CHAPTER 4
A CORRECTION FOR USING BINGHAM PLASTIC MODEL FOR ER FLUIDS IN FLUCTUATING FLOW

4.1 Introduction

Because of its simplicity, Bingham plastic model has been used widely to predict the behaviour of electro-rheological fluid devices such as dampers, valves and clutches. It cannot, however, capture the details of hysteretic behaviour under a dynamic situation due to its inability to account for dynamic flow behaviour. Therefore, a more accurate dynamic model is necessary. The primary objective of this chapter is to clarify the need for a correction term when ER fluids are used in fluctuating flows. This correction is based on the experimental results described in Chapter 3. The work reported in this chapter is performed both at room temperature and at a higher operating temperature. This chapter also incorporates the use of post-yield condition as described in Chapter 2 to consistently predict the ER fluid performance in a valve.

Figure 4.1 has a summary of the journal publications from www.sciencedirect.com. Half of the histogram given in Figure 4.1 is identical to the part of Figure 2.1 and 3.4 obtained with the same key word, ER fluid. From 1985 to present, there are 117 papers related to ER fluid. The most active period of research on ER fluid and model appear from 2000 to present. In this period, 20 papers on ER fluid and model have appeared in the literature. From 1985 to 1995, research on ER fluid and ER damper is relatively inactive; only a few journal publications is listed.
Philips (1969) is among the first to develop a set of non-dimensional variables for ER fluids to determine the pressure gradient in the flow through a duct. This approach is also utilized by Coulter and Duclos (1989), and Gavin et al. (1996a and 1996b) where a quasi-static axisymmetric model is derived based on the Navier-Stokes equations to predict the damper force-velocity behaviour. The model provides a simple and sufficiently accurate estimate of pressure gradients and force levels for design purposes. The model is made for computational purposes and generally fails to explain the details of observed ER behaviour.

Torsten et al. (1999) give an overview on the basic properties of rheological fluids and discuss a brief summary of various phenomenological models of parametric and non-parametric models. Parametric models are represented by a mathematical function.
whose coefficients are determined rheologically, and adjusted until the quantitative results of the models closely match with the experimental data. On the other hand, non-parametric models are entirely based on the experimental validation of ER fluid devices. These are models that have combined viscous, Coulomb, stiffness and inertia effects. These models have enabled the inclusion of dynamic effects, such as fluid compressibility and inertia, and they are quite useful. However, there are practical difficulties in obtaining the parameters due to the complexity of the models.

Kamath and Wereley (1997) present a nonlinear viscoelastic-plastic model using viscous dash-pots in series to account for fluctuating flow. The model uses the experimental data presented by Gamota and Filisko (1991) as the basis for the quantitative study of ER behaviour, and uses an optimization technique to estimate the parameters of the model to reproduce the shape of the shear stress versus shear strain hysteresis loop. The model is able to accurately capture the experimental dynamic shear stress versus shear strain hysteresis loop at a single frequency. However, the model does not account for the pre-yield viscoelastic behaviour which is important in dynamic studies.

Lindler and Wereley (2003) present an improved experimental validation of their existing quasi-steady ER damper using an idealized Bingham plastic model by introducing an intentional leakage effect. The leakage effect allows ER fluid to flow from one side of the piston head to the opposite side without passing through the ER valve. The additional leakage effect rounds-off the corners of the original Coulomb damping shape, improving the slope of the force versus displacement. The leakage results in the removal of the initial yield force. The effect of leakage does not
accurately predict the force versus velocity behaviour, but it is able to closely predict the overall energy dissipation. This paper suggests an alternative design to obtain a fit to the prediction model which is far from quasi-steady flow.

Wang and Gordaninejad (1999) claim that the Bingham plastic model may not be an accurate predictor if ER and MR (Magneto-Rheological) fluids experience shear thinning or shear thickening, since the post yield plastic viscosity is assumed to be constant. The authors propose and adopt the steady flow Herschel – Bulkley model as a design tool for ER and MR fluid dynamics through pipe and parallel plates. Wang and Gordaninejad (1999) demonstrate that the Herschel – Bulkley formulation can collapse to the Bingham plastic model for some particular fluid parameters by comparing the proposed model with Gavin’s (1996) approximated solution of Philips (1969) Bingham visco-plasticity model. Results are presented only for MR fluid.

Wang and Gordaninejad (2001) extend their earlier studies to include the dynamic effect by using the Herschel – Bulkley constitutive equation, to predict behaviour of ER/MR fluid dampers. Excellent agreement is achieved between the experiment and the models, although results are presented only for MR fluid. MR and ER fluids are very similar macroscopically, but they are quite different when examined in a microscopic sense, such as shear thinning exhibited by MR fluids.

Another group of researchers focus entirely on the performance based approach by attempting to reproduce the force-velocity hysteresis. Ehrgott and Masri (1992) use a Chebyshev polynomial fit to approximate the force generated by an ER damper device. Yao et al. (1997) use a recursive least-square algorithm to estimate the viscous
and Coulomb friction force of both multi-plate and single-plate electrode ER damper. Marksmeier et al. (1998) develop a theoretical analysis based on Herschel-Bulkley model to predict the behaviour of an ER grease damper. Whilst the model can agree well with the experimental data, it generally fails to explain the observed behaviour. Although the results are impressive, their practical applications are questionable.

One of the works relevant to this chapter is by Peel and Bullough (1994). These authors develop a non-dimensional base on Bingham plastic constitutive equation to predict the behaviour of an ER damper. Excellent agreement is achieved between the experiment and the predictions, although results are presented with statically measured shear strength and steady Bingham plastic flow assumption. The importance of differentiating between static and dynamic yield strengths is clarified in Section 4.3.1.

It is apparent from the above literature survey that ER damping devices can be made in a variety of configurations and techniques. This has resulted in a diverse range of prototypes and models. These papers provide an excellent overview of the current knowledge and an extensive reference section. So far, there has been little work done on the performance-based approach. Due to its starting assumption of uniform steady flow, a Bingham plastic flow based approach cannot predict the performance of an ER damper in a fluctuating flow. The main contribution of this chapter is to develop a correction model for the uniform-flow Bingham approach to model behaviour in a fluctuating flow. Such a correction is essential for meaningful predictions.
The first part of this chapter is to briefly present the uniform-flow Bingham model from Peel and Bullough (1994). Full derivations of the Bingham model can be found in Appendix D. In the second part, the experimental data obtained in Chapter 3 is used to form a correction for the uniform-flow Bingham model, to account for the dynamic effects and to predict the performance of an ER based damper. The correction model is essentially an empirical equation to describe the pressure loss as a function of frequency, volume flow rate and weight fraction. The model also includes the effects of static strength and operating temperature. The correction is only valid for weight fractions ($\phi$) of 40% to 70%, and for frequencies ranging from 0.5 Hz to 2 Hz.

4.2 Uniform flow Bingham plastic behaviour in valve mode - reminder

Consider a fluid flowing through a section of an annular channel of a gap $h$ as shown in Figure 4.2. ER fluid flows from left to right, between the two conductors indicated with dark thick lines. Application of the electric field to the conductor plates causes the middle region of the flow to solidify and form a “plug” region of thickness $\delta$. The plug region is characterized with a constant velocity distribution, $u$, where the velocity distribution, therefore shear, can only take place outside. The pressure is lost, $\Delta P$, over the length $L$ due to viscosity and due to the solidification of the ER fluid. The assumption for this present condition is for steady flow where a constant flow rate is achieved over the distance $L$. Details of the derivation of the following equations can be found in the Appendix D. Only the relevant expressions are presented here for easy reference.
Figure 4.2. Showing the Bingham plastic valve flow and relevant parameters.

When an electric field is applied, the maximum shear stress occurs at the channel wall, \( \tau_w \)

\[
\tau_w = \frac{h\Delta P}{2L} \quad 4.1
\]

For an ideal Bingham plastic behaviour, \( \tau_w \) can be rewritten as

\[
\tau_w = \tau_b + \mu_P \left( \frac{du}{dy} \right)_{y=0} \quad 4.2
\]

where \( \mu_P \) represents plastic viscosity, in Pa\( \cdot \)s, using the SI unit system, and \( \tau_b \) represents the shear strength for zero strain rate, to indicate the effect of the applied electric field.

Re-arranging Equations 4.1 and 4.2, the velocity gradient at the wall becomes

\[
\left( \frac{du}{dy} \right)_{y=0} = \frac{1}{\mu} \left( \frac{h\Delta P}{2l} - \tau_b \right) \quad 4.3
\]

Taking the coordinate variable \( v \) from the center in Figure 4.2, the velocity gradient at the wall (where \( v = h/2 \)) in Equation 4.3 can be rewritten as

\[
\left( \frac{du}{dv} \right)_{v=h/2} = -\frac{du}{dv} = -\frac{1}{\mu} \left( \frac{v\Delta P}{l} - \tau_b \right) \quad 4.4
\]
Equation 4.4 can be integrated over the cross-sectional area, to obtain the volume flow rate $Q$, and manipulated to give

$$4 \left( \frac{L}{h \Delta P} \right)^3 \tau_b^3 - 3 \left( \frac{L}{h \Delta P} \right) \tau_b + \left( 1 - \frac{12 \mu LQ}{bh^3 \Delta P} \right) = 0 \quad 4.5$$

In this expression, the parameter $b$ represents the depth, normal direction to the view in Figure 4.2. It should be noted that the effect of applied electric field is implied only in terms of the static shear strength $\tau_b$ (Peel and Bullough 1994). Equation 4.5 represents a relationship between the volume flow rate $Q$ (assumed steady) and the resulting pressure loss $\Delta P$, in terms of the geometric parameters ($h, b$ and $L$), viscosity ($\mu$) and the ER response to electric field ($\tau_b$).

The significance of Equation 4.5, is providing the pressure drop over a flow passage, $\Delta P$, when all the geometric and fluid parameters, and the flow rate are prescribed. Hence, if an ER fluid is used as the working fluid of a shock absorber such as that in Chapter 3, the damping force can be obtained from $\Delta P$ by multiplying it with the surface area of the piston of the shock absorber. This damping force may be varied through $\tau_b$, by varying the electric field strength on the ER fluid. Such a shock absorber would have no moving mechanical components to accomplish the desired change in the damping parameters.

