MEASURES OF INTRA-INDUSTRY TRADE AS INDICATORS OF FACTOR MARKET DISRUPTION

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ABSTRACT

Intra-industry trade (IIT) related concepts have often been used as indicators of the extent to which trade growth can be accommodated without factor market disruption. The most commonly used indicators have been movements in the Grubel-Lloyd (GL) index. However, GL-based indicators are sometimes misleading and, at best, they give qualitative information only. We develop two other indicators. The first involves computing changes in IIT. While this method provides a precise measure of the contribution of growth in IIT to total trade (TT) growth, it tends to overestimate the contribution of non-disruptive trade growth. This problem is overcome by our second indicator, dynamic intra-industry trade or matched changes in trade. All our indicators are illustrated with data for 133 Australian manufacturing industries.

Keywords: intra-industry trade, matched trade, factor market disruption.

J.E.L. Classification numbers: F32, F17
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Measures of Intra-Industry Trade as Indicators of Factor Market Disruption*

by

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1. Introduction

International trade has grown faster than income in the postwar period. Nominal exports plus imports as a share of nominal GNP for an average OECD country grew by more than 1 percent per annum over the past three decades. Much of this growth is often attributed to intra-industry trade (IIT). The contribution of growth in IIT to growth in total trade (TT) is of relevance in addressing the issue of adjustment in the context of trade liberalisation or regional trading agreements. If most of the growth in trade is attributable to IIT, then the disruption to factor markets is likely to be lower. This is because IIT does not require inter-industry factor movements. Whereas trade expansion through net trade (NT) requires factor transfer from import-competing industries to export-oriented industries, trade expansion through IIT requires only specialisation within industries. Furthermore, as Krugman (1981) has shown, it is possible for all factors to gain from trade in an IIT setting, thus alleviating adjustment pressures. In this context, Caves (1981) suggests that protectionist pressures are unlikely to grow in proportion to the degree of import competition, thus making it more likely that governments will press ahead with trade liberalisation. Regional trading agreements are more likely to be maintained if governments are not faced with pressures to intervene in order to protect employment in less competitive industries.

There have been numerous studies in which IIT-related concepts have been used as indicators of the extent to which trade growth can be accommodated without factor market disruption.1 The most commonly used indicators (method 1) have been

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movements in the Grubel and Lloyd (GL, 1975) index.\textsuperscript{2} We develop two other methods. Method 2 involves computing changes in IIT. Method 3, which we prefer, is based on computed changes in matched trade or what we call dynamic intra-industry trade (DIIT). Versions of methods 2 and 3 have appeared in earlier papers.\textsuperscript{3} In this paper, we make two contributions. First, we compare and clarify all three methods at a theoretical level. Second, we overcome various limitations in earlier versions of the preferred method based on computations of DIIT.

The paper is organised as follows. Section 2 sets out the theory behind the three methods. Section 3 illustrates our formulas from Section 2 using constant-price data on Australia's trade in manufactures. Section 4 contains concluding remarks.

2. Theory

To explain methods 1 and 2, we start by expressing the level of total trade in commodity $i$ ($TT_i$) in any year as the sum of intra-industry trade ($IIT_i$) and net trade ($NT_i$). That is,

$$TT_i = IIT_i + NT_i,$$

where

$$TT_i = X_i + M_i,$$

$$IIT_i = (X_i + M_i) - |X_i - M_i| = 2 \min (X_i, M_i),$$

and

$$NT_i = |X_i - M_i|.$$

$X_i$ and $M_i$ are exports and imports of commodity $i$ valued in base period f.o.b. prices.

The percentage growth in total trade of commodity $i$ ($tt_i$) over any period is then given by:

$$tt_i = Ciit_i + Cnt_i,$$

where

$$Ciit_i = GL_i it_i.$$

\textsuperscript{(2)} The GL index is sometimes corrected for the aggregate trade imbalance. This correction is motivated by the fact that the greater is the trade imbalance (deficit or surplus), the higher will be the share of NT in TT. Recent studies have avoided this correction because it tends to compound the problem by distorting the GL index. See Menon (1994) for a discussion of this issue.

