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***Abstract*— This paper deals with the problem of robust fault detection for discrete-time switched systems with state delays under arbitrary switching signal. The fault detection filter is used as the residual generator, in which the filter parameters are dependent on the system mode. Attention is focused on designing the robust fault detection filter such that, for unknown inputs, control inputs and model uncertainties, the estimation error between the residuals and the faults is minimized. The problem of robust fault detection is converted into an H_∞ filtering problem. By switched Lyapunov functional approach, a sufficient condition for the solvability of this problem is established in terms of linear matrix inequalities (LMIs). A numerical example is provided to demonstrate the effectiveness of the proposed method.**

***Index Terms*— Fault Detection and Isolation, Switched Systems, LMIs, State Delays.**

I. Introduction

Fault Detection and Isolation (FDI) in dynamic systems has attracted great attention of many researchers over the past decades, and some model-based fault detection approaches have been proposed (see, e.g. [1]–[6] and the references therein). Among these model-based approaches, the most common one is to use state observers or filters to construct a residual signal and compare it with a predefined threshold. When the residual evaluation function has a value larger than the threshold, an alarm is generated. It is well known that unknown inputs, control inputs, model uncertainties and the faults are coupled in many industrial systems, which is a source of false alarms. This means that FDI systems have to be sensitive to faults and simultaneously robust to unknown inputs, control inputs and model uncertainties. Therefore, it is of great significance to design a robust FDI system [7]–[10]. Recently, an H_∞-filtering formulation of FDI problem has been presented to solve the robust FDI problem [11]–[13]. In [11], the problem of Robust Fault Detection Filter Design (RFDFD) for discrete-time Markovain jump linear systems is formulated as an H_∞-filtering problem. In [13], the problem of RFDFD for discrete-time networked systems with multiple state delays and unknown input is transformed into an H_∞-filtering problem for Markovain jumping system. In this paper, the problem of RFDFD for discrete-time switched systems with state delays is cast into an H_∞ filtering problem.

On another research front line, there has been increasing interest in the control problems of switched systems due to their significance both in theory and applications (see, for instance, [14]–[25] and the references therein). Many methods have been developed in the study of switched systems such as multiple Lyapunov functions approach [23], average dwell time technology [24], switched Lyapunov function approach [25] and so on. In [25], a switched Lyapunov function approach is proposed for stability analysis for discrete-time switched systems, but time delays are not considered. In this paper, state delays are involved. However, the existence of time delays in a system may cause instability or bad system performance [26]–[30]. Hence, switched systems with time delay have gained a great deal of attention (see, for example, [19]–[22], [31]–[34] and the references therein). However, to the best of the authors' knowledge, the problem of RFDFD for discrete-time switched systems with state delays has not been investigated yet. This motivates us to study this interesting and challenging problem, which has great potential in practical applications.

This paper deals with the problem of RFDFD by using switched Lyapunov functional approach for discrete-time switched systems with state delays. Firstly, the residual generator is constructed based on the filter. Secondly, by augmenting the states of the original system and the fault detection filter, the problem of RFDFD is formulated as an H_∞ filtering problem. The objective is to make the difference between the faults and the residuals as small as possible, and increase robustness of the residuals to the unknown input. Then, by using switched Lyapunov functional approach, a sufficient condition on the existence of such filters is established in terms of LMIs. The desired filter are constructed by solving the corresponding LMIs. Finally, a simulation example is presented to demonstrate the effectiveness of the proposed method.

The rest of this paper is organized as follows. In Section 2, system descriptions and definitions are presented. A sufficient condition on the existence of a robust fault detection filter for discrete-time switched systems with state delays is presented in terms of LMIs, and the desired filter are constructed in Section 3. To demonstrate the validity of the proposed approach, an example is given in Section 4 which is followed by a conclusion in Section 5.

II. Problem formulation

Consider the following discrete-time switched systems with state delays:

$$x_{k+1} = \sum_{i=1}^{N} \xi_i\left(k\right)\left(A_i x_k + A_{di} x_{k-d} + E_i u_k + B_i d_k + G_i f_k\right)$$
$$y_k = \sum_{i=1}^{N} \xi_i\left(k\right)\left(C_i x_k + C_{di} x_{k-d} + Q_i u_k + D_i d_k + J_i f_k\right) \quad (1)$$

where $x_k \in R^n$ is the state, $y_k \in R^r$ is the measured output, $d_k \in R^p$, $u_k \in R^s$ and $f_k \in R^q$ are, respectively, the unknown input, the control input and the faults which belong to $l_2\left[0,\infty\right)$. The matrices $A_i,\ A_{di},\ B_i,\ C_i,\ C_{di},\ D_i,\ E_i,\ G_i,\ J_i,\ Q_i$ are of the appropriate dimensions, where $A_i = \bar{A}_i + \Delta A_i(k), A_{di} = \bar{A}_{di} + \Delta A_{di}(k), E_i = \bar{E}_i + \Delta E_i(k)$. The modeling errors $\Delta A_i\left(k\right), \Delta A_{di}\left(k\right), \Delta E_i\left(k\right)$ are norm-bounded uncertainties satisfying