### 4.2.1 Using the Bingham Model as a Prediction Tool

The objective of this section is to present the use of Equation 4.5 as a prediction tool for the valve mode of application, such as that for a shock absorber. For such an
application, it is desirable to know the damping coefficient as a function of the geometric parameters and the applied voltage to provide ride comfort and manoeuvring stability. This section briefly highlights the importance of the parameters in Equation 4.5. Full details can be found in Appendix D. The effect of ER response to electric field ($\tau_b$) in Equation 4.5 is also discussed.

If an ER fluid is forced to flow between the electrode plates in the absence of an electric field, the ER fluid will behave like a Newtonian fluid and the flow will give a pressure drop. From Equation 4.5, and for no electric field ($\tau_b = 0$),

$$\left[ 1 - \frac{12 \mu L Q}{b h^3 \Delta P} \right] = 0 \quad \text{4.6}$$

Re-arranging equation 4.6

$$\Delta P = \frac{12 \mu L Q}{b h^3} = \frac{12 L}{b h^3} \mu Q \quad \text{4.7}$$

In Figure 4.3, the slope of the $\Delta P$ versus $Q$ variation may be identified as $\frac{12 L}{b h^3} \mu$.

Remembering that this slope is simply “$\mu$” alone in the shear mode to relate $\tau$ to $\gamma$, the multiplier $\frac{12 L}{b h^3}$ may be interpreted as “the equivalent viscosity” in the valve mode.
Figure 4.3. Variation of $\Delta P$ with $Q$ for a Newtonian fluid.

Figure 4.4. Variation of $\Delta P$ with $Q$ for a Bingham fluid.

When an electric field is applied, the relationship between $\Delta P$ and $Q$ is a third order one in Equation 4.5. One such a nonlinear variation ($\Delta$), for the experimental
parameters $b, h$ and $L$ as specified in Chapter 3 is shown in Figure 4.4. Due to this nonlinear nature of the Bingham behaviour, two values of $\Delta P$ may be obtained for one given value of $Q$. The only exception to this dual $\Delta P$ is when there is no flow. From Equation 4.5, and for $Q = 0$

$$4\left(\frac{L}{h\Delta P}\right)^3\tau_b^3 - 3\left(\frac{L}{h\Delta P}\right)\tau_b = 0 \quad 4.8$$

Re-arranging and cancelling the common term,

$$\Delta P_o = \frac{L\tau_b}{h\sqrt{\frac{3}{4}}} \quad 4.9$$

$$\frac{\tau_b L}{h\sqrt{\frac{3}{4}}}$$ indicates how much pressure difference must be provided before any flow can result in the valve mode.

Now, going back to the multiple $\Delta P$’s for a given $Q$, the lower $\Delta P$ may be neglected easily as this value is physically impossible due to the implication of a negative viscosity in the lower branch of the non-linear $\Delta P$ variation. The upper branch is approximately parallel to the Newtonian behaviour (⦁) with a slope of $\frac{12L}{bh\tau}\mu$.

4.2.2 Yield Shear Stress at zero strain rate

This section presents a discussion on the behavior of an ER fluid under both static and dynamic situations, and to incorporate the influence of these situations in the use of Equation 4.5. This section also points out the importance of correctly distinguishing between the two shear strength values for a zero strain rate.
ER fluid behaviour can be considered in two rheological regions namely, pre-yield and post-yield, as shown in Figure 4.5. The horizontal axis of this figure represents the strain rate ($s^{-1}$) and the vertical axis represents the shear stress. In the pre-yield region, to the left of the vertical dashed line, ER fluids behave like a linear viscoelastic material. For small strains in the pre-yield region, the shear stress increases linearly with the strain rate, with the curve having a slope of $G^*$, the complex shear modulus of the fluid. In the post yield region, the behaviour is characterised by Bingham plastic behaviour [Kim et al. (2003)].

![Figure 4.5. Pre-yield and post-yield regions of an ER fluid in shear][1]

At the yield point (indicated with the vertical dashed line), there are two distinct values of yield shear stress, namely, static and dynamic. At this point, as the strain exceeds the critical strain, the ER fluid moves from pre-yield to post-yield region. However, it is important to notice that post-yield shearing does not occur until the local shear stress is sufficient to break the bonding of the particle chains at $\tau_{\text{static}}$. After the shear stress exceeds the yield level, it quickly drops to its $\tau_{\text{dynamic}}$. Further increase in the rate of strain results in post-yield behaviour.
The dynamic yield shear stress, $\tau_{\text{dynamic}}$, can be obtained by extrapolating the straight line back to the zero strain rate axis. The static yield strength $\tau_{\text{static}}$, on the other hand, may not be directly related to the post-yield process. The dynamic yield shear stress behaviour corresponds to the processes in which breaking and forming of chains occur continuously across the conductors. Thus, it is critically important that this difference is recognised with the particular application involving either the pre-yield or the post-yield condition. If an application involves breaking the chains for the first time, the appropriate strength value is $\tau_{\text{static}}$. Otherwise, $\tau_{\text{dynamic}}$ should be used.

### 4.2.3 Effect of the operating temperature on shear strength

The objective of this section is to incorporate the influence of the operating temperature on ER damper performance. Experimental observations are used to express the effect of the operating temperature. Full details of the experiments can be found in Appendix B.

A schematic representation of the temperature measurements in shock absorber application is shown in Figure 4.6. One thermometer is attached inside the high-pressure tubing and the other thermometer attached outside the cylinder to record the ER fluid’s temperature and the operating temperature of the ER damper. Temperature readings are collected until the heat generation rate becomes equal to the rate of dissipation at a nominally 20°C room temperature. The operating temperature of the ER damper takes approximately 5 minutes to reach 30°C from the room temperature of 20 ºC.
Experimental observations to indicate the variation of the dynamic shear strength for zero strain rate, $\tau_{\text{dynamic}}$, are given in Table 4.1. These results are obtained experimentally as detailed in Appendix B. Yield strength values are given in terms of their variations from the room temperature. Room temperature values are presented in Chapter 2.

**Table 4.1**: Variation of the normalised dynamic yield strength ($\tau_b$) with voltage and operating temperature.

<table>
<thead>
<tr>
<th>Electric field (V/mm)</th>
<th>20 degrees</th>
<th>40 degrees</th>
<th>50 degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.58</td>
<td>2.68</td>
</tr>
<tr>
<td>333</td>
<td>1.00</td>
<td>1.09</td>
<td>0.83</td>
</tr>
<tr>
<td>666</td>
<td>1.00</td>
<td>1.61</td>
<td>1.55</td>
</tr>
<tr>
<td>1000</td>
<td>1.00</td>
<td>1.66</td>
<td>1.50</td>
</tr>
<tr>
<td>1333</td>
<td>1.00</td>
<td>1.61</td>
<td>1.49</td>
</tr>
</tbody>
</table>
Increasing the operating temperature from 20°C to 40°C improves the ER effect by at least 60% for 666 V/mm and higher electric fields. Significant increase at zero electric field is in fact quite marginal in absolute sense, as the value of the intercept is quite small and easily affected by minimal friction losses and curve fitting. For a temperature of 50°C and higher, \( \tau_b \) values decrease. The reason for the decrease in static strength at 50°C is because the cornstarch starts to cook, and its rheology changes irreversibly. The value of \( \tau_b \) at 30°C, for the pressure drop predictions in the flow mode, may be obtained with linear interpolation from those of 20°C and 40°C.

### 4.3 Model for Valve mode

Comparison between the Bingham model and the experimental results, from Chapter 3, are presented in this section with the objective to form a correction model to predict the performance of the ER fluid-based damper. As mentioned earlier, Bingham plastic flow model in Equation 4.5 is derived for a steady flow. However, an ER fluid-based damper experiences a fluctuating flow. Therefore, a correction term to account for the difference may be necessary. The discussion in this section is presented for the 70% weight fraction (\( \phi \)) first. Selective results for other \( \phi \) are then given to indicate dependence on \( \phi \).

The variation of the pressure loss with the volumetric flow rate is shown in Figures 4.7(a) to 4.7(d), for 3mm, 5mm, 10 mm and 15mm stroke lengths (peak-to-peak), respectively, and for 70% weight fraction. In each frame of Figure 4.7, results are presented in two groups. The group of four (in blue) with steeper slopes corresponds to the experimental observations, each of the four indicating an electric field of 0 V/mm (◼), 333 V/mm (▲), 667 V/mm (●), 1000 V/mm (✚) and 1333 V/mm (✦).
The second group of four (in pink) corresponds to the predictions obtained from Equation 4.5.

The experimental volume flow rate is obtained by multiplying the imposed peak velocity of the excitation with the net cross sectional area of the piston. The experimental pressure drop is obtained by dividing the force measurements by the same area of the piston. Frequency of motion is implicit, as it is combined with the stroke length to calculate the volume flow rate. Four values along the horizontal axis in Figure 4.7(a), for instance, are those corresponding to frequencies of 0.5Hz, 1.0Hz, 1.5Hz and 2.0Hz. As mentioned earlier in Chapter 3, the highest frequency and stroke length combinations are missing due to the experimental problems of leakage in Figures 4.7(c) and 4.7(d).

In Figure 4.7(a) for 3mm stroke length and zero Volt (◆), there is a large difference between the experimental and the prediction results. With further increase in the stroke length as shown in Figure 4.7(b) to 4.7(d), the difference between the predicted and the experimental pressure loss at zero field seems to increase. It may be useful to remember that the predicted values correspond to those of a uniform flow assumption. However, the experimental values are the maxima of the harmonically excited fluctuating flow. Hence, it is not surprising to see this difference between the experimental and predicted pressure drops.
Figure 4.7. Comparison of the predicted and measured \( \Delta P \) variation for 70% weight fraction with \( Q \) for 3 to 15mm stroke length. On each line of constant voltage, up to four frequency are given from 0.5Hz to 2Hz.
With an increase in the applied electric field, for instance with the case of 333 V/mm (▲), the pressure loss line seems to shift up and almost parallel to the Newtonian behavior. In addition, the difference in pressure loss between the experiment and prediction is somewhat reduced. With further increase in the applied electric field, the same trend is consistent.

Another significant difference between the experiment and prediction is the implied intercept values. As mentioned earlier in Section 4.3, the dynamic yield strength, $\tau_b$, indicates how much pressure difference must be provided before any flow can result in the valve mode. Observations from Figure 4.7 indicate that this yield strength from the experiment is lower than that of the predictions.