Intra-Industry Trade and Factor Market Disruption

\[ C_{nti} = (1 - GL_i) nt, \quad (7) \]
\[ GL_i = IIT_i / TT_i \quad (8) \]

and \( ii_i \) and \( nt \) are the percentage changes over the period in \( IIT_i \) and \( NT_i \). Note that
\[ GL_i = 1 - \{ |X_i - M_i| / (X_i + M_i) \}, \]
which is the Grubel-Lloyd index of intra-industry trade at the beginning of the period.

In method 2, \( C_{ii} \) is assumed to be the contribution to growth in total trade in commodity \( i \) of growth in intra-industry trade, while \( C_{nt} \) is the contribution of growth in net trade. This assumption is justified if (as in our study in Section 3 of Australian trade)\(^4\) \( ii_i \) is determined independently of \( nt \).

If our purpose is to compute an indicator of the importance of IIT in trade growth, then \( C_{ii} \) (method 2) is the right measure. Certainly the common practice of using movements in GL indices (method 1) is inadequate. For instance, as is clear from (8), \( GL_i \) will increase over a period whenever \( ii_i > nt \). Even under this condition, growth in IIT may make a relatively minor contribution to growth in total trade of product \( i \). More formally:
\[ ii_i > nt \implies GL_i \text{ is increasing,} \]
but if \[ GL_i < nt / (nt + ii_i), \quad (9) \]
and \[ nt + ii_i > 0, \quad (10) \]
then \[ C_{ii} < C_{nt}. \]

Thus movements in the GL index can be misleading when used to infer the importance of growth in IIT.

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(4) Where \( i \) ranges over 133 Australian manufactured products, the correlation coefficient between \( nt \) and \( ii_i \) is 0.004 for the period 1981 to 1986, and -0.066 for the period 1986 to 1991. This finding is consistent with theory, since the factors that determine NT are different from those that drive IIT (see, for instance, Helpman and Krugman, 1985).

(5) Equations (9) and (10) imply that:
\[ GL_i nt + GL_i ii_i < nt_i, \]
i.e. \[ -(1 - GL_i)nt + GL_i ii_i < 0, \]
i.e. \[ C_{ii} < C_{nt}. \]
However, $Ciit_i$ can, itself, be misleading when our ultimate purpose is to discuss adjustment pressures associated with trade growth. In general it will tend to overestimate the contribution of non-disruptive trade growth. Consider the following example. The volume of imports of commodity $i$ over a period grows from 1 to 3 while exports remain at 2. Then $TT_i$ has increased by 66.67 percent (from 3 to 5). All of this growth is accounted for by IIT (i.e. $Ciit_i = 66.67$ percent and $Cnt_i = 0$). Despite this, we might expect considerable disruption to factors employed in industry $i$ because the growth in imports is unmatched by any offsetting growth in exports.

This problem is overcome by adopting method 3. This time we start by explaining the change in total trade in commodity $i$ over any period as the sum of dynamic intra-industry trade ($DIIT_i$) and dynamic net trade ($DNT_i$). $DIIT_i$ is that part of $\Delta TT_i$ which is composed of matched changes in imports and exports. $DNT_i$ is that part of $\Delta TT_i$ consisting of the residual unmatched change in either imports or exports. That is, in the dynamic approach we have:

$$\Delta TT_i = DIIT_i + DNT_i,$$  
(11)

where

$$DIIT_i = 2 \min (\Delta X_i, \Delta M_i)$$  
(12)

and

$$DNT_i = |\Delta X_i - \Delta M_i|.$$  
(13)

From (11) to (13), we decompose the percentage growth in TT of commodity $i$ into the contributions of DIIT and DNT$^6$ according to:

$$tt_i = C_{diit}_i + C_{dnt}_i,$$  
(14)

where

$$C_{diit}_i = 100 \left( \frac{DIIT_i}{TT_i} \right)$$  
(15)

and

$$C_{dnt}_i = 100 \left( \frac{DNT_i}{TT_i} \right).$$  
(16)

Now, in our example, we have $C_{dnt}_i = 67.67$ percent whereas $C_{diit}_i = 0$, indicating that all the growth in trade must be accommodated by factor movements away from industry $i$.