$$\begin{aligned} &\left[\begin{array}{ccc} \Delta A_i\left(k\right) & \Delta A_{di}\left(k\right) & \Delta E_i\left(k\right) \end{array}\right] \\ &= \bar{H}_i \bar{F}\left(k\right)\left[\begin{array}{ccc} \bar{C}_{1i} & \bar{C}_{2i} & \bar{C}_{3i} \end{array}\right] \end{aligned}$$

where $\bar{H}_i, \bar{C}_{1i}, \bar{C}_{2i}, \bar{C}_{3i}$ are known constant matrices, while $\bar{F}(k)$ is unknown time-varying matrices satisfying $\bar{F}^T\left(k\right)\bar{F}\left(k\right) \le I$. The positive integers N and d denote the number of subsystems and the state delay, respectively. $\xi_i\left(k\right)\ :\ Z^+ \to \{0,1\}$ and $\sum\limits_{i=1}^{N} \xi_i\left(k\right) =$

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$1, k \in Z^+, i \in \mathcal{N} = \{1, 2, \cdots, N\}$ is the switching signal which specifies which subsystem is activated at the switching instant.

An FDI system consists of a residual generator and a residual evaluation stage including an evaluation function and a threshold. For the purpose of residual generation, the following fault detection filter is constructed as a residual generator:

$$\begin{aligned} \hat{x}_{k+1} &= \sum_{i=1}^{N} \xi_i(k)\left(A_{fi}\hat{x}_k + B_{fi}y_k\right) \\ r_k &= \sum_{i=1}^{N} \xi_i(k)\left(C_{fi}\hat{x}_k + D_{fi}y_k\right) \end{aligned} \quad (2)$$

where $\hat{x}_k$ is the filter's state, r_k is the residual signal. The matrices A_{fi}, B_{fi}, C_{fi} and D_{fi} are the filter parameters to be determined.

For the purpose of fault detection, it is not necessary to estimate the fault f_k. Sometimes one is more interested in the fault signal of a certain frequency interval, which can be formulated as the weighted fault $\hat{f}(z) = W_f(z)f(z)$ with $W_f(z)$ being a given stable weighting matrix. A minimal realization of $\hat{f}(z) = W_f(z)f(z)$ is supposed to be

$$\begin{aligned} \bar{x}_{k+1} &= A_w\bar{x}_k + B_w f_k \\ \hat{f}_k &= C_w\bar{x}_k + D_w f_k \end{aligned} \quad (3)$$

where $\bar{x}_k \in \mathcal{R}^{\bar{n}}$ is the state of the weighted fault, $f_k \in R^q$ is the original fault and $\hat{f}_k \in R^q$ is the weighted fault. A_w, B_w, C_w and D_w are known constant matrices.

Remark 1: The parameters of filter (2) depend on the system modes. This means that the switching signals in system (1) and filter (2) are the same. In practice, each subsystem of filter (2) is designed for the corresponding subsystem of system (1). Then, each pair of subsystems can be actived by one signal. As in [32], [25], it is assumed that the switching signal is not known a priori, but its instantaneous value is available in real-time implementation. When the swithing signal changes, the jumps between the modes occur simultaneously and in pairs.

Remark 2: Similar to [11], [13], the introduction of $W_f(z)$ could limit the frequency ranges of interest, but the system performance could be improved and the frequency characteristics required to reflect the emphases of different frequency ranges could be captured. Denoting $e_k = r_k - \hat{f}_k$ and augmenting the model of system (1) to include the states of (2), we can obtain the augmented system as follows:

$$\begin{aligned} \tilde{x}_{k+1} &= \sum_{i=1}^{N} \xi_i(k)\left(\tilde{A}_i\tilde{x}_k + \tilde{A}_{di}\tilde{x}_{k-d} + \tilde{B}_i w_k\right) \\ e_k &= \sum_{i=1}^{N} \xi_i(k)\left(\tilde{C}_i\tilde{x}_k + \tilde{C}_{di}\tilde{x}_{k-d} + \tilde{D}_i w_k\right) \end{aligned} \quad (4)$$

where

$$\begin{aligned} \tilde{A}_i &= \begin{bmatrix} A_i & 0 & 0 \\ B_{fi}C_i & A_{fi} & 0 \\ 0 & 0 & A_w \end{bmatrix}, \tilde{x}_k = \begin{bmatrix} x_k \\ \hat{x}_k \\ \bar{x}_k \end{bmatrix}, \\ \tilde{A}_{di} &= \begin{bmatrix} A_{di} & 0 & 0 \\ B_{fi}C_{di} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, w_k = \begin{bmatrix} u_k \\ d_k \\ f_k \end{bmatrix}, \\ \tilde{B}_i &= \begin{bmatrix} E_i & B_i & G_i \\ B_{fi}Q_i & B_{fi}D_i & B_{fi}J_i \\ 0 & 0 & B_w \end{bmatrix}, \\ \tilde{C}_i &= \begin{bmatrix} D_{fi}C_i & C_{fi} & -C_w \end{bmatrix}, \\ \tilde{C}_{di} &= \begin{bmatrix} D_{fi}C_{di} & 0 & 0 \end{bmatrix}, \\ \tilde{D}_i &= \begin{bmatrix} D_{fi}Q_i & D_{fi}D_i & D_{fi}J_i - D_w \end{bmatrix}. \end{aligned}$$

Remark 3: It should be noted from $e_k = r_k - \hat{f}_k$ that the residual r_k generated by filter (2) provides an estimate of the fault $\hat{f}_k$. The stable weighting matrix $W_f(z)$ is given. Thus, detection and isolation of the fault f_k can be achieved by examining the values of the residual r_k. That is, the designed filter not only detects the occurred fault, but also can isolate it.

