hogs), Leuthold and Garcia (1992) applied this approach to live cattle, and were unable to reject the efficient markets hypothesis. The latter authors also computed the Stein (1986) social loss measure for cattle and hogs, and found that this measure was smaller for cattle.

Australia, with 22.4 million head of cattle in 1989, is one of the leading beef producing countries in the world, ranking behind, for example, USA (98m head), Brazil (130m head) and Argentina (57m head). In Australia, the main producing states are
Queensland, New South Wales and Victoria. In 1989, Australia exported 872,000 tonnes of beef, or 55.4% of production, making it the world's leading exporter of beef. The main export markets served are in USA and Japan; indeed 76.7% of Australia's beef exports goes to these two countries (see ABARE, 1993).

Although a cattle contract was introduced on the Sydney Futures Exchange in July 1975, this contract, which called for the delivery of carcases, traded thinly. Revisions were made to this contract in May 1977, providing for the delivery of 10,000 kg live weight of steers every calendar month (28 steers approx.). This revised contract became relatively successful, with average monthly turnover reaching 16,559 contracts in 1981, and 28,007 contracts being traded in March of that year. By 1985, however, average monthly turnover had fallen to 1190 contracts, and in May 1986 this Trade Steers Contract, as it had become known, was replaced by a live cattle contract providing for mandatory cash settlement. This last contract, although retaining 10,000 kg live weight of cattle as the contract unit, provided that contracts were to be settled at the Live Cattle Indicator price, which is itself an average of cash prices for specified cattle types at specified locations. Trading in the cash settlement contract became thin after the end of 1988, and this contract was de-listed in May 1994.

The objective of this paper is to develop, present estimates of, and evaluate a simultaneous, rational expectations model of price determination in the Australian live cattle market. Section II of this paper discusses the specification of the model, while Section III discusses the data employed, presents results for tests for unit roots and cointegration, and discusses the methodology employed for estimation of the model. Results for the intra-sample period are presented and discussed in Section IV, while Section V discusses post-sample simulation by the model, compared with various benchmarks. Some conclusions are presented in Section VI.
MODEL SPECIFICATION

This model contains four functional relationships and a market clearing identity. The first equation explains the combined futures market commitments of both short hedgers and short speculators, while the second relationship refers to the market commitments of both long hedgers and long speculators in futures. The model contains also, a consumption relationship and a spot price equation, and is completed with a futures market clearing identity.

This structure represents a modification of the original model by Peston and Yamey (1960) and of the approach in the empirical models of Giles et al. (1985) and Goss et al. (1992), to deal with the case of non-storables. The combined nature of the first two equations arises because Australian futures market data on commitments of traders are not disaggregated into hedging and speculation components, as are data for Reporting Traders provided by the Commodity Futures Trading Commission in the USA.

Although the ideas of Working (1953, 1962) on discretionary hedging were developed for storable commodities, such as grains, his analysis of the motives for hedging is applicable to the case of non-storables. Two of the major types of hedging distinguished by Working (1953, 1962) are carrying charge hedging and selective hedging. On the first hypothesis, the market commitments of short hedgers, who gain from a reduction in the forward premium, can be expected to vary directly with the current forward premium (futures price less spot price), and negatively with the expected forward premium. If, on the other hand, short hedgers are selective hedgers, where a proportion only of their spot market commitments is hedged, then their futures market commitments would be expected to vary directly with the current futures price, and negatively with the expected futures price. Preliminary estimation for short hedgers in this market, such as beef producers, favoured the latter of these two hypotheses.
The market commitments of short speculators, who expect the futures price to fall can be expected to vary directly with the current futures price, and negatively with the expected futures price and marginal risk premium. This specification is based on the equilibrium condition for short speculators (see Goss (1972, p. 23)). The traditional view that the coefficient of the marginal risk premium is negative (e.g. see Kaldor (1953, p. 23) and Brennan (1958, p. 54)), has been challenged recently by Stein (1986, pp. 48-52), who argues, in terms of his “hedging pressure theory”, that an increase in the risk premium may have a positive or negative effect on the futures price, and hence on the market commitments of speculators.

The supply of futures contracts by short hedgers and short speculators combined may be expected to be a function of the sum of the influences outlined above. The specification of this function (HSS) is therefore:

\[
HSS_t = \theta_1 + \theta_2 P_t + \theta_3 P_{t+1}^* + \theta_4 r_t + e_{tt}
\]

where \( P_t \) = current futures price;

\( P_{t+1}^* \) = rational expectation of the futures price for \((t + 1)\), formed in period \(t\);

\( r_t \) = marginal risk premium;

\( e_{tt} \) = error term

and \( \theta_1 \) = constant; \( \theta_2 > 0; \theta_3 < 0; \theta_4 < 0 \);

This specification suggests a predominance of speculative, rather than hedging, elements.