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(6) As with $Ciit_i$ and $Cnt_i$, $C_{diit}_i$ and $C_{dnt}_i$ are legitimate contribution measures only if $DIIT_i$ is determined independently of $DNT_i$. In our study of 133 Australian manufacturing products, the correlation coefficient between $DIIT_i$ and $DNT_i$ is -0.051 for the period 1981 to 1986, and 0.101 for the period 1986 to 1991.

(7) Hamilton and Kniest (1993) use ratios of $\Delta M$ and $\Delta X$ as indicators of non-disruptive trade growth. They recognise that this ratio approach is inappropriate when either $\Delta M$ or $\Delta X$ is negative. The importance of this limitation is emphasised by Greenaway et al. (1994). Our approach, based on $C_{diit}$ and $C_{dnt}$, is not subject to this limitation.
In general,

\[ C_{iit} \geq C_{dit} , \]  

(17)

with the strict inequality applying whenever

either \[ X_i > M_i \text{ but } \Delta M_i > \Delta X_i , \]  

(18)
or \[ M_i > X_i \text{ but } \Delta X_i > \Delta M_i . \]  

(19)

The proof of this proposition is in the appendix. What the proposition means is that the change in intra-industry trade over a period is at least as great as the change in matched or dynamic intra-industry trade, and may often be greater.

A corollary of this proposition is that

\[ C_{nt} \leq C_{dnt} . \]  

(20)

That is, \( C_{nt} \) underestimates the contribution of factor-disruptive trade growth to total trade growth. Consistent with being an indicator of required inter-industry factor movement,

\[ C_{dnt} \geq 0 . \]  

(21)

\( C_{dnt} \) is necessarily non-negative because it indicates the part of trade growth which must be accommodated either by movement of factors out of or into industry \( i \). \( C_{nt} \), on the other hand, can have either sign.

3. Illustrative Application: Australian Manufacturing Trade

3.1 Data Issues

The definition of "industry" employed in compiling the data base is potentially important to the measurement of our IIT-related concepts. Sceptics such as Finger (1975), Lipsey (1976) and Pomfret (1985) have argued that almost all measured IIT is a statistical artefact brought about by trade data having been grouped in heterogeneous categories. In a sense they are right. At an extremely fine level of disaggregation, there will be no IIT.

However, as explained in Section 1, our interest is in IIT-related indicators of factor market disruption associated with trade growth. For looking at such problems, we need industry categories that have the following property. They should be defined so that the cost of intra-industry factor movements is low relative to inter-industry movements. This means that the categories must be neither too fine nor too broad. With very fine categories, there will be inter-industry factor movements which are
barely more costly than intra-industry movements. With categories which are too broad, intra-industry movements may be just as costly as inter-industry movements.

With these considerations in mind, we judged that disaggregation at the 3-digit SITC level was appropriate. At this level, we have industries such as inorganic acids (SITC 523), paints (SITC 533), paper and paperboard (SITC 641), glass (SITC 664), glassware (SITC 665), tractors (SITC 722), television receivers (SITC 761) and furniture (SITC 821). Activities within such industries tend to have similar capital and skill requirements. Furthermore, it is often true that each firm produces the full range of the industry’s products. For example, chemical firms usually produce most types of inorganic acids. Thus it is reasonable to assume that factor re-allocations within 3-digit industries are relatively cheap. On the other hand, movements of factors between industries such as inorganic acids, paints etc. are likely to be quite costly.

At the 3-digit level, we had data for Australia from the United Nations' COMTRADE data base covering 133 manufacturing industries belonging to SITC 5-8 less 67-68 (metals). These data are in current prices and denominated in US dollars. To analyse issues related to adjustment costs and factor market disruption, we need data measured in constant prices. Consequently, we applied two transformations to the COMTRADE data. First, we converted the data into Australian dollars using the $A/$US average annual exchange rate indexes in the IMF’s *International Financial Statistics*. Then we deflated using $A$ import and export price series using unpublished data from the Australian Bureau of Statistics.