In Figure 4.8, the pressure loss-flow rate variation is presented in an identical format to that in Figure 4.7, but for a smaller of $\phi = 0.50$. Weight fraction, $\phi$, changes from 0.5 to 0.4 in Figure 4.9. As the value of $\phi$ decreases, and as the strength of the ER fluid decreases, the difference between the experiment and the prediction seems to be further apart. The maximum pressure loss for $\phi = 0.70$ is approximately $8 \times 10^5$ Pa. This maximum changes to $7 \times 10^5$ Pa and $6 \times 10^5$ Pa for the smaller $\phi$, respectively.
Figure 4.8. Similar to Figure 4.7 but for 50% weight fraction.
Figure 4.9. Similar to Figure 4.7 but for 40% weight fraction.
4.3.1 Correction model

Following the discussion in the preceding section, the difference between the predicted and experimental pressure loss, $\Delta P$, is grouped according to the applied electric field, stroke length and frequency. The following simple expression has been found to apply for a constant electric field

$$Z(Q, f) = aQ^2 + bf^2 \quad (4.10)$$

where $Z$ represents the additional pressure loss to what is the predicted by equation 4.5, $f$ represents frequency in Hz, $Q$ represents volume flow rate in (l/min). Stroke length is implicitly expressed in the volume flow rate $Q$.

The surface generated by imposing the form suggested in Equation 4.10 is shown in Figures 4.10(a) to 4.10(e) for 70% weight fraction with varying electric field from 0 V/mm to 1333 V/mm, respectively. The horizontal axis represents the flow rate $Q$ in l/min, and the depth axis represents the frequency $f$ in Hz. The true values of the experimental $\Delta P$ and predicted $\Delta P$ are marked with ($\bullet$).

The two coefficients $a$ and $b$ in Equation 4.10 and the corresponding correlation coefficients, $R^2$, are presented in Table 4.2 for different electric fields. The correlation coefficient varies between 0.77 (0 V/mm) and 0.94 (1333 V/mm), suggesting a relatively close agreement with the expected form of Equation 4.10.
Figure 4.10. Variation of the difference between the predicted and measured pressure loss for 70% weight fraction and different electric fields of (a) 0 V/mm and (b) 333 V/mm (c) 667 V/mm (d) 1000 V/mm and (e) 1333 V/mm.
Table 4.2. Variation of parameters in Equation 4.10 and the regression coefficient with the applied electric field, and for 70% weight fraction.

<table>
<thead>
<tr>
<th></th>
<th>0 V/mm</th>
<th>333 V/mm</th>
<th>666 V/mm</th>
<th>1000 V/mm</th>
<th>1333V/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \text{ Pa/(l/min)}^2$</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>$b \text{ Pa/(Hz)}^2$</td>
<td>16000</td>
<td>16000</td>
<td>16000</td>
<td>16000</td>
<td>16000</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.77</td>
<td>0.89</td>
<td>0.94</td>
<td>0.92</td>
<td>0.82</td>
</tr>
</tbody>
</table>

In Table 4.2, parameter “a” represents the effect of the volume flow rate, and parameter “b” represents the effect of the frequency. Parameters “a” and “b” are constant for all electric fields, indicating that the difference between the experiment and prediction is not a function of the electric field.

Hence, the correction model from Equation 4.10 and from Table 4.2 for 70% weight fraction becomes

$$Z(Q, f) = 1500Q^2 + 16000f^2$$  \hspace{1cm} (4.11)

4.3.2 Correction Model with varying weight fractions

A similar procedure as described for 70% weight fraction, is repeated here for 50% and 40% weight fractions. The surfaces generated by imposing the form suggested in Equation 4.10 is shown in Figure 4.11 and Figure 4.12 for 50% and 40% weight fractions, respectively. The variation of parameters “a” and “b” is plotted against weight fraction as shown in Figure 4.13 to complete the correction model to predict the performance of the ER damper.
Figure 4.11. Similar to Figure 4.10 but for 50% weight fraction,
Figure 4.12. Similar to Figure 4.10 but for 40% weight fraction,
Figure 4.13. Variation of parameters $a$ (♦) and $b$ (■) with weight fraction.

In Figure 4.13, parameter “a”, as shown in blue (♦), seems to be approximately constant as a function of weight fraction. The correlation coefficient variation with $\phi$ is approximately 0.96. Parameter “b” is decreases with $\phi$, indicating a weaker contribution of the frequency as the solid ratio of the ER suspension increases. The correlation coefficient for parameter, b, is approximately 0.996. Hence, the correction model for the quasi-steady Bingham plastic flow to cyclic flow becomes

$$Z(Q, f, \phi) = (-32\phi + 1507)Q^2 + (-12857\phi + 24857)f^2$$  \hspace{1cm} (4.12)

It should be remembered that the model in Equation 4.12 is only tested for a range of weight fractions from 40% to 70%

$$0.4 \leq \phi \leq 0.7$$ \hspace{1cm} (4.13)
Equation 4.12 can be used as a design tool to predict the pressure loss for an ER fluid-based damper if the operating condition (Q) and the geometric parameters are given within a reasonable scope of the range of variables considered in this work. Very importantly, the standard Bingham plastic flow model does not seem to need correction for the applied electric field. As mentioned earlier, predicting pressure drop is significant in the prediction of the equivalent damping coefficient for applications where variation of this coefficient with electric field is beneficial for vibration control.

4.3 Conclusions

The standard Bingham plastic flow model is not suitable to use for describing behavior in applications involving non-steady flows. Here, the standard model is extended for a fluctuating flow, such as that in a shock absorber, by incorporating a correction term. The correction term involves only the frequency and the volume flow rate of application, but not the applied electric field. The correction term is expressed in such a form that it can be readily used by providing the system parameters and the operating conditions.
CHAPTER 5
CONCLUSIONS

The subject matter of this thesis is electro-rheological (ER) fluids, in particular modeling of their behavior in such a way to be useful for engineering design purposes. Hence, a significant proportion of the investigation is of experimental nature, leading to attempts to model dynamic behaviour for applications.

ER fluids are usually suspensions made up of some insulating oils and some semi-conducting particles. These suspension liquids respond to being exposed to electric fields in such a way that the semi-conducting particles form chains parallel to the electric field which cause a change of phase from a liquid to a solid-like gel. As the chains become stronger, in response to increasing field strength, they are able to bear considerable shear stress under static conditions. The phase transformation is seemingly reversible, and it takes place over milliseconds. Hence, ER fluids hold considerable promise to enhance motion control applications in engineering.

As presented in Chapter 2, the primary objective of studying the shear mode of operation is to develop a prediction tool useful for applications involving this particular mode such as those of clutches and brakes. To this end, a comprehensive experimental investigation has been performed to form an empirical expression to predict the shear stress bearing capacity of the particular corn starch-mineral oil ER fluid. The parameters of importance are the strength of the applied electric field, rate of strain imposed by the particular application and the solid-to-liquid weight ratio of
the ER fluid. Amongst these parameters, including the weight ratio in the model is the first attempt in the literature to provide a design point-of-view.

Although numerous similar attempts to that in Chapter 2 of this thesis, have been reported in the literature, the work presented in this chapter is unique in such a way that the proposed model is employed to predict the performance of a clutch which uses the ER fluid as its power transmission agent. The accuracy of the reported prediction has an error in the order of 20% in the most useful range of operation. The advantage of such a clutch design is quick reaction time, low power consumption and avoiding the inherent friction related problems of conventional designs.

The flow mode of operation is the subject of Chapters 3 and 4. In Chapter 3, an ER valve is presented to provide a variable damping coefficient for a shock absorber. With the particular composition of the ER fluid, it is demonstrated with extensive measurements that it is possible to affect a 50% increase in the damping coefficient with the application of electric field. The advantages of such a shock absorber over a conventional one, are similar to those of the clutch of Chapter 2, namely fast operation, low power consumption and no moving mechanical components to achieve the desired effect.

Chapter 4 attempts to address a fundamental difficulty associated with using the standard Bingham Plastic Flow model for some applications using an ER fluid in the valve mode. The point here is the fact that the standard Bingham model assumes a uniform flow to make the necessary constitutive expression available. There is no doubt of the significance of this expression as it relates the amount of (given) volume
flow rate to the amount of (required) pressure drop which employs the strength of the applied electric field as the control parameter. Such a pressure drop may then be converted to relevant damping force acting on the given surfaces of the ER device. However, as in the case of the shock absorber application, such applications involve flow conditions drastically different than the starting uniform-flow assumption. Hence, a correction term to reflect the severity of the flow conditions may be necessary. The severity of the flow conditions is represented by the frequency and stroke length of oscillations. It is quite significant to note that the required correction does not involve an effect of the applied electric field. It appears that the correction is necessary only for the presence of dynamic flow conditions. As there is a number attempts in the literature which report close agreement between experimental observations and empirically-constructed models based on the standard Bingham approach, caution may be necessary to interpret their validity.

Finally, it should be mentioned that both the amount of torque transmission, as reported in Chapter 2, and the amount of variation possible in the equivalent damping coefficient, as reported in Chapter 3, may be quite small for an immediate industrial application. However, ER fluids which are available commercially, may be as much as an order of magnitude more powerful than the corn starch-mineral oil composition used in this investigation (this fact has been confirmed with measurements at the initial stages of the reported work). Hence, the reported outcomes should be taken as just what is possible with these controllable fluids, rather than the limit of their performance.
ER fluids hold significant promise as control agents. Although the scope of this work is limited to motion-related applications, there is no reason to restrict their application to such a small area. There is no doubt that their seemingly reversible and practically instantaneous capability to change phase, should find numerous other applications for ER fluids. Applications involving the control of sound absorption/reflection coefficients, or applications involving the control of various heat transfer properties, may be good candidates to extend the usefulness of these fluids. As indicated above, as a further extension of this work, the two models developed in Chapters 2 and 4, can be re-evaluated using ER fluids of higher shear strength, different particle composition and suspension oils. Such extensions of this study, can lead to universal models.
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APPENDIX A

USING FREE AND RESTRICTED SLOSHING WAVES FOR CONTROL OF STRUCTURAL OSCILLATIONS

A1. Introduction

This appendix contains results of an experimental study conducted during the course of the investigation presented as the Ph.D. thesis. Although the Electro-rheological effect of the particular fluid is not used to generate these results, the presented work still constitutes an engineering application of the fluid for vibration control purposes. Hence it has been found appropriate to include it here.