The rational expectations hypothesis, which is employed in this model, originated with Muth's observation that mean expectations in an industry are as accurate as "elaborate
equation systems" and his suggestion that "rational expectations are the same as the predictions of the relevant economic theory (Muth, 1961, p. 316). Much has been written on the assumptions, implications and formation of rational expectations, and summaries of this literature can be found in Sheffrin (1985), Minford and Peel (1986), Goss (1991) and Goss et al. (1992). While these summaries will not be repeated here, some important points deserve to be emphasized. The first of these is that the rational expectations hypothesis (REH) implies that agents have the particular economic model, under review, in mind in forming their expectations, so that any test of the REH is a joint test of the expectations hypothesis and of the appropriateness of the model (Maddock and Carter 1982). The REH implies, therefore, that the model which agents believe determines returns is the same as the model driving returns in practice; otherwise abnormal returns would occur (Minford and Peel, 1986, p. 122). Second, the question of the likelihood of agents learning to form rational expectations may still be open, although some pessimistic notes (e.g. Frydman, 1983) and some optimistic notes (e.g. Bray and Savin 1986) have been struck. The question of how agents learn to form rational expectations has been discussed by several authors, including Blume et al. (1982) who referred to agents using the same forecasting rule for a long period, and Stein (1986) whose asymptotically rational expectations converge to Muth rational expectations with repeated sampling. Third, there is experimental evidence on the convergence of prices to rational expectations equilibrium in futures and asset markets, in the work of Plott and Sunder (1982), Friedman et al. (1983) and Harrison (1992). It is the view of the present authors that experimental evidence suggests that a rational expectations equilibrium can be achieved in a comparatively short time, especially with futures markets operating. Finally, support for the REH has been found in models of this type for storable commodities (see Giles et al. (1985), Goss et al. (1992)).

The market commitments of long hedgers, such as meat processors and beef exporters
traditionally, have been regarded as the mirror image of those of short hedgers (e.g. see Stein, 1961). We would expect the positions of these agents, therefore, to vary negatively with the current forward premium, directly with the expected forward premium, and directly with measures of the market commitments of these agents, such as planned consumption and planned exports. The market positions of long speculators, who expect the futures price to rise, can be expected to vary negatively with the current futures price, directly with the expected futures price, and negatively with the marginal risk premium. The combined functional relationship for these two groups of agents could be expected to reflect the sum of these influences. Preliminary estimation suggested that the price spread variables were more important than the level form price variables, and that the planned change in consumption should replace the planned level of consumption. This last change is a consequence of the unit root and cointegration tests reported in Section III. The demand function for futures contracts (HSL), therefore is

\[ HSL_t = \theta_5 + \theta_6(P_t - A_t) + \theta_7(P_{t+1} - A_{t+1})^* + \theta_8 \Delta C_{t+1}^* + \theta_9 X_{t+1}^* + \theta_{10} r_t + \epsilon_{2t} \]  

(2)

where 

- \( A_t \) = current spot price;
- \( (P_{t+1} - A_{t+1})^* \) = rational expectation of the forward premium in \((t+1)\) formed in period \( t \);
- \( \Delta C_{t+1}^* = C_{t+1}^* - C_t^* \) = planned change in consumption next period;
- \( X_{t+1}^* \) = planned exports in period \((t+1)\);
- \( r_t \) = marginal risk premium;

and \( \theta_6 < 0, \theta_7, \theta_8, \theta_9 > 0; \theta_{10} > 0 \).
This specification contains a mixture of hedging and speculative elements, although it does suggest a predominance of hedging activity. It is, however, consistent with the view that speculators take straddle positions.

The demand for live cattle is derived from the demand for dressed beef. The demand function for live cattle, therefore, can be seen as dependent upon the spot price of live cattle, parameters of the demand for the end product, and parameters of the supply of other inputs. In this case, expected real income next period, and the spot prices of two substitute meats, lamb and pork, have been employed as parameters of the demand for dressed beef. The spot price of feed grain, a complementary input with live cattle, is used as a proxy for the supply of other inputs. The demand for live cattle, therefore, can be expected to vary negatively with the spot price of live cattle and the price of oats, and directly with expected real income, the price of lamb and the price of pork. The resulting specification of this function is

\[ C_t = \theta_{11} + \theta_{12}A_t + \theta_{13}Y_{t+1}^* + \theta_{14}A_t^L + \theta_{15}A_t^P + \theta_{16}A_t^G + e_{3t} \]  

where

- \( C_t \) = consumption of live cattle in period \( t \).
- \( Y_{t+1}^* \) = planned real income in period \((t+1)\);
- \( A_t^L \) = spot price of lamb;
- \( A_t^P \) = price of pork in period \( t \);
- \( A_t^G \) = spot price of feed grain;

and \( \theta_{12}, \theta_{16} < 0; \theta_{13}, \theta_{14}, \theta_{15} > 0 \).

The model contains also a spot price equation, in which the spot price of live cattle is specified first, as a direct function of the current futures price, on the ground that changes
in these two prices are expected to be closely correlated. Secondly, it is postulated that the spot price is negatively related to the number of store cattle in current yardings for sale, because an increase in yardings can be expected to lead to an increase in the number of finished live cattle, and hence to a decline in the spot price. The spot price equation is written as

$$A_t = \theta_{17} + \theta_{18} P_t + \theta_{19} N_t + e_{4t} \quad (4)$$

where $N_t$ = number of cattle in current yardings; and $\theta_{18} > 0$; $\theta_{19} < 0$.