### 3.2 Results

In Tables 1 and 2 we have aggregated our results for the 133 manufacturing industries into SITC 1-digit classifications and total manufacturing. The aggregation formulas are in the notes at the end of the tables. We consider two periods: 1981 to 1986 (Table 1) and 1986 to 1991 (Table 2). Apart from being able to use constant-price data at the 3-digit level, there are several other reasons why we

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(8) The detailed results for the 133 industries are available on request.

(9) The weights used are industry shares in sectoral aggregates. With these weights, $C_{diit}(j)$ and $C_{dnt}(j)$, for instance, refer to contributions of DIIT and DNT to growth in sectoral trade. As emphasised by Milner (1988) and Greenaway *et al.* (1994), alternative representations of contributions are possible. For example, by adopting different scaling, we could present sectoral contributions of DIIT and DNT to growth in *total* manufacturing. These results are available on request.
chose the Australian experience over these periods to illustrate the various IIT-related measures: (i) manufacturing trade as a share of GDP rose from 14 to 15 percent between 1981 and 1986, and then more rapidly to 17 percent by 1991; (ii) protection levels in the manufacturing sector rose between 1981 and 1986, but then fell markedly between 1986 and 1991; and (iii) the Closer Economic Relations trading agreement with New Zealand, one of the most comprehensive agreements in the world, was signed in 1983 and further expanded in 1988.

Over the period 1981 to 1986, the average GL indexes fell in all 1-digit sectors and in total manufacturing. The opposite was true for the period 1986 to 1991. What this indicates is that over the first period TT grew at a faster rate than IIT, whereas in the second period IIT grew faster than TT. However, what the GL indexes cannot tell us for either period is how important IIT growth was in TT growth. This information is provided by $\text{Ciit}$. In the first period, trade growth was overwhelmingly NT. This occurred because most of the growth in trade was import growth in net import industries.$^{10}$ Overall, the contribution of IIT to growth in TT was 7.45 percent (i.e. 2.21 out of 29.68). At the 1-digit level, the greatest percentage contribution of IIT to TT growth was for Chemicals, 15.27 percent (3.82 out of 25.01).

In the second period, IIT was much more important in TT growth than in the first. Growth in IIT contributed more than half the growth in TT for total manufacturing (19.97 out of 39.17). At the 1-digit level, IIT contributed more to the growth in TT in 3 out of the 4 sectors. The increase in the importance of IIT during this period occurred because of strong export growth in net import industries.

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$^{10}$ Formulas that measure the contributions of growth in imports and exports to the growth in total, intra-industry and net trade are presented in Menon and Dixon (1994).
### Table 1: GL Indexes and Contributions Measures, 1981 to 1986

<table>
<thead>
<tr>
<th>Product Description</th>
<th>100.GL(j)(81)</th>
<th>100.GL(j)(86)</th>
<th>tt(j)</th>
<th>Ciit(j)</th>
<th>Cnt(j)</th>
<th>tt(j)</th>
<th>Cdiit(j)</th>
<th>Cdnit(j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SITC 5 Chemicals</td>
<td>32.82</td>
<td>29.31</td>
<td>25.01</td>
<td>3.82</td>
<td>21.19</td>
<td>25.01</td>
<td>-8.04</td>
<td>33.06</td>
</tr>
<tr>
<td>SITC 6 Materials</td>
<td>28.89</td>
<td>26.83</td>
<td>20.02</td>
<td>3.31</td>
<td>16.72</td>
<td>20.02</td>
<td>-0.08</td>
<td>20.10</td>
</tr>
<tr>
<td>SITC 7 Machinery, transport equip.</td>
<td>19.76</td>
<td>15.80</td>
<td>25.83</td>
<td>0.12</td>
<td>25.71</td>
<td>25.83</td>
<td>-14.25</td>
<td>40.09</td>
</tr>
<tr>
<td>SITC 8 Miscellaneous</td>
<td>23.41</td>
<td>19.02</td>
<td>57.06</td>
<td>6.46</td>
<td>50.60</td>
<td>57.06</td>
<td>5.32</td>
<td>51.74</td>
</tr>
<tr>
<td>Total manufacturing</td>
<td>23.69</td>
<td>19.97</td>
<td>29.68</td>
<td>2.21</td>
<td>27.47</td>
<td>29.68</td>
<td>-7.59</td>
<td>37.27</td>
</tr>
</tbody>
</table>