In many practical applications, it is critical to suppress liquid sloshing to avoid its detrimental effects. Alternatively, sloshing may be induced intentionally to control excessive structural vibrations. This concept was suggested earlier by Fujii et al. (1990), Abe et al. (1996), Kaneko and Yoshida (1994), and Seto and Modi (1997). These works dealt with shallow liquid levels which result in a traveling sloshing wave. Traveling waves are preferable to standing waves because of their better energy dissipation characteristics. The poor energy dissipation characteristics of standing waves have been reported earlier by Seto and Modi (1997) and Anderson et al. (2000).

Deep liquid levels are important practically as they more realistically represent storage containers. Again, it is practically important to use these containers for structural control purposes as they may already exist as part of the structure to be controlled. The control effort may then be reduced to designing these containers for improved control purposes, rather than prescribing additional components. What is reported here is a summary of extensive experiments to investigate the effective
control parameters of a standing-wave type sloshing absorber. To the best of the author’s knowledge, this work is only the second attempt in this direction, following that of Anderson et al. (2000).

**Figure A1.** Schematic drawing showing a single degree of freedom oscillator with sloshing absorber of unrestricted free surface.

In Figure A1, a simple mechanical oscillator is shown with a container partially filled with a liquid, representing the sloshing absorber. It is desired to design the container dimensions such that there is strong interaction between the sloshing liquid and the oscillator to achieve an effective control. A standing-wave type sloshing absorber normally has poor energy dissipation even when there is a strong interaction. This poor energy dissipation leads to a beating envelope in the response of both the structure and the liquid [Anderson et al. (2000)]. A cap placed above the free surface of the liquid may partially restrain the surface wave, possibly improving the rate of energy dissipation. In Figure A2, this suggested configuration is illustrated. Experimental observations are reported in this paper to determine the effect of such restraints in suppressing structural response quickly.

The next section includes a brief description of the parameters of the structure, the sloshing absorber and the experimental procedure. Then, typical results are discussed to indicate important parameters.
Distance from the free surface

Figure A2. Showing the sloshing container with the cap

A2. Experiments

An isometric view of the experimental setup is shown in Figure A3(a). A rigid platform is cantilevered with light aluminium strips to form a simple oscillator. The platform also provides a flat surface to mount the container (75x105x100 mm) of the sloshing absorber which has two vertical adjusting strips to allow variable gaps of the surface plate. An approximately 30-mm depth of water produces strong interaction with the structural response. The fundamental frequency of the structure is 2.7 Hz. The mass of the sloshing liquid is maintained to be 10% of the mass of the structure to be controlled. Structural parameters and fluid properties are summarized in Table 1.

Figure A3. Experimental setup, (a) isometric and (b) schematic view. 1: Keyence, LB-12 laser displacement transducer; 2: Keyence LB-72 amplifier and DC power supply; 3: DataAcq A/D conversion board, and Personal Computer
Table A1. Structural and fluid properties (at room temperature).

<table>
<thead>
<tr>
<th></th>
<th>mass (± 0.01kg)</th>
<th>Stiffness (N/m)</th>
<th>$\xi_{eq}$</th>
<th>Viscosity (Pa.s)</th>
<th>Density (g/ml)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>4.0</td>
<td>1150</td>
<td>0.002 ±0.001</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Water</td>
<td>0.4</td>
<td>--</td>
<td>--</td>
<td>0.001 [6]</td>
<td>1.0</td>
</tr>
<tr>
<td>Mineral oil</td>
<td>“”</td>
<td>--</td>
<td>--</td>
<td>0.010 ± 0.005</td>
<td>0.84</td>
</tr>
<tr>
<td>ER-susp.</td>
<td>“”</td>
<td>--</td>
<td>--</td>
<td>0.53 ± 0.01</td>
<td>0.78</td>
</tr>
</tbody>
</table>

The experimental procedure consisted of observing the free decay of the structure after giving it an initial displacement of $13 \pm 0.5$ mm. The displacement history of the structure was sensed with a non-contact laser transducer and amplified before it was recorded in a personal computer (items 1, 2 and 3 in Figure A3(b)). The experiment was repeated from a zero gap to a large enough value to ensure free surface response. Water, a light mineral oil and mineral oil suspension with cornstarch (45% solid content) were used as working liquids in the absorber.

![Uncontrolled displacement history of the structure without sloshing.](image)

**Figure A4.** Uncontrolled displacement history of the structure without sloshing.
Displacement history of the oscillator without sloshing is shown in Figure A4. The structure had relatively poor dissipation with an equivalent viscous damping ratio ($\zeta_{eq}$) of approximately 0.2%, taking well over a minute before its oscillations could settle. The objective of the experiments was to determine the parameters of the sloshing absorber to shorten this long settling time.

**A3. Results**

Typical displacement histories are shown in Figure A5 when water was used as the working fluid of the sloshing absorber. Figure A5(a) represents the case with zero gap where no sloshing is allowed in the container. Apparent improvement in the response of the structure as compared to the uncontrolled one in Figure A4, is primarily due to the added mass effect. In Figure A5(b), the best performance case is given when the gap above the free water surface is 5mm. For this case, oscillations of the structure stop in less than 15 seconds.

For the 5 mm gap case, rising water waves hit the plate above the free surface violently. As a wave descends, its preceding strong interaction with the plate produces surface breaks to dissipate energy. As a consequence of this dissipation, the control action on the structure is quite effective.

When the gap is increased to 25 mm, a drastic deterioration of the control is observed with oscillations taking over 30 seconds to stop, as illustrated in Figure A5(c). With increased gap, the interaction of a rising wave with the plate becomes milder than the case in Figure A5(b). Further increases in the gap changes in the response of the structure only marginally (not shown here for brevity). In Figure A5 (d), free surface case is shown with a quite similar settling time to that in Figure A5(c). For this case, a
rising sloshing wave can reach heights of up to 60 mm with large velocity gradients at the free surface. One significant phenomenon to notice in Figure A5 (d), as compared to the other three frames, is the presence of a beat in the envelope of the structure’s response. This beat is an indication of tuning between the fundamental sloshing frequency and the structural natural frequency.

Figure A5. Displacement history of the structure with water and with (a) zero gap, (b) 5mm gap, (c) 25 mm gap and (d) free surface.

In Figure A6, results are presented in an identical format to that in Figure A5, but for a mineral oil as the working liquid instead of water. As expected, Figure A6(a) is similar to Figure A5(a). In Figure A6(b), a 5 mm gap again indicates effective attenuations. However, the interaction of the mineral oil with the top plate is much
milder than that with water. As a result, descending waves display fewer surface discontinuities than in the case with water. The free surface with the mineral oil still shows some mixing but without surface breaks. Therefore, despite a comparable control effect on the structure, the mechanism of energy dissipation with the mineral oil is quite different than it is with water. It is expected that energy is lost through viscous dissipation with the mineral oil due to its increased viscosity by an order of magnitude. Fluid properties are listed in Table A1.

Another significant difference between the mineral oil and water is in the sensitivity of the control to varying gap. As the gap increases to 25 mm in Figure A6(c), the change in the response of the structure is marginal, with similar settling times even in the free surface case in Figure A6(d). For the free surface case, rising sloshing waves reaches a maximum of 50 mm, about 20% lower than that with water. Although velocity gradients of the mineral oil are smaller than those of water, there is a large enough viscous dissipation to maintain effective control. As a consequence of this inherent energy dissipation, no beat is apparent in Figure A6(d).
In Figure A7, the results are shown when the same mineral oil is used to suspend cornstarch with a 40% weight ratio. This liquid mixture was noticed in an earlier study of the authors for its high viscosity, as indicated in Table A1. Cornstarch suspension in mineral oil is an electrorheological (ER) fluid which can reversibly change its phase from liquid to a solid-like gel when a large enough electric potential (about 1kV/mm) is applied to it.
Figure A7. Same as in Figure A5 but with ER fluid.

The response of the structure with the ER fluid is similar to that of the mineral oil. Differences can be observed, however, in the surface patterns of the sloshing liquid. With a 5 mm gap, the interaction of a rising wave with the plate is even milder than that with the mineral oil, producing no surface discontinuities at all. Figure A2 is a close representation of the ER fluid wave with the restricting plate. As the gap increases, again similar to the case of the mineral oil, the change in the structure’s response is marginal.

A summary of results is given in Figure A8 where the 10% settling times of the controlled cases, $t$, are compared with that of the uncontrolled structure, $t_0$. 10 %
settling time corresponds to the duration required for the peak displacement to decay 1/10 of the initial displacement. Hence, the ratio of $t/t_0$, should be small to indicate effective control. The horizontal axis represents the variation of the gap, $d$, non-dimensionalised with the initial displacement $X_0$.

![Graph](image)

**Figure A8.** Variation of settling time ratio with gap

The smallest $t/to$ of water ($◊$) is about 0.17 representing 83% attenuation. Sloshing water is quite sensitive to the variation in the gap, deteriorating to about 70% attenuation at $d/X_0$ of 2.5 and larger. Both mineral oil ($☐$) and ER fluid ($∆$) are more effective than water with best attenuations close to 90%. More importantly, the effectiveness of control is maintained at about 85% attenuation for all values of $d/X_0$ larger than 1. This relative insensitivity is a great advantage from a design point of view where maintaining a critical $d/X_0$ may not be possible practically in the field.
A4. Conclusions

Some typical results are presented from an extensive experimental investigation to enhance the energy dissipation characteristics of standing-wave type sloshing absorbers. Restraining plates, placed above the free surface of the sloshing liquid, are used to induce dissipation. When water is used, best attenuation is about 80% with a 10% mass ratio between the absorber and the structure. Energy dissipation is accomplished through the strong interaction of rising waves with the restraining plate, and severe surface breaks of descending waves. Other two liquids, a light mineral oil and ER fluid, gave best attenuation of 90%. Moreover, these two liquids are much less sensitive to the variations of the gap between the restraining plate and the free surface. With these two liquids, energy dissipation is expected to occur more through viscous dissipation than interaction with the plate. This work is part of an ongoing study to investigate the mechanisms of energy dissipation with different sloshing liquids for structural control.
APPENDIX B

EFFECT OF OPERATING TEMPERATURE ON THE SHEAR STRENGTH OF ER FLUID

B1. Introduction

Some applications of an ER fluid may require that the temperature of operation is higher than the normal room temperature. A shock absorber presents one such case where the dissipated energy through the damping action causes the temperature of the working fluid to rise. In a shock absorber, ER fluid operates in the flow mode where its solidification causes varying pressure losses. As it was already discussed in the main text of this thesis, predictions with the Bingham Model for performance of an ER fluid in the flow mode requires the corresponding zero-strain-rate (or static) shear strength of the fluid at the designated electric field.