This model, with five endogenous variables (HSS, HSL, C, P, A) and four equations, is completed with the futures market clearing identity

$$HSS_t = HSL_t \quad (5)$$

Conventional identification conditions do not apply to linear multi-equation models with forward rational expectations (Pesaran, 1987, p. 119). The model developed here, however, fulfils the identification conditions developed by Pesaran (1987, pp. 156-60) for such models.

III

DATA, UNIT ROOTS, COINTEGRATION TESTS AND ESTIMATION

Data

The sample period for the results reported in this paper, after allowance for leads and lags, is 1980(05) to 1985(12), comprising a total of 68 monthly observations; the post-sample forecast period, again after allowance for leads and lags, is 1986(03) to 1988(12), which is a total of 34 observations. Data are discussed in this section under the headings "Endogenous Variables" and "Exogenous Variables".
**Endogenous Variables**

Futures price data (P) are futures prices of live steers, on the median trading day of the month, for a contract two months prior to delivery (the most heavily traded contract), in Australian cents per kg live weight from the Sydney Futures Exchange *Statistical Yearbook* 1980-88. Spot price data (A) for the period 1980(05) to 1986(06), during which time the Trade Steer Contract (deliverable) traded on the SFE, are prices in Australian cents per kg live weight, for "futures type steers", on the median trading day of the month, provided by the New South Wales Meat Industry Authority. Data on spot prices for the period 1986(07) to 1988(12), when the Live Cattle (cash settlement) Contract replaced the previous contract, are SFE Live Cattle Indicator prices, on the median trading day of the month, in Australian cents per kg live weight, provided by the SFE. The Live Cattle Indicator price is a five day average of cash prices for specified cattle types at specified selling centres. At maturity, positions in the Live Cattle Contract are settled at the Indicator price.

The total supply of, and total demand for futures contracts (HSS = HSL) are measured by the open positions (or commitments) of traders, in number of contracts, on the median trading day of the month, for a futures contract two months from maturity. The data on commitments of traders, therefore, are synchronized with the data on spot and futures prices.

Data on consumption (C) are Australian consumption of beef and beef meat products, per quarter, in thousand tonnes, from Australian Bureau of Statistics (ABS) *Livestock and Livestock Products* (Catalogue 7221.0). These data were interpolated to monthly observations using the program TRANSF (Wymer 1977).

**Exogenous Variables**

Exports of beef (X) are measured by exports of beef meat, fresh chilled or frozen, in tonnes per month, from ABS *Exports, Australia. Monthly Summary Tables* (Cat. 5432) and ABS
Exports of Major Commodities and Their Principal Markets (Cat. 5403). Exports of live beef cattle from Australia are insignificant and are not included.

Real income (Y) is Australian household disposable income per quarter in million Australian dollars from ABS, divided by the Consumer Price Index (quarterly), also from ABS. These data were interpolated to monthly observations.

The marginal risk premium (r) is the monthly average 90 day bank accepted bill rate, in per cent per annum, minus the monthly average 90 day Treasury Bill rate, in per cent per annum; observations on both these rates are taken from the Reserve Bank of Australia Statistical Bulletin. This treatment of the risk premium is consistent with Stein (1991, p. 39).

The spot price of lamb (AL) is the monthly average saleyard price, in Australian cents per kg for lambs (16kg to 19kg) on a dressed weight basis. Similarly, the spot price of pork (AP) is the monthly average saleyard price, in Australian cents per kg for pigs (60kg to 70kg) on a dressed weight basis. Observations on both these prices were taken from Australian Meat and Livestock Corporation, Statistical Review of Livestock and Meat Industries, and ABARE (1993). The spot price of feed grain (AG) is the monthly average price of oats, in Australian dollars per tonne, from ABARE Situation and Outlook: Coarse Grains. The number of cattle in current yardings (N) is the total number per month of beef cattle in current yardings listed for sale from ABS Livestock and Livestock Products.

UNIT ROOTS AND COINTEGRATION TESTS

To obtain meaningful estimates of the parameters of the model, it is necessary that the residuals of the estimating equations are stationary. This condition will be fulfilled if all the variables in these equations are stationary (i.e. integrated of order I(0)), or alternatively, if some of these variables are integrated of order I (1) or higher order, this condition will be
fulfilled only if the non-stationary variables are integrated of the same order and are cointegrated. The first step in this procedure is to determine the order of integration of the variables in the model.

In the autoregressive representation of the time series

\[ Z_t = \rho Z_{t-1} + e_t \]  

where \( Z \) is an economic variable, \( \rho \) is a real number, and \( e_t \) is \( N(0, \sigma^2) \), if \( |\rho| < 1 \), \( Z_t \) converges to a stationary series as \( t \to \infty \). On the other hand, if \( \rho = 1 \), there is a single unit root and \( Z_t \) is non-stationary, while if \( |\rho| > 1 \), the series is explosive. Tests of the hypothesis \( H(\rho = 1) \) in (6), and for variations of this model with constant and time trend, were developed by Dickey and Fuller (1979, 1981). Critical values for these tests are given in Fuller (1976) and Dickey and Fuller (1981). These tests were extended by Said and Dickey (1984) to accommodate autoregressive processes in \( e_t \) of higher but unknown order.

In this latter case the model is augmented by lagged first differences in \( Z \) to render \( e_t \) as \( N(0, \sigma^2) \), and the hypothesis \( H(\rho = 1) \) is tested by the Augmented Dickey-Fuller Test (ADF).