Now, the problem of RFDFD can be transformed into an H_∞ filtering problem for system (4): to develop filter (2) for system (1) such that the augmented system (4) under arbitrary switching signal is asymptotically stable when $w_k = 0$ and, under zero-initial condition, the infimum of γ is made small in the feasibility of

$$\sup_{w_k \neq 0, w_k \in l_2[0,\infty)} \frac{\|e_k\|_2}{\|w_k\|_2} < \gamma, \gamma > 0 \quad (5)$$

After designing the residual generator, the remaining important task is to evaluate the generated residual. One of the widely adopted approaches is to select a threshold and a residual evaluation function. In this paper, the residual evaluation function is chosen as

$$J_L(r) = \|r_k\|_{2,L} = \left(\sum_{k=l_0}^{l_0+L} r_k^T r_k\right)^{1/2} \quad (6)$$

where l_0 is the initial evaluation time instant, L is the evaluation time window.

Remark 4: In fact, the length of the evaluation window L is limit since it is desired that the faults will be detected as early as possible, while an evaluation of residual signal over the whole time range is not practical. This point has been mentioned in [**?**], [10]–[12].
Once the evaluation function has been selected, we are able to determine the threshold. Since the faults can be detected by using the following logical relationship:

$$\begin{aligned} &J_L(r) > J_{th} \Rightarrow Faults \Rightarrow Alarm \\ &J_L(r) \leq J_{th} \Rightarrow No\ Fault \end{aligned}$$

it is reasonable to choose the threshold as

$$J_{th} = \sup_{d \in l_2,\ u \in l_2,\ f=0} \|r_k\|_{2,L}. \quad (7)$$

It is clear that the computation of J_{th} involves the determination of the unknown inputs d_k and the control inputs u_k on the residuals r_k.

III. H_∞ Fault Detection Filter Design

In this section, a sufficient condition on the existence of the robust fault detection filters would be given and a desired filer could be constructed.

Lemma 1 [35]: Let Y, H and C be matrices with appropriate dimensions. Supppose that Y is symmetric and $\Delta(k)^T\Delta(k) \leq I$, then,

$$Y + H\Delta(k)C + C^T\Delta(k)H < 0$$

if and only if

$$Y + \varepsilon^{-1}HH^T + \varepsilon C^T C < 0$$

where ε is a given positive scalar.

Lemma 2: For a given scalar $\gamma > 0$, system (4) under arbitrary switching signals is asymptotically stable when $w_k = 0$ and, under zero-initial conditions, guarantees the performance index (5) for all nonzero $w_k \in l_2[0,\infty)$, if there exist the positive definite symmetric matrices P_i and Q_i, $i \in \mathcal{N}$ such that the following inequality holds,

$$\begin{bmatrix} -P_j^{-1} & \tilde{A}_i & \tilde{A}_{di} & \tilde{B}_i & 0 & 0 \\ * & -P_i & 0 & 0 & \tilde{C}_i^T & I \\ * & * & -Q_l & 0 & \tilde{C}_{di}^T & 0 \\ * & * & * & -\gamma^2 I & \tilde{D}_i^T & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -Q_i^{-1} \end{bmatrix} < 0 \tag{8}$$

Proof. First, the asymptotical stability of system (4) with $w_k = 0$ is established. The following Lyapunov functional is constructed by:

$$\begin{aligned} V_k &= \tilde{x}_k^T \left(\sum_{i=1}^{N} \xi_i(k) P_i \right) \tilde{x}_k \\ &+ \sum_{s=k-d}^{k-1} \tilde{x}_s^T \left(\sum_{i=1}^{N} \xi_i(k) Q_i \right) \tilde{x}_s \end{aligned} \tag{9}$$

where P_i and Q_i are the positive define symmetric matrices. Then, we get

$$\begin{aligned} \Delta V_k &= V_{k+1} - V_k \\ &= \tilde{x}_{k+1}^T \left(\sum_{i=1}^{N} \xi_i(k+1) P_i \right) \tilde{x}_{k+1} \\ &- \tilde{x}_k^T \left(\sum_{i=1}^{N} \xi_i(k) P_i \right) \tilde{x}_k + \tilde{x}_k^T \left(\sum_{i=1}^{N} \xi_i(k) Q_i \right) \tilde{x}_k \\ &- \tilde{x}_{k-d}^T \left(\sum_{i=1}^{N} \xi_i(k-d) Q_i \right) \tilde{x}_{k-d} \end{aligned} \tag{10}$$