The purpose of this appendix is to present experimental observations to determine the effect of raised temperatures on the static shear strength of the corn starch – mineral oil ER fluid. To this end, the experiments are described in the next section. Then, the results are reported. The objective is to obtain data consistent with the temperature of operation for prediction of the pressure loss in the shock absorber application. A further objective is to determine if the temperature may be a design factor to enhance performance.

B2. Experimental Setup

A simple environmental chamber as shown in Figure B1 was used to repeat the tests described in Chapter 2 at elevated temperatures. The chamber was built with standard 3mm thick hardware store Perspex sheets cut to form a cube of approximately 400mm
to contain the test rig. Corrugated packaging cardboard was used to provide some insulation around the walls. Insulation was used to line the inside walls of the chamber. Up to four standard light bulbs (100 Watts each) were used to provide the heat source. Lights were left on until the heat generation rate became equal to the rate of dissipation at a nominally 20°C room temperature. This temperature equalization took approximately 5 hours, before the shear stress readings were taken.

**Figure B1.** Test rig for high temperature measurements.

For each strain rate and weight fraction, three values of temperature were applied initially, namely 20°C, 40°C and 60°C. It was noticed that 60°C caused the cornstarch to cook and change its rheology irreversibly. The temperature of 47°C has been determined to be the limit by which the cornstarch is able to maintain its integrity.
B3. Experimental Observations

The variation of the zero-strain rate shear strength of the ER fluid is given in Table B1 as a function of the temperature for the 70% weight fraction between the cornstarch and the mineral oil. To indicate the change clearly, the results at higher temperatures are normalized with those at the room temperature. Again, same as in Chapter 2, these results are obtained after fitting a linear variation to the shear stress – strain rate variations, and reading the corresponding intercept on the vertical axis for each different voltage.

In Table B1, for 2000 V and higher, the strength of the ER fluid seems to increase by at least 60% at 40°C. Then, there is a marginal drop in the last column. The seemingly significant percentage change for the 0-V case, corresponds to quite small values in absolute sense, and due to experimental nature of the reported observations to cause slight deviations from the ideal value of zero.

Table B1. Variation of the normalized zero strain-rate shear strength with voltage and operating temperature.

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>20 degrees</th>
<th>40 degrees</th>
<th>47 degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.58</td>
<td>2.68</td>
</tr>
<tr>
<td>1000</td>
<td>1.00</td>
<td>1.09</td>
<td>0.83</td>
</tr>
<tr>
<td>2000</td>
<td>1.00</td>
<td>1.61</td>
<td>1.55</td>
</tr>
<tr>
<td>3000</td>
<td>1.00</td>
<td>1.66</td>
<td>1.50</td>
</tr>
<tr>
<td>4000</td>
<td>1.00</td>
<td>1.61</td>
<td>1.49</td>
</tr>
</tbody>
</table>

The variation of the zero strain-rate strength is plotted in Figure B2 in the form of a surface chart and for different weight fractions. As expected, as the weight fraction decreases from frame (a) to frame (d), the change in the ER fluid strength becomes less pronounced, as a result of the diminishing ER effect.
Figure B2. Variation of shear strength with voltage and temperature for (a) 70%, (b) 60%, (c) 50% and (d) 40% weight ratio.
Figure B2. Continued.
C1. Introduction

In Chapter 3 of this thesis, experimental observations are reported in the flow mode of an ER fluid. These experiments are designed to investigate the possibilities of implementing a variable damping shock absorber. Motivation for such a variable damping device is the fact that a shock absorber requires different levels of damping to provide its best performance as the frequency content of the disturbance changes. Hence, the performance of a constant-parameter shock absorber is always a compromise.

Experiments reported in Chapter 3, suggest that a variable damping shock absorber is certainly possible. This fact has also been demonstrated in number of earlier publications [Hartsock et al. (1991), Choi et al. (1998), Gavin (2001), Choi and Kim (2000), Lindler and Wereley (2003)]. It should be mentioned, however, that the amount of damping range reported in Chapter 3, should not be interpreted as the limit of such possible range, but only as an experimental demonstration of the viability of the concept. With this last qualifying statement in mind, results of a numerical study are presented in this section to search for an effective procedure of damping variation for best shock isolation.

Effective shock isolation is a necessity for dynamic systems exposed to transient disturbances to retain the integrity of the intended operation. Ground vehicles represent one such example where the suspension system has the task of isolating the suspended mass of the vehicle from the road irregularities. A soft suspension in a vehicle provides a
comfortable ride on well-paved roads. However, a soft suspension also produces large static deflections resulting in a shortened dynamic stroke. In addition, the suspended mass with a soft suspension may take a long time to dissipate the energy imparted by the road disturbances. These long durations not only result in discomfort, but also large variations in the vehicle's stability when cornering, accelerating and decelerating [Mizuguchi et al. (1984)]. A hard suspension, on the other hand, is dynamically stable. It dissipates the effect of a road disturbance rapidly. However, comfort is greatly sacrificed as the resulting large accelerations produce a jerky ride, and large inertial forces on the vehicle's components. Hence, designing the parameters of a passive shock isolator is a compromise between either restraining the excessive dynamic deflections or accelerations of the suspended mass.

An active shock isolator offers a solution to the conflicting requirements of a passive design. An active controller has two primary components. First component measures the response and decides on the control action. The second component implements the decision taken by the first component. Implementing the control is usually accomplished with hydraulic or pneumatic force actuators. However, force generators are generally quite complex and they may have limitations in their force capacity and frequency. With the required power supplies for sensing and actuation, active controllers may be bulky, and certainly more delicate and costlier than passive components [Karnopp (1990) and Bastow (1988)].

The industry has the responsibility of improving performance while providing this improvement at a reasonable cost. Hence, this prime responsibility naturally leads to compromises of the effective control action possible with fully active systems. A
compromise between the simplicity of the passive and the versatility of the active shock absorbers is a semi-active system. Objective with a semi-active system is to at vary the parameters of a passive system to implement the control, as opposed to imposing the control forces with actuators. This approach stems from the fact that passive controllers are quite effective when their design parameters are “tuned” specifically for the operating parameters. As the operating parameters change, however, tuning loses effectiveness. Therefore, it should be possible to maintain tuning by varying the parameters of the otherwise passive components according to a predetermined criterion. Since the control is accomplished by varying the parameters of the system, a semi-active controller requires much less effort than an active controller.

A simple procedure to implement the parameter change of a semi-active controller, is modulating the orifice opening of the damper of a shock isolator [Bastow (1988), Tavner et al. (1997), Nagai et al. (1996), Crosby and Karnopp (1973) and Karnopp (1990), Alonoly and Sanka (1987), Hrovat et al. (1988)]. In such a semi-active controller, a control strategy decides how this modulation should be implemented. This control strategy needs to know the displacements and velocities of the suspended mass, in order to evaluate the control parameters [Bastow (1988), Alonoly and Sanka (1987)]. Although the control action requires minimal effort to execute, system response still has to be monitored in order to determine the required control action. Therefore, providing the parameter change without the need to monitor system response would clearly simplify the control.

A simplified method is suggested here to attenuate the vibrations of a suspended mass when exposed to a transient base disturbance. This simplified suggested method uses a
"predetermined" sequence of damping changes in an attempt to improve the shock absorption. Hence, unlike the other semi-active controllers, sensing is required only to trigger the sequence of damping changes, but not during the implementation.

The numerical procedure and the response of a passive shock isolator will be discussed briefly next in order to establish a comparison base for the effectiveness of the suggested control procedure. Then the response of the simplified shock isolator will be compared to that of the passive system, and the advantages of the simplified system will be discussed. Comments will be provided for the implementation relevant to the results presented in Chapter 3.

**C2. Numerical Approach**

A schematic of the single degree-of-freedom (SDOF) quarter vehicle model is shown in Figure C1. The equation of motion of the suspended mass, m, is

\[ m\ddot{x} + c\dot{z} + kz = 0 \]  \hspace{1cm} (C1)

where c and k represent the viscous damping coefficient and the stiffness. The relative coordinate z is defined as

\[ z = x - y \]  \hspace{1cm} (C2)

The half-sinusoid road disturbance is

\[ y(t) = Y\sin\omega t \]  \hspace{1cm} for \hspace{0.5cm} 0 \leq t \leq t_0 \]

and

\[ y(t) = 0 \]  \hspace{1cm} for \hspace{0.5cm} t \geq t_0 \]  \hspace{1cm} (C3)
Figure C1. SDOF model and the half-sinusoidal base disturbance.

Hence, the complete solution to equation (C1) for $0 \leq t \leq t_o$ and for $t > t_o$, respectively, may be written as

$$z(t) = Z \sin(\omega_n t - \theta) + e^{-\xi \omega_d t} (c_{11} \sin \omega_d t + c_{21} \cos \omega_d t) \quad \text{(C4)}$$

and

$$z(t) = e^{-\xi \omega_d t} (c_{12} \sin \omega_d t + c_{22} \cos \omega_d t) \quad \text{(C5)}$$

where the standard parameters are

$$z = \frac{(m \omega_n Y)}{k \sqrt{(1-r^2)^2 + (2 \xi r)^2}}, \quad \theta = \tan^{-1} \left( \frac{2 \xi r}{1-r^2} \right), \quad r = \frac{\pi}{t_o \omega_n}$$

$$\omega_n = \sqrt{\frac{k}{m}}, \quad \omega_d = \omega_n \sqrt{1-\xi^2}, \quad \xi = \frac{c}{2m \omega_n} \quad \text{and} \quad t' = t - t_o$$

The integration constants $C_{11}$, $C_{21}$ and $C_{12}$, $C_{22}$ need to be obtained from the initial conditions of the suspended mass at the instant when the disturbance is encountered, and from the solution of equation (C4) at time $t_o$, respectively. Closed form solutions (C4) and (C5), together with (C2), may be enumerated at discrete time.
intervals to produce the displacement, \( x(t) \). Velocity and the acceleration expressions may be obtained from the first and the second time derivatives of the same expressions straightforwardly.