In this paper the following models were estimated by ordinary least squares (OLS) to test the hypothesis of a unit root in all endogenous and exogenous variables in the structural model:

\[
\Delta Z_t = \mu + \gamma Z_{t-1} + \phi \Delta Z_{t-1} + e_t \tag{7}
\]

\[
\Delta Z_t = \mu + \gamma Z_{t-1} + \phi \Delta Z_{t-1} + \phi_2 \Delta Z_{t-2} + e_t \tag{8}
\]

\[
\Delta Z_t = \mu + \beta t + \gamma Z_{t-1} + \phi \Delta Z_{t-1} + e_t \tag{9}
\]

\[
\Delta Z_t = \mu + \beta t + \gamma Z_{t-1} + \phi_1 \Delta Z_{t-1} + \phi_2 \Delta Z_{t-2} + e_t \tag{10}
\]
where $\mu = \text{constant};$

$$\beta, \phi, \phi_1, \phi_2,$$ are coefficients to be estimated;

$e_t$ is assumed to be $NID(0, \sigma^2)$.

Models (9) and (10) contain a time trend, (7) and (9) contain a single lagged value of $\Delta Z_t$, and (8) and (10) contain two such lagged values. In each case, (7) was estimated first, the other models being estimated as necessary to whiten $e_t$. The hypothesis $H(\rho = 1)$ is addressed by testing the hypothesis $H(\gamma = 0)$ in (7) - (10). This is executed by the ADF test, although it is now preferable to refer to critical values of MacKinnon (1991), which are based on more replications than the original Dickey-Fuller tables. Calculated ADF statistics, together with 5 per cent and 10 per cent critical values from MacKinnon (1991), are provided in Appendix 1 for all variables in the model. Notwithstanding the low power of these tests (see Evans and Savin, 1981), for only two variables (consumption of beef $C$ and the spot price of pork $A^p$) is it not possible to reject the hypothesis of a single unit root; these tests support the view that all other variables in the model are stationary.

To address the issue of higher order autocorrelation in (6), the method of Phillips and Perron (1988) makes a non-parametric correction to the estimated test statistic, to allow for the autocorrelation which would otherwise be present in the residuals. Asymptotically, the same limiting distributions apply as in the Dickey-Fuller case, and the same critical values may be employed. Phillips-Perron tests for unit roots were conducted for all variables in this paper, and the results of these tests, presented in Appendix 2, suggest that spot and futures prices $(A, P)$, consumption $(C)$, price of pork $(A^p)$, income $(Y)$, and the price of grain $(A^g)$ are $I(1)$ at the $10\%$ level. The Phillips-Perron procedure, however, is thought to suffer greater size distortion than the Said-Dickey tests (i.e. rejects a true hypothesis with a proportion
greater than nominal size), in the presence of negative moving average components in the errors (see Banerjee et al, pp. 108-109, 113, 129). Nevertheless, the present series do not exhibit negative MA errors, and this disability does not appear to have affected the outcome of the present tests. It is generally agreed that the Phillips-Perron tests have greater power than the augmented Dickey-Fuller tests (see Banerjee et al, p. 113), and in this case these tests have been taken as definitive. In equation (2) of this model, there is one non-stationary variable, $C_{t+1}$, and in order to render the residuals in (2) stationary, the first difference of this variable is taken. In equation (2), therefore, the planned consumption proxy employed for long hedgers' commitments is $\Delta C_{t+1}$.

Equations (1) and (4) each contain two I(1) variables, while equation (3) contains five I(1) variables. While $C$, $A$, $Y^*$, $A^P$, $A^G$ in (3) are non-stationary, it is possible that a linear combination of these variables may be stationary, i.e. they may be cointegrated, in which case the residuals of (3) will be stationary. A similar statement may be made about (1) and (4). To investigate whether these I(1) variables are cointegrated, the cointegration test analysed by MacKinnon (1991), which is based on the work of Engle and Granger (1987), could be employed. The Engle-Granger technique is adequate in the case of (1) and (4), because the question of cointegration in each of these equations, refers to two variables only. In (4) for example, this test requires first that a relationship between the I(1) variables, such as the following, be estimated by OLS.

$$A_t = \alpha_0 + \alpha_1 P_t + \alpha_2 t + u_t$$ (11)

The hypothesis of no cointegration in (11) is addressed by testing the hypothesis that the series of estimated values of residuals ($\hat{u}_t$) from (11) contains a unit root. To test the
hypothesis of a unit root in \( u_t \), the following model can be estimated

\[ \Delta \hat{u}_t = \gamma \Delta \hat{u}_{t-1} + \phi \Delta \hat{u}_{t-1} + v_t \]  

(12)

and the hypothesis \( H(\gamma = 0) \) can be tested, using the Augmented Engle-Granger (AEG) test.