As this has to be satisfied under arbitrary switching signals, it follows that this has to hold for the configuration $\xi_i(k) = 1$, $\xi_{r\neq i}(k) = 0$, $\xi_j(k+1) = 1$, $\xi_{r\neq j}(k+1) = 0$, $\xi_l(k-d) = 1$ and $\xi_{r\neq l}(k-d) = 0$, along the solution of system (4) with $w_k = 0$, we have

$$\Delta V_k|_{w_k=0} = \tilde{\eta}_k^T \Lambda_i \tilde{\eta}_k \tag{11}$$

where

$$\begin{aligned} \tilde{\eta}_k^T &= \begin{bmatrix} \tilde{x}_k^T & \tilde{x}_{k-d}^T \end{bmatrix}, \\ \Lambda_i &= \begin{bmatrix} \tilde{A}_i^T \\ \tilde{A}_{di}^T \end{bmatrix} P_j \begin{bmatrix} \tilde{A}_i & \tilde{A}_{di} \end{bmatrix} + \begin{bmatrix} Q_i - P_i & 0 \\ * & -Q_l \end{bmatrix}. \end{aligned}$$

By using Schur complement, it follows from (8) that $\Lambda_i < 0$. Thus, from (11) we have $\Delta V_k|_{w_k=0} < -\rho \|x_k\|^2$ for a sufficiently small $\rho > 0$ and $x_k \neq 0$, which establishes the asymptotical stability of system (4).

Secondly, we consider the following performance index:

$$J = \sum_{k=0}^{K-1} \left[e_k^T e_k - \gamma^2 w_k^T w_k \right] \tag{12}$$

where K is an arbitrary positive integer. For any nonzero $w_k \in l_2[0,\infty)$ and under zero-initial condition $\tilde{x}_0 = 0$, one has

$$\begin{aligned} J &= \sum_{k=0}^{K-1} \left[e_k^T e_k - \gamma^2 w_k^T w_k + \Delta V_k|_{(4)} \right] - V_K \\ &\leq \sum_{k=0}^{K-1} \left[e_k^T e_k - \gamma^2 w_k^T w_k + \Delta V_k|_{(4)} \right] \end{aligned} \tag{13}$$

where $\Delta V_k|_{(4)}$ defines ΔV_k along the solution of system (4). It is noted that

$$e_k^T e_k - \gamma^2 w_k^T w_k + \Delta V_k|_{(4)} = \eta_k^T \Pi \eta_k \tag{14}$$

where

$$\begin{aligned} \Pi &= \begin{bmatrix} \tilde{A}_i^T \\ \tilde{A}_{di}^T \\ \tilde{B}_i^T \end{bmatrix} P_j \begin{bmatrix} \tilde{A}_i & \tilde{A}_{di} & \tilde{B}_i \end{bmatrix} \\ &+ \begin{bmatrix} -P_i + Q_i & 0 & 0 \\ * & -Q_l & 0 \\ * & * & -\gamma^2 I \end{bmatrix} \\ &+ \begin{bmatrix} \tilde{C}_i^T \\ \tilde{C}_{di}^T \\ \tilde{D}_i^T \end{bmatrix} \begin{bmatrix} \tilde{C}_i & \tilde{C}_{di} & \tilde{D}_i \end{bmatrix}, \ \eta_k = \begin{bmatrix} \tilde{x}_k \\ \tilde{x}_{k-d} \\ w_k \end{bmatrix} \end{aligned}$$

It follows from (8) and Schur complement that $\Pi < 0$, which implies $J < 0$. Then, one has that for any nonzero $w_k \in l_2[0,\infty)$, $\|e_k\|_2 < \gamma \|w_k\|_2$, which completes the proof.

Lemma 3: For a given scalar $\gamma > 0$ and the augmented system (4), LMI (8) is feasible, if there exist the positive definite symmetric matrices R_i, Φ_i, and matrices Ω_i, $i \in \mathcal{N}$ such that the following inequality holds,

$$\begin{bmatrix} -R_j & \tilde{A}_i\Omega_i & \tilde{A}_{di}\Omega_i & \tilde{B}_i & 0 & 0 \\ * & \bar{R}_i & 0 & 0 & \Omega_i^T \tilde{C}_i^T & \Omega_i^T \\ * & * & \bar{\Phi}_l & 0 & \Omega_i^T \tilde{C}_{di}^T & 0 \\ * & * & * & -\gamma^2 I & \tilde{D}_i^T & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\Phi_i \end{bmatrix} < 0. \tag{15}$$

where $\bar{R}_i = R_i - \left(\Omega_i^T + \Omega_i\right)$ and $\bar{\Phi}_l = \Phi_l - \left(\Omega_i^T + \Omega_i\right)$.

Proof. Assume that (15) is feasible, then it is easy to see that $R_i - \left(\Omega_i^T + \Omega_i\right) < 0$ which means that Ω_i is nonsingular. Since $R_i > 0$, we have $\left(R_i - \Omega_i\right)^T R_i^{-1} \left(R_i - \Omega_i\right) \geq 0$ which implies $-\Omega_i^T R_i^{-1} \Omega_i \leq R_i - \left(\Omega_i + \Omega_i^T\right)$. Similarly, one has $-\Omega_i^T \Phi_l^{-1} \Omega_i \leq \Phi_l - \left(\Omega_i + \Omega_i^T\right)$. Then, (15) is transformed into