### C3. Passive Shock Isolator

The non-dimensional displacement, \( x(t) / Y \), and acceleration, \( \dot{x}(t) / (\omega^2 Y) \), histories of the suspended mass in Figure C1, are presented in the first and the second columns of Figure C2. Here, \( Y \) and \( \omega \) are the peak displacement and the frequency of the base half-sinusoid disturbance, respectively. The non-dimensional base disturbance, \( y(t)/Y \) (- - -), is also presented in the first column for reference. Non-dimensional time, \( t/T_n \), is indicated along the horizontal axis where \( T_n \) is the undamped natural period of the suspended mass. Each row corresponds to a different value of \( r \), namely 0.2, 0.5, 1.0, 2.0, 5.0 and 10.0 in a descending order. \( r \) is the frequency ratio of the disturbance \( r = \pi / (t_0 \omega_n) \). In addition, for each value of \( r \), the response of the mass \( m \) is presented for four different damping ratios, \( \xi \). A very lightly damped shock absorber is represented with \( \xi = 0.01 \) (- - -), whereas an absorber with \( \xi = 0.99 \) (-----) is virtually critically damped. A 10\% (------) and a 30\% (-- --) damping ratio, on the other hand, are included to demonstrate the system response for a light and moderate damping, respectively.

It may be seen in the first three rows of the first column of Figure C2 (when \( r \leq 1 \)), that the largest displacement amplitudes are produced for the lightest damping case with \( \xi = 0.01 \). As expected, the peak displacement amplitude is monotonically reduced as the critical damping ratio is increased from 0.01 to 0.10, 0.30 and finally to, the almost critically damped, 0.99.
Generally, the system behaves like a stiff system when $r \leq 1$, following the ground disturbance relatively closely.

It may also be noticed in Figures C2(c) and C2(e), for $r = 0.5$ and $r = 1$, that the first peak displacement is delayed as the damping gets lighter. This trend may be explained by considering the two components of the total response of the oscillator separately, namely the transient response and the forced (or steady state) response. Initially, the transient component of the response is opposite in phase with the forced component of the response. A lightly damped system produces a large amplitude transient component which enables it to virtually suppresses the effect of the forced response at the start. As a result, a lightly damped system reaches its peak response during the second half of its natural period when the transient component is in phase with the forced component. For a heavily damped system, on the other hand, the transient component is smaller and easily overwhelmed by the forced component of the response. Therefore, peak response follows the disturbance with a short time delay for large damping. This particular trend will be of importance for the effectiveness of the suggested control in the next section.

In addition to the most favourable displacement response, the case with $\xi = 0.99$ rapidly dissipates the residual transient vibrations after the disturbance is over. But, this fast energy dissipation capability has an adverse effect on the acceleration response. The steep slope of the displacement, as soon as the half-sinusoid bump is encountered, results in a sharp rise of acceleration. In addition, the instant when the disturbance is over is also exaggerated. In spite of its discontinuous nature, however, the acceleration amplitudes
for $\xi = 0.99$, are either comparable or smaller than those of the other three lightly damped systems.

For $r > 1$, in the last three rows of Figure C2, the fast energy dissipation capability of high damping produces quite unfavourable results. As the damping ratio increases, the transient component becomes smaller, and the response is primarily dictated by the forced component. In addition, the amplitude of this forced component increases with $r$, as the value of damping increases. In Figure C2(g) when $r = 2.0$, the peak displacements of the smaller three $\xi$ are comparable to that of $\xi = 0.99$. In the last two rows of Figure 2 when the frequency of excitation is further increased to $r = 5.0$ and $r = 10.0$, the maximum displacement of $\xi = 0.99$ case is larger than those of the lightly damped cases.

Acceleration response for cases when $r > 1$, follows the same trend as the displacement response. For $\xi = 0.99$ and $r > 1$, the first rapid change of state from rest, produces a significantly large acceleration amplitude as soon as the disturbance is encountered. With a lightly damped system, on the other hand, this large acceleration peak is virtually eliminated because of the slow build-up rate at the start. However, the slow rate of energy dissipation of a lightly damped system, is not capable of dissipating the residual transient vibrations effectively. Hence, the remaining transient component after the disturbance is over, takes a long time to decay as shown in Figures C2(g) to C2(l). This last deficiency of a lightly damped system makes it a very poor choice as an effective shock absorber of ground vehicles.

In summary, light damping produces a small acceleration response. Displacement response of a lightly damped system is quite acceptable when $r > 1$. For $r \leq 1$, however,
light damping results in large displacements. In addition, transient oscillations take a long time due to poor energy dissipation with light damping. On the other hand, large damping produces quick dissipation of transient oscillations, and small displacement response at $r \leq 1$. But large damping produces large accelerations, and large displacement response at $r > 1$. Therefore, no passive shock absorber is capable of performing favourably at all values of $r$. 
Figure C2. Histories of displacement and acceleration of the suspended mass of a passive system when (a), (b) $r = 0.2$; (c), (d) $r = 0.5$; (e), (f) $r = 1.0$; (g), (h) $r = 2.0$; (i), (j) $r = 5.0$; (k), (l) $r = 10.0$

---, -------: indicates $\xi = 0.01$, $\xi = 0.10$, $\xi = 0.30$, $\xi = 0.99$, respectively.

---, indicates the base disturbance in the first column.
Figure C2. Histories of displacement and acceleration of the suspended mass of a passive system when (a), (b) $r = 0.2$; (c), (d) $r = 0.5$; (e), (f) $r = 1.0$; (g), (h) $r = 2.0$; (i), (j) $r = 5.0$; (k), (l) $r = 10.0$. 

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Figure C2. Histories of displacement and acceleration of the suspended mass of a passive system when (a), (b) $r = 0.2$; (c), (d) $r = 0.5$; (e), (f) $r = 1.0$; (g), (h) $r = 2.0$; (i), (j) $r = 5.0$; (k), (l) $r = 10.0$

- indicates $\xi = 0.01$, $\xi = 0.10$, $\xi = 0.30$, $\xi = 0.99$, respectively.
- indicates the base disturbance in the first column.
A variable parameter damper which starts from an advantageous value of $\xi$, as soon as a disturbance is encountered, and experiences a certain sequence of change, may be able to produce a good compromise between the high dissipation rate of a heavily damped system and a lightly damped system with a small acceleration response. From the results presented in Figures C2(g) to C2(l), for $r > 1$, the initial value of $\xi(t)$ should certainly be small in order to avoid the excessively large accelerations at the start. However, $\xi(t)$ should increase rapidly in time in order to limit the displacement amplitudes from becoming too large for $r \leq 1$, and also to produce an acceptable rate of decay during transient vibrations.

![Diagram](image)

**Figure C3.** The suggested sequence of $\xi$ update with a constant increment and even interval.

To demonstrate the suggested technique, a 10% and a 99% critical damping ratios are chosen (quite arbitrarily) to be the starting and the final values, respectively. This choice of the starting and the final values fixed the range of variation as $\Delta \xi = 0.89$. As shown in Figure C3, once the control is triggered, the damping ratio is started from $\xi(t) = 0.10$, and it is maintained at this value for $\frac{T_n}{N}$ seconds. Then $\xi(t)$ is incremented by a
constant “Δξ / M” at every T_n/N seconds until the final value of ξ(t) = 0.99 is reached. Damping ratio is kept at 0.99 until the effect of the external disturbance diminishes. Once the two control parameters N and M (representing the integer fractions of the undamped natural period and the fixed range of Δξ = 0.89, respectively) are chosen, the numerical solution of such a semi-active system may be obtained in a similar manner to that of a passive system. However, the damping coefficient in these two equations has to be updated at every T_n/N seconds.

C4.1. Results

The effectiveness of the variable parameter shock absorber was evaluated by performing numerical simulations for different integer values of N and M varying between 1 to 12 and 1 to 10, respectively. The case discussed in this section, was picked for its simplicity and effectiveness in minimizing the peak displacement and acceleration along with a reasonably rapid decay rate over the range of r from 0.2 to 10.0.

The response of the suggested shock absorber is shown in Figure C4, in the same format as in Figure C2. The control parameters are N = 10 and M = 1. Hence, the critical damping ratio is started from 0.10 as soon as the control is triggered due to the half-sinusoid bump. This value of damping is maintained for a duration of T_n/10 seconds (N = 10). Consequently, the critical damping ratio is changed to 0.99, in one step (M = 1), and maintained at this value. The maximum displacement, maximum acceleration and the settling time obtained with this shock absorber are compared to those of the constant parameter absorber with ξ = 0.99 in Table C1. The settling time in the third column of Table C1, is defined as the time required for the amplitude of the
displacement of the suspended mass, to be smaller than a threshold displacement. This threshold value was taken to be the 5% of the peak displacement of the constant parameter system with a moderate $\xi = 0.30$ at $r = 1.0$. It should be noted that the choice of threshold displacement, is quite arbitrary. The settling time values are reported only for comparison purposes.

As may be seen in this Table C1, the suggested absorber maintained the effectiveness of the heavily damped constant parameter absorber for $r \leq 1$. For $r > 1$, on the other hand, the peak displacement of the constant parameter absorber is improved by 40% to 50%. The improvement of the peak acceleration is even more significant than that of the peak displacement, reaching up to 89%. This improvement is not surprising, considering that starting with $\xi = 0.10$ allows for completely bypassing the instant of the large acceleration peak of the constant parameter case with $\xi = 0.99$. Although, there is another rather large discontinuity at the instant when the half-sinusoid bump is over, the acceleration amplitude corresponding to this second discontinuity is quite small. In contrast with the displacement and acceleration, the settling time response of the suggested absorber is elongated by up to 2.33 times as compared to that of the constant parameter absorber. This elongated settling time may be attributed to the sluggish start of oscillations due to light damping. Although switch to the large damping is activated quickly, the slow start results in a wider spread of the system's response. Despite this relative deterioration in the settling time, transient oscillations are still dissipated fairly rapidly in absolute sense.
Table C1. Ratio of the peak displacement, peak acceleration and settling time of the variable parameter suspension to those of the constant parameter suspension with $\xi = 0.99$.