The Engle-Granger procedure has been criticized inter alia on the grounds first, that the distribution of the test statistics is not independent of the nuisance parameters of the particular application, and second, that it is capable of estimating one cointegrating vector only (which varies according to the normalization). The procedure of Johansen (1988) and Johansen and Juselius (1990) overcomes these difficulties, and their likelihood ratio test is capable of identifying all cointegrating vectors in a set of I(1) variables. Two tests have been developed by these authors: the first is the "trace" test, which tests the hypothesis that the number of cointegrating vectors \( m \) is at most equal to \( q \) (where \( q < n \), the number of I(1) variables in the relationship), against the general alternative that \( m \leq n \). The second test, the "\( \lambda \) max" test, tests the hypothesis that \( m \leq q \), against the specific alternative \( m = q + 1 \). In this paper, both these tests have been employed. The results of the trace test, reported in Appendix 3, suggest that, in equations (1) and (4), there are two cointegrating vectors, while in equation (3), this test suggests that there are 5 cointegrating vectors, at the five per cent level. The results of the \( \lambda \) max test for equation (3), reported in Appendix 4, also suggest that there are five cointegrating vectors in this equation, at the five per cent level. The unit root and cointegration tests discussed in this section were executed with program E-Views-Micro TSP (Hall, Lilien and Johnston, 1994).

In summary, these cointegration tests support the view that in equation (1) the current and expected futures prices are cointegrated, and in equation (4) the current spot and futures prices are cointegrated, as we would expect, while in equation (3) these tests support the view that all I(1) variables are cointegrated. The implications of these tests are that no changes to
the specification of (1), (3) or (4) are necessary.

Estimation

Full information estimators of simultaneous models with forward rational expectations, while potentially more efficient, are less robust to specification errors, and are computationally more demanding than limited information methods (Pesaran, 1987, p. 162). For these reasons the model presented here is estimated by the instrumental variables (IV) method of McCallum (1979). This requires that an instrument is obtained, by OLS, for the unobservable expectation of an endogenous variable, such as $P_{t+1}$ in (1), as a fitted value on the information set at time $t$ ($\phi_t$) comprising all exogenous and predetermined variables (including lagged endogenous variables) in the model. That is

$$P_{t+1} = E(P_{t+1} | \phi_t) \quad \text{and}$$

$$P_{t+1} = E(P_{t+1} | \phi_t) + \eta_t$$

(13)

where $E(\eta_t) = 0$ and $\eta_t$ is uncorrelated with the variables in $\phi_t$, under rational expectations. $E(P_{t+1} | \phi_t)$ is taken to be linear in the elements of $\phi_t$. The structural equations can then be estimated by IV, and if the residuals of those equations are not serially correlated, this method will produce consistent estimates. This procedure is discussed in McCallum (1979) and is summarized in Giles et al. (1985, pp. 754-55). This procedure has been used for equation (2) in this model; $Y_{t+1}^*$, $X_{t+1}^*$, $C_{t+1}^*$ also were obtained as fitted values on $\phi_t$).

When serial correlation is present, however, a simple autoregressive (AR) correction with IV estimation will not produce consistent estimates, as Flood and Garber (1980) pointed
out. In this case an AR transformation has been made, and each of the variables in the transformed equation was regressed on the elements of the relevant information set, using OLS. The fitted values so obtained were substituted in the transformed equation (see McCallum (1979, p. 67-68)), and consistent estimates of the parameters in that equation were obtained by non-linear least squares, using the option LSQ in TSP (Hall et al., 1993). This method, which is discussed by Cumby et al. (1983), has been employed for equations (1) and (3) in this model.  

Equation (4), which does not contain any expectational variables but includes an endogenous regressor, was estimated by IV, with a correction for first order serial correlation.  

IV

RESULTS: INTRA-SAMPLE PERIOD

Estimates of the parameters of the model are provided in Table 1, together with their asymptotic \( t \) values. It will be seen that estimates of all 15 structural parameters have the expected signs and all are significant at the five per cent level (one tail test), thereby providing strong support for the model specification discussed above. There are, however, several features of the results for individual equations, which deserve comment. First, the clear significance of \( \hat{\theta}_3 \) and \( \hat{\theta}_7 \), the coefficients of the expected futures price and expected price spread respectively, appears to provide support for the rational expectations hypothesis (see also Section V). Moreover, the results for equation (1) support the view that HSS is essentially a speculative relationship. Similarly, the results for equation (2) suggest that commitments on the long side of the market are a combination of hedging and speculative elements, with a strong discretionary component in the hedging activities.

Second, the positive estimates of \( \theta_4 \) and \( \theta_{10} \), the coefficients of the marginal risk
premium in equations (1) and (2) respectively, support an interpretation different from the Kaldor (1953) - Brennan (1958) view of the risk premium. In equation (1) the positive sign of $\hat{\theta}_4$ can be explained as follows: an increase in the marginal risk premium $cet par$ will lead to an increase in the equilibrium futures price, and hence to an increase in the market commitments of short speculators. In equation (2), an increase in the marginal risk premium $cet par$ will lead to a decrease in the equilibrium price spread, and hence to an increase in the market commitments of long speculators. These explanations are similar to the "hedging pressure theory" of Stein (1986, pp. 48-52), although Stein's argument is directed to the effect of a change in the risk premium on price alone.