$$\begin{bmatrix} -R_j & \tilde{A}_i\Omega_i & \tilde{A}_{di}\Omega_i & \tilde{B}_i & 0 & 0 \\ * & \hat{R}_i^{-1} & 0 & 0 & \Omega_i^T \tilde{C}_i^T & \Omega_i^T \\ * & * & \hat{\Phi}_l^{-1} & 0 & \Omega_i^T \tilde{C}_{di}^T & 0 \\ * & * & * & -\gamma^2 I & \tilde{D}_i^T & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\Phi_i \end{bmatrix} < 0 \tag{16}$$

where $\hat{R}_i^{-1} = -\Omega_i^T R_i^{-1} \Omega_i$ and $\hat{\Phi}_l^{-1} = -\Omega_i^T \Phi_l^{-1} \Omega_i$. Premultiplying $diag\left\{I, \Omega_i^{-T}, \Omega_i^{-T}, I, I, I\right\}$ and postmultiplying $diag\left\{I, \Omega_i^{-1}, \Omega_i^{-1}, I, I, I\right\}$ to (16) yields

$$\begin{bmatrix} -R_j & \tilde{A}_i & \tilde{A}_{di} & \tilde{B}_i & 0 & 0 \\ * & -R_i^{-1} & 0 & 0 & \tilde{C}_i^T & I \\ * & * & -\Phi_l^{-1} & 0 & \tilde{C}_{di}^T & 0 \\ * & * & * & -\gamma^2 I & \tilde{D}_i^T & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\Phi_i \end{bmatrix} < 0 \tag{17}$$

Letting $R_i = P_i^{-1}$ and $\Phi_i = Q_i^{-1}$, by means of Schur Complement, we can see that (17) is equivalent to (8). The proof is completed.

Remark 5: We can see from (8) in Lemma 2 that it is difficult to deal with the problem of RFDFD due to the existence of P_j^{-1}, which leads to some product terms between P_j and $\tilde{A}_i$, $\tilde{A}_{di}$ and $\tilde{B}_i$. To overcome the difficulties, an auxiliary slack matrix Ω_i is introduced in Lemma 3 such that these product terms are decoupled. That is, R_i and R_j are not involved in any product with $\tilde{A}_i$, $\tilde{A}_{di}$ and $\tilde{B}_i$ in (15). This implies it is more tractable to cope with the problem of RFDFD.

Now, we will present our main results in this paper as follows.

Theorem 1: For a given scalar $\gamma > 0$, the problem of RFDFD for system (1) is solvable, if there exist the positive definite symmetric matrices R_{1i}, R_{4i}, R_{6i}, X_{1i}, X_{4i}, X_{6i}, and metrics R_{2i}, R_{3i}, R_{5i}, X_{2i}, X_{3i}, X_{5i}, Z_i, Y_i, W_i, H_i, M_i, L_i, N_i, S_i and a scalar $\varepsilon > 0$, $i \in \mathcal{N}$ such that the following LMI holds,

$$\begin{bmatrix} -\Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & 0 & 0 & \Psi_{17} \\ * & \hat{\Psi}_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25}^T & \Psi_{26}^T & 0 \\ * & * & \hat{\Psi}_{33} & \Psi_{34} & \Psi_{35}^T & 0 & 0 \\ * & * & * & \hat{\Psi}_{44} & \Psi_{45}^T & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\Psi_{66} & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \quad (18)$$