### Table 2: GL Indexes and Contributions Measures, 1986 to 1991

<table>
<thead>
<tr>
<th>Product Description</th>
<th>100.GL(j)(86)</th>
<th>100.GL(j)(91)</th>
<th>tt(j)</th>
<th>Ciit(j)</th>
<th>Cnt(j)</th>
<th>tt(j)</th>
<th>Cdiit(j)</th>
<th>Cdnit(j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SITC 5 Chemicals</td>
<td>29.31</td>
<td>37.67</td>
<td>44.06</td>
<td>24.95</td>
<td>19.11</td>
<td>44.06</td>
<td>5.13</td>
<td>38.93</td>
</tr>
<tr>
<td>SITC 6 Materials</td>
<td>26.83</td>
<td>30.22</td>
<td>31.18</td>
<td>12.82</td>
<td>18.36</td>
<td>31.18</td>
<td>4.56</td>
<td>26.62</td>
</tr>
<tr>
<td>SITC 7 Machinery, transport equip.</td>
<td>15.80</td>
<td>27.15</td>
<td>45.72</td>
<td>23.75</td>
<td>21.96</td>
<td>45.72</td>
<td>12.82</td>
<td>32.90</td>
</tr>
</tbody>
</table>

**Notes:**

Using these equations, we obtain:

\[
\begin{align*}
tt(j) & = \sum_{i} tt(\frac{TT}{TT(j)}) \\
nit(j) & = \sum_{i} nt(\frac{NT}{NT(j)}) \\
Cn(j) & = GL(j) - nt(j) \\
Cdiit(j) & = 100 \left( \frac{DIIT(j)}{TT(j)} \right) \\
Cdnit(j) & = 100 \left( \frac{DNT(j)}{TT(j)} \right)
\end{align*}
\]

(1) In all the formulas below, the \(s(j)\)'s are sets of products. For example, in the first row of each table, \(j = \text{SITC 5, Chemicals}\). To obtain these sectoral aggregates, we begin by defining the following:

\[
\begin{align*}
TT(j) & = \sum_{i} TT(i) \\
IIT(j) & = \sum_{i} IIT(i) \\
NT(j) & = \sum_{i} NT(i) \\
GL(j) & = \sum_{i} GL \left( \frac{TT(i)}{TT(j)} \right) \\
DII(j) & = \sum_{i} DIII(i) \\
DNI(j) & = \sum_{i} DNII(i)
\end{align*}
\]
In Section 2 we showed, as a theoretical possibility, that movements in GL indexes can be misleading when used to make inferences about the importance of IIT in trade growth. In Tables 1 and 2, we see that most of the GL movements point in the right direction. Nevertheless, consistent with our theory, there are exceptions. For example, in Table 2, the GL index rises for Materials, yet the contribution of IIT to TT growth is only about two thirds of that of NT (12.82 compared with 18.36). At the 133 industry level, there were 18 cases in the first period in which the GL index increased despite $C_{iit}$ being less than $C_{nt}$ and 41 such cases in the second period. In both periods, there were 2 cases of the GL index falling despite $C_{iit}$ being greater than $C_{nt}$.

How reliable is $C_{iit}$ as an indicator of factor market disruption? In Section 2, we showed theoretically that $C_{iit}$ may overestimate the contribution to total trade growth of non-disruptive trade growth. Our preferred indicator is $C_{diit}$. In Tables 1 and 2, we see that $C_{iit}$ does indeed exceed $C_{diit}$ by a considerable margin in nearly all cases. For total manufacturing in the first period, $C_{diit}$ is -7.59 percent compared with 2.21 percent for $C_{iit}$. In the second period, the difference between $C_{diit}$ and $C_{iit}$ for total manufacturing is even greater.

The large difference between $C_{diit}$ and $C_{iit}$ in Tables 1 and 2 arise because the strict inequality conditions, (18) and (19), often apply. In the first period, they applied to 30 industries accounting in 1981 for 21 percent of total trade. Over the period, 27 of these 30 industries experienced greater export growth than import growth ($\Delta X_i > \Delta M_i$) despite starting the period as net importers ($M_i > X_i$). The remaining 3 industries were net exporters ($X_i > M_i$) which experienced greater growth in imports than exports ($\Delta M_i > \Delta X_i$).