<table>
<thead>
<tr>
<th>$r$</th>
<th>Displacement</th>
<th>Acceleration</th>
<th>Settling Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.94</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>1</td>
<td>0.75</td>
<td>0.91</td>
<td>1.05</td>
</tr>
<tr>
<td>2</td>
<td>0.39</td>
<td>0.82</td>
<td>1.99</td>
</tr>
<tr>
<td>5</td>
<td>0.51</td>
<td>0.15</td>
<td>2.33</td>
</tr>
<tr>
<td>10</td>
<td>0.47</td>
<td>0.11</td>
<td>2.20</td>
</tr>
</tbody>
</table>
Figure C4. Histories of displacement and acceleration of the suspended mass of a variable parameter system with $\omega_0^2, c_0, \alpha_0, \beta_0, n_0 = 0.5, r = 0.5, \lambda = 1$, for $r = 0.2, 0.5, 1.0, 2.0, 5.0, 10.0$, $N = 10$, $M = 1$, $\lambda$ indicates the variable parameter suspension's response and base disturbance, respectively.
Figure C4. Histories of displacement and acceleration of the suspended mass of a variable parameter system when (a), (b) \( r = 0.2 \); (c), (d) \( r = 0.5 \); (e), (f) \( r = 1.0 \); (g), (h) \( r = 2.0 \); (i), (j) \( r = 5.0 \); (k), (l) \( r = 10.0 \). N = 10, M = 1

\( \gamma \), \( \gamma \), indicates the variable parameter suspension’s response and base disturbance, respectively
C5. On Application

Ford Mondeo has been made available with an adaptive suspension system based on two-state dampers [Tavner et al. (1997)]. The state of these dampers is controlled electronically in response to body acceleration, the road and the driving conditions. A design of an ER damper suggested in Chapter 3 is of similar configuration to that of Tavner et al. (1997). A simple schematic view of the suggested ER damper is shown in Figure C5. When there is no electric field is applied, the damper behaves as a “soft” damper. ER fluid is allowed to flow freely through the valve. When an electric field is applied, however, there is increased resistance against the flow. As a consequence of the increased resistance, the damper behaves as a “hard” damper. The actuator of the semi-active control suggested in this work would normally maintain soft damping until a disturbance is sensed. Then the required electric field is applied on the ER valve to provide the high damping soon after.

Implementing a variable damping strategy with an ER fluid has the very obvious advantage of not requiring moving mechanical parts. In addition, extending a two-state variation into a multi-state one, requires no special additional hardware with ER fluids. A multi-state, predetermined sequence of damping variation could extend the capability of the suggested control from transient vibrations alone to forced vibration cases where the excitation is provided by the random road roughness in the case of ground vehicles.
Figure C5. A schematic representation of a variable damping absorber with ER fluid.

Figure C6 is repeated from Chapter 3 to indicate the range variation with the corn starch-mineral oil ER fluid used in this study. Particularly at the low frequencies and at the largest available electric potential of 4 kV (corresponding to 1.33 kV/mm of electric field), that could be applied safely. It is possible to obtain a damping coefficient of about three times larger than the case of no voltage. Considering the positive slope of the variation with the electric field, it is anticipated that this three-fold increase may increase significantly with higher applied potential. In addition, an extensive listed of different (and more costly) chemical compositions is reported by Makela 1999 and commercially available ER fluid from Advanced Fluid Systems Ltd are up to an order of magnitude stronger than the ER fluid used in this study. Hence, it is possible to obtain a damping variation by a factor of 10 and larger with the application of electric potential. It is anticipated that such an ER dampers will be in application soon not only in laboratories and experimental prototypes but in mass production for commercially available vehicles.
C6. Conclusions

A two-state viscous damper is suggested as a controller of the excessive transient vibrations of a SDOF oscillator with a transient base disturbance. The technique is based on a step change in the damping ratio, over a predetermined duration. Hence, it is a significant simplification over the existing semi-active techniques. This simplified variable parameter isolator is capable of effectively controlling both the displacement and the acceleration response of the suspended mass. In addition, the settling times obtained with this suggested technique are quite comparable to those of a critically damped passive system.

It should be mentioned here that these results are strictly limited to a single event transient disturbance. In cases where multiple transient disturbances are encountered, similar to a case of
road roughness, several additional measures have to be taken to assure satisfactory performance. These measures may include determining proper thresholds to trigger the increase in damping after a disturbance, and to return to a light damping phase, after the oscillations decay to an acceptably small levels.

The present study assumes linear response from both the spring and the damper of the shock-isolator. However, both of these components used in the automotive suspensions may behave non-linearly [Bastow (1988), Tavner et al. (1997), Nagai et al. (1996)] with varying hardening and softening characteristics. Dampers may respond significantly differently during extension (rebound) and compression (bump), generally producing larger resistances during extension. In addition, suspension springs often show hysteresis. Hence, conclusions of this study certainly need to be verified against such non-linearities.
APPENDIX D

BINGHAM PLASTIC BEHAVIOUR OF AN ELECTRORHEOLOGICAL FLUID

The focus of this appendix is to give some understanding of an ER fluid’s idealized behaviour. The first part briefly discusses the shear stress - strain rate behaviour of a Newtonian fluid. Then, the Bingham behaviour of an ER fluid in the shear mode is presented, followed by the concept of an ER fluid in the valve flow mode. Further objective of this appendix is to relate the two types of operation, namely, the shear and valve modes.

D1. Viscosity of a Newtonian Fluid

Consider a thin layer of fluid between two parallel plates as shown in Figure D1 below.

![Figure D1](image)

Figure D1. The configuration used in the derivation of the viscosity expression.

The bottom plate is fixed, and a shearing force \( F \) is applied to the moving top plate. The fluid which is in contact with the top disk moves with its velocity, and the fluid which is in contact with the fixed bottom plate has a zero velocity. The fluid between
the two plates moves with a velocity as a function of the distance y from the fixed plate, \( u = u(y) \). This velocity distribution may be assumed to be linear if the separating distance h is small. If the rate of strain is expressed in terms of the velocity gradient, \( \frac{du}{dy} \), then the constant of proportionality to express the resulting shear stress, \( \tau \), becomes the fluid viscosity of a Newtonian fluid.

\[
\tau = \mu \frac{du}{dy} \quad \text{(D1)}
\]

Here, \( \mu \) is the absolute viscosity of the fluid, with units of Pa.s in the SI unit system.

**D2. Bingham Plastic Behaviour in Shear Mode**

In the shear mode of operation, the ER fluid is contained in the same arrangement as shown in Figure D1, and the top and bottom plates are used as the conductor plates. The expected shear stress versus strain rate relationships is illustrated in Figure D2.

![Figure D2. Ideal shear stress versus strain rate behaviour for a Bingham plastic ER fluid.](image-url)
When there is no electric field applied, an ER fluid behaves as a Newtonian fluid, and this behaviour may be represented by line a. Any applied shear stress causes the fluid to flow when there is no electric field. When an electric field is applied, E, Line b shows the Bingham plastic behaviour which is characterized by a straight line with an intercept \( \tau_b \) on the shear stress axis. This yield stress, \( \tau_{b1} \), is the yield shear strength at zero strain rate which must be exceeded before the flow starts. The plastic viscosity, \( \mu_p \), is the slope of line b above the yield stress, \( \tau_{b1} \). Below this value, any applied stress will strain the fluid but not cause it to flow.

Further increase in applied electric field will shift line b upward and approximately parallel to itself to become line c, and the zero strain rate yield stress changes from \( \tau_{b1} \) to \( \tau_{b2} \). This type of behavior is an ideal Bingham plastic behaviour [Simonds 1991 and Wilkinson 1960], and it can be expresses as.

\[
\tau = \tau_b + \mu_p \gamma \quad \text{(D2)}
\]

where \( \mu_p \) is the plastic viscosity in Pa.s and \( \tau_b \) is the shear yield stress in Pa in the SI unit system. \( \tau_b \) represents the effect of the applied electric field E.

**D3. Bingham Plastic Behaviour in Valve Mode**

Bingham plastic behaviour in valve mode is similar to that of the shear mode. In the shear mode of operation, the ER fluid is subjected to a uniform shear, whereas in the valve mode of operation, the fluid ER fluid is squeezed between the electrodes as it flows. Thus, when the ER fluid develops a yield strength in response to an electric field, the effect of this yield strength is to produce pressure drop along the length of the two plates.
The purpose of this section is to develop an equivalent equation to Equation D2 for the valve mode. The shear stress and strain rate terms in Equation D2 would be equivalent to pressure drop and flow rate, respectively, in the valve mode. The following paragraphs are to establish these two parameters [Peel et al. (1994)].

Consider a fluid flowing through a section of an annular channel of a constant gap \( h \) as shown in Figure D3. Assuming that \( h \) is sufficiently small, and the mean annular radius \( R \) is sufficiently large, \( h \ll R \), the curvature effect on the flow will be small. Also, the length is assumed to be sufficiently long for the end effects to be small (\( R < L \)). With no edge effects caused by a finite width, this flow tends to be that between two parallel flat plates [Peel et al. (1994)].

![Figure D3. Showing the Bingham plastic valve flow and relevant parameters.](image)

Take an element of fluid of length \( L \) and gap \( h \) as shown in Figure D4. When an electric field is applied, the maximum shear stress occurs at the channel wall. No shear occurs at the center of the channel. This no shear region is called the plug region, and it is represented by the dashed line in Figure D4. For a steady flow, the pressure drop \( \Delta P = P_1 - P_2 \) is along the electrode length \( L \).
The balance of wall shear and pressure forces becomes

\[ 2 L b \, \tau_w = \left( P_1 - P_2 \right) h b \quad \text{(D3)} \]

where \( 2Lb \) is the wall surface area and \( hb \) is the cross sectional area, and the subscript “w” represents the wall value.

Re-aranging Equation D3, the shear stress at the wall, \( \tau_w \), becomes

\[ \tau_w = \frac{h \Delta P}{2L} \quad \text{(D4)} \]

The term \( h/L \) is effectively a multiplier acting on the available yield stress in the ER fluid to produce the pressure drop. The factor of 2 arises because both the top and bottom valve plates contribute to the shear resistance.

For a Bingham plastic flow, as discussed earlier, \( \tau_w \) can be rewritten as

\[ \tau_w = \tau_b + \mu \left( \frac{du}{dy} \right)_{y = 0} \quad \text{(D5)} \]
Substituting Equation D4 into D5, the velocity gradient at the wall becomes

\[
\left( \frac{du}{dy} \right)_{y=0} = \frac{1}{\mu} \left( \frac{h\Delta P}{2l} - \tau_b \right) \tag{D6}
\]

To apply a similar procedure to the plug alone, the control volume in Figure D5 is considered next.