Thirdly, in equation (3), the consumption relationship, the signs of $\hat{\theta}_{14}$ and $\hat{\theta}_{15}$ are consistent with a substitution relationship between lamb and beef and between pork and beef, respectively. Moreover, the significance of $\hat{\theta}_{13}$, $\hat{\theta}_{14}$ and $\hat{\theta}_{15}$ suggests that expected real income and the prices of lamb and pork are parameters of the demand for beef. Moreover, the negative sign and significance of $\hat{\theta}_{16}$ suggest that live cattle and feed grain are indeed complementary inputs.

A further test of model performance is the ability of the model to forecast the endogenous variables within the sample period, according to specified criteria. Table 2 presents an evaluation of the (static) intra-sample simulation of the three key variables, P, A and C, according to the correlation coefficient, Theil's inequality coefficient, and per cent root mean square error. Concentrating on the per cent RMSE criterion, it will be seen that the best forecast is that of consumption, while the better forecast of the two prices is that of the futures price of live cattle (P). The simulation errors of HSS (= HSL) (not reported here) are somewhat larger than those for consumption and the futures and spot prices. This may be
due, in part, to the thinness of the futures market in the latter part of the sample period.

V

POST-SAMPLE SIMULATION

A more stringent test of model performance is the ability of the model to forecast key endogenous variables, against pre-determined criteria, outside the sample period, especially in comparison with alternative forecasts. Table 3 presents an evaluation of (dynamic) two months ahead forecasts of consumption and the futures and spot prices, for the post-sample forecast period 1986(03) to 1988(12), comprising 34 monthly observations. Concentrating on the per cent RMSE criterion, it will be seen first, that the best forecast is again that of consumption, which has deteriorated compared with the intra-sample forecast, and second that the better of the two price forecasts is again that of the futures price, and both these price forecasts have improved significantly compared with intra-sample simulations. This last outcome with respect to prices provides substantial support for the validity of the model.

The question is then how does the model perform, compared with alternative price forecasts. Table 4 presents an evaluation of post-sample forecasts of the spot price, two months ahead, by the model (AS: the same as A in Table 3), compared with three alternative forecasts. The first alternative forecast is the futures price lagged two months prior to maturity \( P_{t-2} \), the second is a random walk forecast two months ahead, and the third is a complex ARIMA model of MA terms with lags of one and five months, and an AR term with a lag of five months. The two latter forecasts are conventional benchmarks in assessing the forecasting performance of economic models. Table 4 shows that the model developed in this paper outperforms all the alternative forecasts of the spot price, according to the per cent RMSE criterion. The difference between the per cent RMSE's for the model (AS) and
the random walk (AWALK 2), however, which is the best of the alternative forecasts, is not statistically significant, at the five per cent level, according to a test of the type proposed in Granger and Newbold (1986, pp. 278-79).

Turning to a comparison of the spot price forecasts provided by the model (AS) and by the lagged futures price \( P_{t-2} \), it should be noted that in executing the model-derived post-sample forecasts, the parameter estimates of the model were updated by one month following each forecast. Hence, the model and the futures price were placed always on the same informational footing during the post-sample period. While the model outperforms the futures price in making a two-month ahead forecast of the spot price, according to the per cent RMSE criterion, again this difference is not significant according to the test employed above. It must be inferred therefore, that the semi-strong efficient markets hypothesis should not be rejected, because the model evidently does not contain any publicly available information, which is not reflected in the futures price (see Leuthold and Hartmann, 1979, and Leuthold and Garcia, 1991). This outcome is consistent with the rational expectations hypothesis employed above, for this assumption implies that agents know the true economic model driving returns in practice.

VI

CONCLUSIONS

This paper develops and presents estimates of a simultaneous rational expectations model of the Australian finished (non-storable) live cattle market, using information from both spot and futures markets. Published studies of simultaneous rational expectations models of such markets are extremely rare, and only two countries, Australia and the US, have produced data sets for the estimation of such models.
Australia is the world's leading beef exporting country. The model developed in this paper contains functional relationships for short hedgers and short speculators combined (there is no disaggregation of hedging and speculative positions in Australian market commitments data), long hedgers and long speculators combined, and consumers. The model contains also a spot price equation, and is completed with a futures market clearing identity.

Augmented Dickey-Fuller and Phillips-Perron tests for unit roots yield ambiguous results, and the Phillips-Perron tests, taken as definitive, suggest that spot and futures prices, consumption of beef, expected real income, the price of pork and the price of grain are I(1), all other variables being stationary. Johansen cointegration tests suggest that the I(1) variables in each of the structural equations, are cointegrated; in the case of the long hedging-long speculation relationship, the first difference of the only I(1) variable, expected consumption is employed.

Instruments for expectational variables are obtained as fitted values on the set of predetermined variables in the model. The structural equations are estimated by instrumental variables in the absence of serial correlation of the error term, and by non-linear least squares, if a correction for serial correlation is necessary.