where

$$\begin{aligned}
&\hat{\Psi}_{22} = \Psi_{22} - \Psi_{26} - \Psi_{26}^T, \hat{\Psi}_{33} = \Psi_{33} - \Psi_{26} - \Psi_{26}^T,\\
&\hat{\Psi}_{44} = -\gamma^2 I + \varepsilon C_{3i}^T C_{3i}, \hat{Y}_i = Y_i \bar{A}_i + H_i C_i,\\
&\hat{Y}_{di} = Y_i \bar{A}_{di} + H_i C_{di}, Y_{ei} = Y_i \bar{E}_i + H_i Q_i,\\
&Y_{bi} = Y_i B_i + H_i D_i, Y_{gi} = Y_i G_i + H_i J_i,\\
&\bar{C}_{12i} = \varepsilon \bar{C}_{1i}^T \bar{C}_{2i}, \bar{C}_{13i} = \varepsilon \bar{C}_{1i}^T \bar{C}_{3i}, \bar{C}_{23i} = \varepsilon \bar{C}_{2i}^T \bar{C}_{3i},\\
&\Psi_{11} = \begin{bmatrix} R_{1j} & R_{2j} & R_{3j} \\ * & R_{4j} & R_{5j} \\ * & * & R_{6j} \end{bmatrix}, \Psi_{66} = \begin{bmatrix} X_{1i} & X_{2i} & X_{3i} \\ * & X_{4i} & X_{5i} \\ * & * & X_{6i} \end{bmatrix},\\
&\Psi_{12} = \begin{bmatrix} Z_i \bar{A}_i & Z_i \bar{A}_i & 0 \\ \hat{Y}_i + L_i & \hat{Y}_i & 0 \\ 0 & 0 & W_i A_w \end{bmatrix}, \Psi_{17} = \begin{bmatrix} Z_i \bar{H}_i \\ Y_i \bar{H}_i \\ 0 \end{bmatrix},\\
&\Psi_{13} = \begin{bmatrix} Z_i \bar{A}_{di} & Z_i \bar{A}_{di} & 0 \\ \hat{Y}_{di} & \hat{Y}_{di} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \Psi_{23} = \begin{bmatrix} \bar{C}_{12i} & \bar{C}_{12i} & 0 \\ \bar{C}_{12i} & \bar{C}_{12i} & 0 \\ 0 & 0 & 0 \end{bmatrix},\\
&\Psi_{14} = \begin{bmatrix} Z_i \bar{E}_i & Z_i B_i & Z_i G_i \\ Y_{ei} & Y_{bi} & Y_{gi} \\ 0 & 0 & W_i B_w \end{bmatrix}, \Psi_{24} = \begin{bmatrix} \bar{C}_{13i} & 0 & 0 \\ \bar{C}_{13i} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\\
&\Psi_{22} = \begin{bmatrix} R_{1i} + \varepsilon \bar{C}_{1i}^T \bar{C}_{1i} & R_{2i} + \varepsilon \bar{C}_{1i}^T \bar{C}_{1i} & R_{3i} \\ * & R_{4i} + \varepsilon \bar{C}_{1i}^T \bar{C}_{1i} & R_{5i} \\ * & * & R_{6i} \end{bmatrix},\\
&\Psi_{26} = \begin{bmatrix} Z_i & Z_i & 0 \\ Y_i + M_i & Y_i & 0 \\ 0 & 0 & W_i \end{bmatrix}, \Psi_{34} = \begin{bmatrix} \bar{C}_{23i} & 0 & 0 \\ \bar{C}_{23i} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\\
&\Psi_{33} = \begin{bmatrix} X_{1l} + \varepsilon \bar{C}_{2i}^T \bar{C}_{2i} & X_{2l} + \varepsilon \bar{C}_{2i}^T \bar{C}_{2i} & X_{3l} \\ * & X_{4l} + \varepsilon \bar{C}_{2i}^T \bar{C}_{2i} & X_{5l} \\ * & * & X_{6l} \end{bmatrix},\\
&\Psi_{25} = \begin{bmatrix} N_i C_i + S_i & N_i C_i & -C_w \end{bmatrix},\\
&\Psi_{35} = \begin{bmatrix} N_i C_{di} & N_i C_{di} & 0 \end{bmatrix},\\
&\Psi_{45} = \begin{bmatrix} N_i Q_i & N_i D_i & N_i J_i - D_w \end{bmatrix}.
\end{aligned}$$

then, a robust fault detection filter (2) can be constructed by

$$\begin{bmatrix} A_{fi} & B_{fi} \\ C_{fi} & D_{fi} \end{bmatrix} = \begin{bmatrix} V_i^{-1} L_i M_i^{-1} V_i & V_i^{-1} H_i \\ S_i M_i^{-1} V_i & N_i \end{bmatrix} \quad (19)$$

where $V_i \in \mathcal{R}^{n \times n}$ is any invertible matrix (for example, V_i could be set as I).

Proof. By Lemmas 2 and 3, the augmented system (4) under arbitrary switching signals is asymptotically stable when $w_k = 0$ and, under zero-initial conditions, guarantees (5) for all nonzero $w_k \in l_2[0, \infty)$, if LMI (15) holds.
Note that from (18), we have

$$\begin{bmatrix} Z_i + Z_i^T & Z_i + Y_i^T + M_i^T & 0 \\ * & Y_i + Y_i^T & 0 \\ * & * & W_i + W_i^T \end{bmatrix} > 0 \quad (20)$$

which means that Z_i, Y_i and W_i are nonsingular.
Premultiplying $\begin{bmatrix} I & -I & 0 \end{bmatrix}$ and postmultiplying $\begin{bmatrix} I & -I & 0 \end{bmatrix}^T$ to (20), one has $-M_i - M_i^T > 0$, which implies that M_i is nonsingular. And thus, if (18) holds, there exist nonsingular matrices V_i and U_i satisfying $M_i = V_i U_i$.

Now, introduce

$$F_i^T = \begin{bmatrix} Z_i & 0 & 0 \\ Y_i & V_i & 0 \\ 0 & 0 & W_i \end{bmatrix}, \ \Omega_i = \begin{bmatrix} I & I & 0 \\ U_i & 0 & 0 \\ 0 & 0 & I \end{bmatrix} F_i^{-1},$$

and define

$$\begin{aligned}
&H_i \triangleq V_i B_{fi}, \ L_i \triangleq V_i A_{fi} U_i, \ S_i \triangleq C_{fi} U_i, \ N_i \triangleq D_{fi},\\
&M_i \triangleq V_i U_i, R_j \triangleq F_i^{-T} \Psi_{11} F_i^{-1}, \ R_i \triangleq F_i^{-T} \Psi_{22} F_i^{-1},\\
&\Phi_l \triangleq F_i^{-T} \Psi_{33} F_i^{-1}, \ \Phi_i \triangleq F_i^{-T} \Psi_{66} F_i^{-1}.
\end{aligned} \quad (21)$$

By using (4), (18) and (21), we can get

$$\begin{aligned}
&F_i^T \tilde{A}_i \Omega_i F_i = \Psi_{12}, \ F_i^T \tilde{A}_{di} \Omega_i F_i = \Psi_{13}, \ F_i^T \tilde{B}_i = \Psi_{14},\\
&F_i^T \Omega_i F_i = \Psi_{26}, \ \tilde{C}_i \Omega_i F_1 = \Psi_{25}, \ \tilde{C}_{di} \Omega_i F_i = \Psi_{35}.
\end{aligned} \quad (22)$$

Then, by using (22), Lemma 1 and Schur complement, performing a congruence transformation to (15) via $diag\{F_i, F_i, F_i, I, I, F_i\}$ yields (18), which implies that (15) holds. Meanwhile, we know from (21) that the parameters of a admissible filter are given by (19). The proof is completed.