In the second period, conditions (18) and (19) applied to 45 industries which in 1986 accounted for 34 percent of total trade. Over the period, 44 of these 45 industries experienced greater export growth than import growth ($\Delta X_i > \Delta M_i$) despite starting the period as net importers ($M_i > X_i$). The remaining industry was a net exporter ($X_i > M_i$) which experienced greater growth in imports than in exports ($\Delta M_i > \Delta X_i$).
4. Concluding Remarks

There have been numerous studies in which IIT-related concepts have been used as indicators of the extent to which trade growth can be accommodated without factor market disruption. The most commonly used indicators have been movements over time in the GL index. However, as illustrated in Section 3, GL-based indicators are sometimes misleading and, at best, can give qualitative information only.

In this paper, we develop two other indicators. The first involves computing changes in IIT. While this method provides a precise measure \( \Delta \text{IIT} \) of the contribution of growth in IIT to TT growth, it too can be misleading when the aim is to discuss adjustment pressures associated with trade growth. In general, \( \Delta \text{IIT} \) will tend to overestimate the contribution of non-disruptive trade growth. This bias is overcome by our indicator, \( \Delta \text{DIIT} \), which is the contribution to growth in trade of dynamic intra-industry trade (matched changes in trade). In the Australian case, we found that the bias in \( \Delta \text{IIT} \) is considerable.

Although we think that \( \Delta \text{DIIT} \) is superior to other indicators of non-disruptive trade growth, we should emphasise that our argument is theoretical. In common with other writers in this area, we have not provided empirical evidence linking presumed indicators of non-disruptive trade growth with estimates of factor market disruption. This would be a major task involving the construction of a model containing detailed estimates of the costs of factor transfers between industries, regions and occupations. With such a model, we could simulate the effects of trade liberalisation, regional trading agreements or other shocks affecting trade growth. Then we could correlate movements implied by the model for indicators of non-disruptive trade with the model’s estimates of the costs of disruption. However, until we have a model of suitable detail and empirical content, we must make do with theoretical justifications of our indicators.
Appendix

Proposition:

\[ C_{it} \geq C_{dit} , \]

that is

\[ \min \{ X(1), M(1) \} - \min \{ X(0), M(0) \} \geq \min \{ \Delta X, \Delta M \} \quad (A1) \]

where the arguments 0 and 1 indicate the initial and final years of the period under consideration. (For convenience, we omit the product subscript \( i \).)

Proof:

Case 1: Assume \( X(0) \geq M(0) , \quad X(1) \geq M(1) . \)

In this case, \( \text{L.H.S.}( A1 ) = M(1) - M(0) = \Delta M . \)

Thus, \( \text{L.H.S.}( A1 ) \geq \text{R.H.S.}( A1 ) \)

with the strict inequality applying when \( \Delta M > \Delta X . \)

Case 2: Assume \( X(0) \geq M(0) , \quad X(1) < M(1) . \)

\[ \text{L.H.S.}( A1 ) = X(1) - M(0) \]

\[ = X(1) - X(0) + \{ X(0) - M(0) \} \geq \Delta X \]

\[ \text{R.H.S.}( A1 ) = \Delta X \]

Thus, \( \text{L.H.S.}( A1 ) \geq \text{R.H.S.}( A1 ) . \)

(Notice in case 2 that \( \Delta M > \Delta X \) and that \( \text{L.H.S.}( A1 ) > \text{R.H.S.}( A1 ) \) if \( X(0) > M(0) \).)

The other two possibilities are

\[ M(0) \geq X(0) , \quad M(1) \geq X(1) , \]

and

\[ M(0) \geq X(0) , \quad M(1) < X(1) . \]

These two cases are similar to cases 1 and 2 with the roles of \( X \) and \( M \) interchanged.
This not only completes the proof of our proposition, but we can also see that

\[ \text{L.H.S.}(A1) > \text{R.H.S.}(A1) \]

if either \( X(0) > M(0) \) and \( \Delta M > \Delta X \)

or \( M(0) > X(0) \) and \( \Delta X > \Delta M \).
References