All parameter estimates have the expected signs, and all are statistically significant at the five per cent level. The signs and significance of the estimated coefficients of the price and expected price variables in the combined hedger-speculator relationships for the futures market, provide support for the rational expectations hypothesis. Moreover, the parameter estimates for the equation referring to short market commitments suggest that this relationship is essentially speculative; furthermore, there is support for a rival hypothesis of the risk premium, of the type discussed by Stein (1986, pp. 48-52). Parameter estimates suggest also that market commitments on the long side of the futures market are predominantly those of discretionary long hedgers in the sense of Working (1953), probably beef exporters and meat
processors. Intra-sample the model simulates the futures price with a per cent RMSE of 4.5% and the spot price with per cent RMSE of 8.6%, while for consumption of beef the corresponding figure is 0.7%. Post-sample, the forecast errors for futures and spot prices decline to 2.8% and 4.6% respectively, while that for consumption becomes 2.3%. In post-sample forecasts of the spot price, the model outperforms rival forecasts such as a random walk (% RMSE 5.1%), an ARIMA model (6.7%) and a lagged futures price (5.8%), although none of these differences in per cent RMSE between the model and alternative predictors is significant. The result in this last comparison, between the model and the lagged futures price, implies that the semi-strong efficient markets hypothesis cannot be rejected, for there is no evidence that the model contains information which is not reflected in the futures price. This outcome, however, is consistent with the employment of the rational expectations hypothesis.
Endnotes

1. Fuller (1976) has shown that the limit distribution of the t statistic for \( \hat{\gamma} \) is independent of the number of lags of \( \Delta Z \) in the equation.

2. For three variables (\( A, A^0, Y \)) this rejection is made at the 10 per cent level, using the most appropriate model from the group (7) - (10).

3. The instruments employed for the IV estimation of equation (2) are:

\[ X_{t-2}, Y_{t-2}, r_{t-2}, r_{t-1}, A_{t-1}, A_{t-1}^L, A_{t-1}^G, N_{t-1}, (P-A)_{t-2}, \left( \frac{P-A}{P-A} \right)_{t-1}^\Lambda, \Delta C_{t-1}, \]

\( HSL_{t-1}, HSL_{t-2} \).

4. Tests were executed for the presence of ARCH effects (see Engle 1982, 1983) in this model. An ARCH (\( p \)) process postulates that, conditional on information at time (\( t-1 \)), an error term \( e_t \) is assumed to be normally distributed with mean zero and variance

\[ \sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \ldots + \alpha_p e_{t-p}^2 \]

Although graphs of the partial autocorrelation functions for the squared residuals revealed various significant lags for the HSS and HSL equations (only), tests of the hypothesis \( H(\alpha_p = 0) \) suggested that the coefficients were significant at lags \( p = 1, 2, 3, 4, 5, 7, 8, 9, 10 \) for HSS only (see Appendix 5, available on request). It is well known that high order ARCH effects may be represented by a GARCH (1, 1) process (see Bollerslev, 1986) where

\[ \sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \]

Accordingly, the HSS equation was re-estimated by maximum likelihood, with the error term assumed to follow a GARCH (1, 1) process. While the estimates of the
structural parameters have the expected signs, and are significant at the 5% level (see Appendix 6, available on request), the estimates of \( \alpha \) and \( \beta \) are not significant. These last two estimates support the view that the GARCH effects in the HSS equation are not significant, and therefore the results in Table 1 are retained.

5. The instruments for the IV estimation of equation (4) are:

\[ X_{t-1}, X_{t-2}, Y_{t-2}, r_{t-2}, r_{t-1}, A_{t-1}^G, N_{t-1}, A_{t-1}^P, A_{t-1}^L, C_{t-1}, P_{t-2}, P_{t-3}, \]

\[ A_{t-1}, A_{t-2}, A_{t-3}, HSS_{t-1}, HSS_{t-2}. \]

6. Theil's inequality coefficient and per cent RMSE are defined in Pindyck and Rubinfeld (1981, pp. 362, 364).

7. Random walk forecasts of the spot price two months ahead were obtained by estimating the following model by ML: \( A_t = \beta A_{t-2} \) where \( \beta \) is a parameter to be estimated. From these estimates fitted values \( \hat{A}_t \) were obtained, which acted as forecasts.
Table 1

PARAMETER ESTIMATES

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient</th>
<th>Variable</th>
<th>Estimate</th>
<th>Asymp. t Value</th>
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<td>$P_1$</td>
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<td>2.611</td>
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<td>$P_{r-1}$</td>
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<tr>
<td></td>
<td>$\theta_4$</td>
<td>$r_1$</td>
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<td>2.029</td>
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<td></td>
<td>$\rho_1$</td>
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<td>-0.511</td>
<td>-4.410</td>
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<td></td>
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<td>$P_{r-1}^* - A_{r-1}^*$</td>
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<td>$\Delta C_{r-1}^*$</td>
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<td>$\theta_{12}$</td>
<td>$A_i$</td>
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<td>$Y_{r-1}^*$</td>
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<td>$\theta_{15}$</td>
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<td>$\theta_{16}$</td>
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<td></td>
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<td>$P_i$</td>
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<td>5.165</td>
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<td></td>
<td>$\theta_{19}$</td>
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<td></td>
<td>$\rho_4$</td>
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<td>0.643</td>
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Table 2

INTRA-SAMPLE SIMULATION

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation Coefficient</th>
<th>Theil's IC</th>
<th>% RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>0.9307</td>
<td>0.0213</td>
<td>4.5066</td>
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<tr>
<td>$A$</td>
<td>0.8875</td>
<td>0.0377</td>
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<tr>
<td>$C$</td>
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<td>0.6735</td>
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Table 3