Remark 6. Note that in the derivation of [11] and [13], not only a block-diagonal Lyapunov matrix is used, but also it is required that a diagonal block is the same for different Markov modes. This can bring conservatism in the filter design. The main reasons for introducing such block-diagonal Lyapunov matrix rather than the symmetric Lyapunov matrix are:

1) Coupling the parameter matrices of the original systems and the filter with those of estimator of the fault can result from the existence of the non-diagonal block of the latter.
2) The former can make the filter design feasible by introducing less dimension of some auxiliary matrices than one of the latter.

However, the conservatism could be reduced by our proposed approach. That is, the symmetric Lyapunov matrix is introduced and those problems mentioned above are solved by using F_i^T and $\Omega_i F_i$. Thus, our approach could improve the results in [11] and [13].

Remark 7: Stability criteria for delayed systems can be classified into two categories: delay-independent and delay-dependent. As time delay is not considered during stability analysis, our results in this paper are delay-independent. That is, our results can be applicable to unknown value of time delay, or even time-varying delay. On the contrary, as time delay is taken into consideration in delay-dependent approach, stability result is less conservative comparatively, especially when the value of time delay is small. To obtain delay-dependent results would be one of our future topics.

Remark 8: For discrete-time systems with state delay, one can transform a delayed system into a delay-free system by using state augmentation methods. Stability of delay-free systems can be tested by employing classical results. But delay-free systems will become much complex and thus difficult to analyze with the increase of the size of delays. Moreover, such method is not applicable to time-varying case due to the limitation of available tools. On the contrary, Lyapunov approaches can be particularly good to deal with discrete-time systems with time-varying delay.

Remark 9: Based on [36], it is possible to extend the main results to uncertain stochastic systems with missing measurements.

IV. Illustrative example

In this section, an example is provided to illustrate the effectiveness of the proposed method.

Consider discrete-time switched system (1) consisting of two subsystems with parameters:

$$\begin{aligned}
A_1 &= \begin{bmatrix} 0.2 & -0.1 \\ 0 & 0.4 \end{bmatrix}, & A_{d1} &= \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.3 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}^T, & E_1 &= \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}^T, \\
G_1 &= \begin{bmatrix} 1.3 & 1.6 \end{bmatrix}^T, & C_1 &= \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \\
C_{d1} &= \begin{bmatrix} 0 & 0.1 \end{bmatrix}, & \bar{H}_1 &= \begin{bmatrix} 0.01 & 0.1 \end{bmatrix}^T, \\
\bar{C}_{11} &= \begin{bmatrix} 0.1 & 0.01 \end{bmatrix}, & \bar{C}_{21} &= \begin{bmatrix} 0.01 & 0.01 \end{bmatrix}, \\
\bar{C}_{31} &= 0.01,\ D_1 = 1.1, & J_1 &= 1.4,\ Q_1 = 1.0.
\end{aligned}$$

$$\begin{aligned}
A_2 &= \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, & A_{d2} &= \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 0.2 & 0.6 \end{bmatrix}^T, & E_2 &= \begin{bmatrix} 0.3 & 0.2 \end{bmatrix}^T, \\
G_2 &= \begin{bmatrix} 1.5 & 1.2 \end{bmatrix}^T, & C_2 &= \begin{bmatrix} 0 & 0.1 \end{bmatrix}, \\
C_{d2} &= \begin{bmatrix} 0.1 & 0 \end{bmatrix}, & \bar{H}_1 &= \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T, \\
\bar{C}_{12} &= \bar{C}_{22} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, & \bar{C}_{32} &= 0.1, D_2 = 1.2, \\
J_2 &= 1.5, Q_2 = 1.1, & \bar{F}(k) &= 1, \tau = 1.
\end{aligned}$$

The weighted matrix of the fault is supposed to be $W_f(z) = 0.5z/(z-0.5)$ with the minimal realization: $A_w = 0.5, B_w = 0.25, C_w = 1.0, D_w = 0.5$. For a given $\gamma = 1.2$, by solving (18), we can have a feasible solution. Furthermore, by (19), it follows from Theorem 1 that:

$$\left[\begin{array}{c|c} A_{f1} & B_{f1} \\ \hline C_{f1} & D_{f1} \end{array}\right] = \left[\begin{array}{cc|c} -0.1065 & -0.4050 & 0.0386 \\ 0.2451 & 0.6430 & -0.2548 \\ \hline -0.2042 & 0.0671 & 0.4552 \end{array}\right]$$

$$\left[\begin{array}{c|c} A_{f2} & B_{f2} \\ \hline C_{f2} & D_{f2} \end{array}\right] = \left[\begin{array}{cc|c} 0.2392 & 0.0403 & -0.0126 \\ -0.0657 & 0.1803 & -0.2144 \\ \hline -0.1760 & -0.0792 & 0.4089 \end{array}\right]$$