**Figure D5.** Forces acting on a “plug”, (a) side view and (b) front view.

The pressure force,

\[ F_P = (P_1 - P_2) \delta b \]

is balanced with the shear force in the plug:

\[ F_S = \tau_b 2(Lb) \]

where the subscripts S and b refer to total shear and bulk, respectively.

Therefore, the balance of forces on the plug is

\[ 2l\tau_b = \Delta P \delta \]

Re-arranging Equation D7,

\[ \tau_b = \frac{\Delta P \delta}{2L} \tag{D8} \]

Dividing Equations D8 and D4 side by side,

\[ \frac{\tau_b}{\tau_w} = \frac{\Delta P \delta}{h \Delta P} = \frac{\delta}{h} \tag{D9} \]
Equation D9 suggest a linear shear stress variation between the wall and the plug, Equation D9 also suggests that as \( \delta \) approaches to \( h \), the plug tends to fill the channel as the flow diminishes.

### D3.1 Velocity Profile in Valve Flow

Taking the coordinate \( v \) from the center in Figure D3, the velocity gradient at the wall \((v = h/2)\) in Equation D6 can be rewritten as,

\[
\left( \frac{du}{dv} \right)_{v = \frac{h}{2}} = -\frac{du}{dy}_{y = 0} = -\frac{1}{\mu} \left( \frac{v\Delta P}{L} - \tau_b \right)_{v = \frac{h}{2}} \tag{D10}
\]

Integrate Equation D10 for the region outside of plug with respect to \( v \), with \( h/2 \) replaced by \( v \) to indicate the linear variation as implied in Equation D9:

\[
u(v) = \int -\frac{1}{\mu} \left( \frac{\Delta P v}{L} - \tau_b \right) dv + C \quad \text{for} \quad \frac{\delta}{2} \leq v \leq \frac{h}{2} \tag{D11}
\]

Evaluate \( C \) from the boundary condition, \( u(v = \frac{h}{2}) = 0 \), the velocity profile outside of the plug becomes,

\[
u = \frac{1}{\mu} \left[ \frac{\Delta P}{2L} \left( \frac{h^2}{4} - v^2 \right) - \tau_b \left( \frac{h}{2} - v \right) \right] \quad \tag{D12}
\]

In the plug region, the velocity is constant at its largest value:

\[
u_{\text{max}} = u \left( v = \frac{\delta}{2} \right)
\]

\[
u_{\text{max}} = \frac{1}{\mu} \left[ \frac{\Delta P}{8L} \left( h^2 - \delta^2 \right) - \frac{\tau_b}{2} \left( h - \delta \right) \right] \quad \text{for} \quad 0 \leq v \leq \frac{\delta}{2} \tag{D13}
\]
The volume flowrate outside of plug can be computed by integrating Equation D12 over the elementary area of the channel, bdv,

\[ Q_1 = 2 \int u (v) bdv \quad \text{equation (D14)} \]

Since the velocity gradient outside of the plug consists of the upper and lower regions, the volume flowrate is multiplied by 2. Evaluating the integration in Equation D14, the flow rate outside the plug becomes,

\[ Q_1 = \frac{b}{24\mu} \left\{ \frac{\Delta P_{eo}}{L} \left( 2h^3 - 3h^2 \delta + \delta^3 \right) - 6\tau_b \left( h - \delta \right)^2 \right\} \quad \text{equation (D15)} \]

The volume flowrate within the plug can be computed by multiplying Equation D13 by the cross sectional area of the channel, \( \delta b \):

\[ Q_2 = u_{\text{max}} \delta b \quad \text{equation (D16)} \]

\[ Q_2 = \frac{b\delta(h - \delta)}{8\mu L} \left\{ \Delta P_{eo} \left( h + \delta \right) - 4\tau_b L \right\} \quad \text{equation (D17)} \]

The total volume flowrate within the channel is the sum of \( Q_1 \), outside the plug, and \( Q_2 \), inside the plug:

\[ Q_{\text{total}} = Q_1 + Q_2 \quad \text{equation (D18)} \]

\[ Q_{\text{total}} = \frac{b}{24\mu} \left\{ \frac{\Delta P_{eo}}{L} \left( 2h^3 - 3h^2 \delta + \delta^3 \right) - 6\tau_b \left( h - \delta \right)^2 \right\} + \]

\[ \frac{b\delta(h - \delta)}{8\mu L} \left\{ \Delta P_{eo} \left( h + \delta \right) - 4\tau_b L \right\} \quad \text{equation (D19)} \]
Substitute $\delta = \frac{2 L \tau_b}{\Delta P_{co}}$ from Equation D8 into Equation D19 and rearrange,

$$Q_{Total} = \left( \frac{bL^2}{3 \mu \Delta P^2} \right) \tau_b^3 - \left( \frac{bh^2}{4 \mu} \right) \tau_b + \frac{\Delta Pb h^3}{12 \mu L} \quad \text{(D20)}$$

To simplify Equation D20, multiply both sides of the equation by $\frac{\Delta Pb h^3}{12 \mu L}$ and rearrange, the Bingham plastic equation for valve flow becomes,

$$4 \left( \frac{L}{h \Delta P} \right)^3 \tau_b^3 - 3 \left( \frac{L}{h \Delta P} \right) \tau_b + \left( 1 - \frac{12 \mu L Q}{bh^3 \Delta P} \right) = 0 \quad \text{(D21)}$$

where $L$, $b$ and $h$ are the system parameters, and $\mu$ is the Newtonian viscosity. $\Delta P$ and $Q$ are the pressure drop and volumetric flow rate. $\tau_b$ in Equation D21 is the yield shear strength at zero strain rate, and it is given in Equation D2.

**D4. Using the Bingham Model as a Prediction Tool**

Equation D21 represents the Bingham plastic behaviour in the valve mode configuration. The purpose of this section is to present the use of Equation D21 as a prediction tool for valve mode applications, such as for a shock absorber. For such an application, it is desirable to know the damping coefficient as a function of the geometric parameters and the applied voltage.

The pressure drop, $\Delta P$, in Equation D21 represents the damping force for a given stroke length, frequency and applied voltage. The stroke length and frequency are represented by $Q$ in Equation D21. The applied voltage is represented by $\tau_b$. 
From Equation D21, and for no electric field \((\tau_b = 0)\),

\[
\left[ 1 - \frac{12 \mu L Q}{b h^3 \Delta P} \right] = 0 \quad \text{(D22)}
\]

Re-arranging Equation D22

\[
\Delta P = 12 \frac{\mu L}{b h^3} Q = \frac{12 L}{b h^3} \mu Q = \mu' Q \quad \text{(D23)}
\]

**Figure D6.** Variation of \(\Delta P\) with \(Q\) for Newtonian behaviour.

In Figure D6, the slope of the \(\Delta P\) versus \(Q\) variation may be identified as \(\frac{12L}{bh^3} \mu\).

Remembering that this slope is simply “\(\mu\)” alone in the shear mode to relate \(\tau\) to \(\gamma\), the multiplier \(\frac{12L}{bh^3} \mu\) represents the equivalent viscosity term in the valve mode.
In Equation D21, when an electric field is applied on the ER fluid, the variation of $\Delta P$ versus $Q$ is a cubic one. One such variation, for the experimental parameters $b, h$ and $L$, as specified in Chapter 3 and 4, is shown in Figure D7. The value of $\tau_b$ in Figure D7 is 524 Pa, corresponding to an electric field of 2kV/mm. The variation of $\Delta P$ suggested in Figure D6 is also repeated in this frame for reference.

Figure D7. Variation of $\Delta P$ with $Q$ for a Bingham fluid.

Due to the nonlinear nature of the Bingham behaviour, two distinct values of $\Delta P$ may be obtained for a given value of $Q$. The only exception to this dual $\Delta P$ is when $Q = 0$. From Equation D21 and for no flow,

$$4\left(\frac{L}{h\Delta P}\right)^3 \tau_b^3 - 3\left(\frac{L}{h\Delta P}\right)\tau_b = 0 \quad \text{(D24)}$$
re-arranging and cancelling the common \( \frac{L}{h \Delta P_0} \tau_b \) term,

\[
4 \left[ \frac{L}{h \Delta P_0} \right] \tau_b^2 - 3 = 0
\]

\[
\left[ \frac{L}{h \Delta P_0} \right]^2 = \frac{3}{4 \tau_b^2}
\]

\[
\frac{L}{h \Delta P_0} = \sqrt{\frac{3}{4}} \tau_b
\]

\[
\Delta P_0 = \frac{L}{h \sqrt{\frac{3}{4}}} \tau_b
\]

Hence, \( \Delta P_0 \) indicates how much pressure difference must be provided before any flow can result in the valve mode. Similar to the equivalent viscosity term earlier, the group \( \frac{L}{h \sqrt{\frac{3}{4}}} \) times \( \tau_b \), represents the equivalent \( \tau_b \) in the valve mode. The similarity of the shear and valve mode operations is further illustrated in Figure D8.

**Figure D8.** Bingham plastic behaviour in (a) shear and (b) valve modes.
As indicated earlier, multiple values of $\Delta P$ exist for a given $Q$. The lower $\Delta P$ may be neglected easily as this value is physically impossible. The justification of such a procedure is the implication of a negative viscosity in the lower branch of the non-linear $\Delta P$ variation. The upper branch, on the other hand, is approximately parallel to the Newtonian behaviour with a slope of $\frac{12L}{bh^3} \mu$. Hence, the higher $\Delta P$ should be used.

In a similar fashion to the shear mode, now the pressure drop can be expressed as

$$\Delta P = \Delta P_o + \mu'Q$$

Or

$$\Delta P = \frac{L}{h\sqrt{\frac{3}{4}}} \tau_b + \frac{12\mu LQ}{bh^3}$$ \hspace{1cm} (D26)

In this final expression, once the geometric parameters ($b$, $h$, $L$) are specified, for a given fluid ($\mu$) and operating condition ($Q$), the pressure drop can be estimated in terms of $\tau_b$. $\tau_b$, of course, is the direct consequence of the applied electric field. Hence, it is now possible to predict the pressure drop as a function of the control parameter, applied electric field. As mentioned earlier, predicting pressure drop will result in the prediction of the equivalent damping coefficient for applications where variation of this coefficient with electric field is beneficial for vibration control.
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