POST-SAMPLE SIMULATION: SPOT AND FUTURES PRICES

<table>
<thead>
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<th>Variable</th>
<th>Correlation Coefficient</th>
<th>Theil's IC</th>
<th>% RMSE</th>
</tr>
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<tbody>
<tr>
<td>$P$</td>
<td>0.9437</td>
<td>0.0132</td>
<td>2.8355</td>
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<tr>
<td>$A$</td>
<td>0.8946</td>
<td>0.0203</td>
<td>4.6297</td>
</tr>
<tr>
<td>$C$</td>
<td>0.9421</td>
<td>0.0113</td>
<td>2.3476</td>
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Table 4

POST-SAMPLE SPOT PRICE FORECASTS

<table>
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<tr>
<th>Variable</th>
<th>Correlation Coefficient</th>
<th>Theil's IC</th>
<th>% RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AS$</td>
<td>0.8946</td>
<td>0.0203</td>
<td>4.6297</td>
</tr>
<tr>
<td>$P_{r2}$</td>
<td>0.7608</td>
<td>0.0270</td>
<td>5.7983</td>
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<tr>
<td>$AWALK \ 2$</td>
<td>0.8338</td>
<td>0.0235</td>
<td>5.0507</td>
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<tr>
<td>ARIMA*</td>
<td>0.7992</td>
<td>0.0298</td>
<td>6.6541</td>
</tr>
</tbody>
</table>

* This is a complex ARIMA model: see text.
REFERENCES


Minford, P. & D. Peel (1986). *Rational Expectations and the New Macroeconomics,*


### Appendix 1

#### UNIT ROOT TESTS: ADF

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Calculated ADF Statistic</th>
<th>5% Critical Value</th>
<th>10% Critical Value</th>
<th>Integration Order</th>
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<tbody>
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<td>P</td>
<td>9</td>
<td>-3.8095</td>
<td>-3.4527</td>
<td>-3.1516</td>
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<td>A</td>
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<td>-3.3090</td>
<td>-3.4527</td>
<td>-3.1516</td>
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<tr>
<td>HSS (= HSL)</td>
<td>9</td>
<td>-4.7421</td>
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<td>C</td>
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<td>-1.8444</td>
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<tr>
<td>(P - A)</td>
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<td>X</td>
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<td>-2.8892</td>
<td>-2.5813</td>
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<td>Y</td>
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<td>-3.4527</td>
<td>-3.1516</td>
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<td>r</td>
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<td>-3.4531</td>
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<tr>
<td>A&lt;sup&gt;L&lt;/sup&gt;</td>
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<td>-5.1816</td>
<td>-2.8889</td>
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<tr>
<td>A&lt;sup&gt;P&lt;/sup&gt;</td>
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<td>-2.1550</td>
<td>-2.8892</td>
<td>-2.5813</td>
<td>I(1)</td>
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<td>N</td>
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<td>A&lt;sup&gt;G&lt;/sup&gt;</td>
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<td>-2.7462</td>
<td>-2.8889</td>
<td>-2.5812</td>
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Appendix 2

**UNIT ROOT TESTS: PHILLIPS-PERRON**

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<th>Variable</th>
<th>Calculated Test Statistic</th>
<th>Probability of Rejection</th>
<th>Order of Integration</th>
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<tr>
<td>$P$</td>
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<td>.3073</td>
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</tr>
<tr>
<td>$A$</td>
<td>-12.5246</td>
<td>.2892</td>
<td>I(1)</td>
</tr>
<tr>
<td>$HSS = HSL$</td>
<td>-50.0937</td>
<td>.0095 $\times 10^{-2}$</td>
<td>I(0)</td>
</tr>
<tr>
<td>$C$</td>
<td>-4.1430</td>
<td>.8782</td>
<td>I(1)</td>
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<tr>
<td>$P - A$</td>
<td>-21.3211</td>
<td>.0541</td>
<td>I(0)</td>
</tr>
<tr>
<td>$X$</td>
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<tr>
<td>$Y$</td>
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<td>$A^g$</td>
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<td>.3467</td>
<td>I(1)</td>
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### Appendix 3

**JOHANSEN COINTEGRATION PROCEDURE: TRACE TEST**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Variables</th>
<th>Calculated Test Statistic</th>
<th>Probability of Rejection</th>
<th>No. of Cointegrating Vectors: m</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$p_t$, $\dot{p}_{t-1}$</td>
<td>8.9470</td>
<td>.0022</td>
<td>$m \leq 1$</td>
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<tr>
<td>(3)</td>
<td>$c_t$, $a_t$, $\dot{y}_{t+1}$, $a_t^p$, $a_t^g$</td>
<td>5.4694</td>
<td>.0174</td>
<td>$m \leq 4$</td>
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<td>(4)</td>
<td>$a_t$, $p_t$</td>
<td>9.5650</td>
<td>.0015</td>
<td>$m \leq 1$</td>
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### Appendix 4

**JOHANSEN COINTEGRATION PROCEDURE: $\lambda$ MAX TEST**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Variables</th>
<th>Calculated Test Statistic</th>
<th>5% Critical Value</th>
<th>No. of Cointegrating Vectors: m</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3)</td>
<td>$c_t$, $a_t$, $\dot{y}_{t+1}$, $a_t^p$, $a_t^g$</td>
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