Furthermore, an unknown input is assumed to $d_k = 0.01exp(-0.04k)cos(0.03\pi k)$. The control input u_k is the unit step function. The fault signal f_k is simulated as a square wave of unit amplitude occurred from 20 to 40 steps. The switching signal is generated randomly and shown in Fig.1. The generated residual r_k is shown in Fig.2. The threshold can be determined as $J_{th} = 15.8092$ for $l_0 = 0$ and $L = 100$. Fig.3 shows the evolution of residual evaluation function $J_L(r)$, where the dashed line is fault-free case, the solid line is the case with the fault f_k. The simulation results show that $J_L(r) = 15.9613 > 15.8092$ for $L = 32$, which means that the fault f_k can be detected twelve time steps after its occurrence.

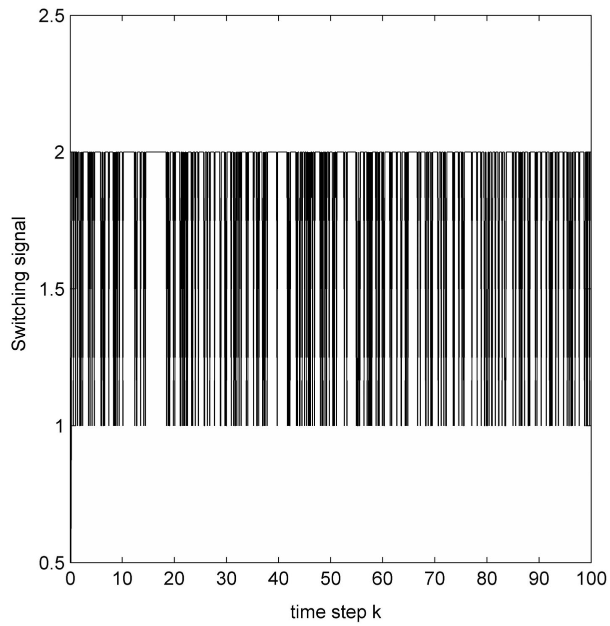

Fig. 1. Switching signal

V. Conclusion

The problem of RFDFD for discrete-time switched systems with state delays has been investigated. A mode-dependent filter is constructed as a residual generator. By augmenting the states of the original system and the filter, the problem of RFDFD has been cast into an H_∞ filtering problem. By using switched Lyapunov functional approach, a sufficient condition for the solvability of this problem is established in terms of LMIs and the desired filter has been constructed. An example has been given to show the effectiveness of the proposed methods.

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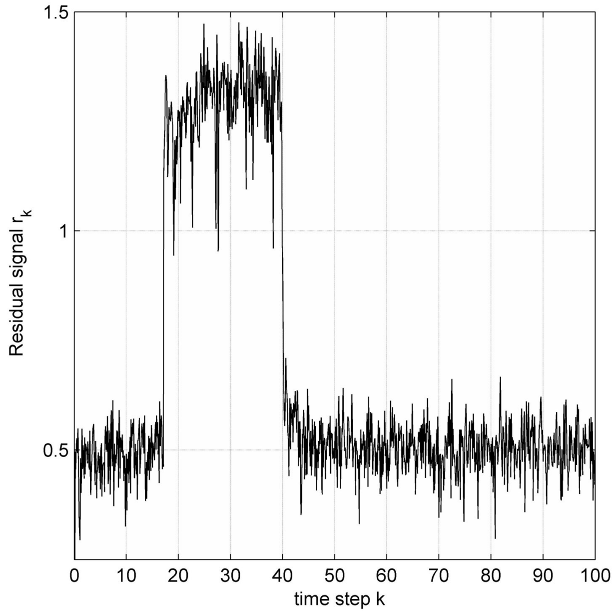

Fig. 2. Residual signal

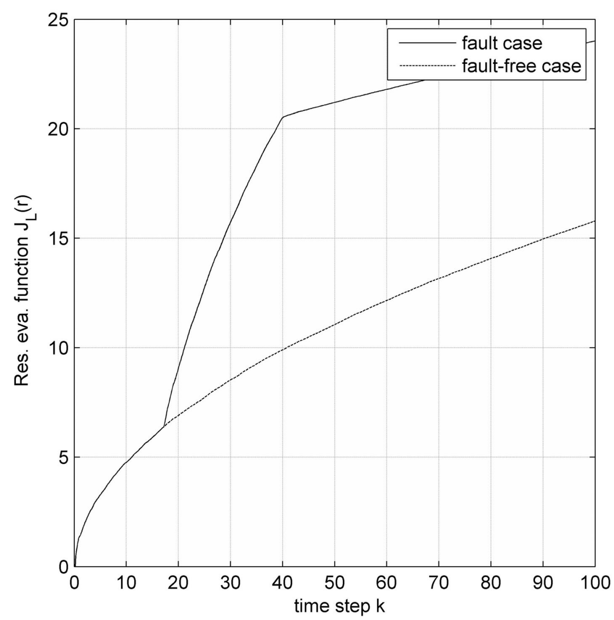

Fig. 3. Evolution of residual evaluation function

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