

# Observer-based fault-tolerant control for a class of networked control systems with transfer delays <sup>\*</sup>

Zehui Mao<sup>†‡</sup>, Bin Jiang<sup>‡</sup>, Peng Shi<sup>§</sup>

**Abstract:** In this paper, an observer-based fault-tolerant control (FTC) method is proposed for a class of networked control systems (NCSs) with transfer delays. Markov chain is employed to characterize the transfer delays. Then, such kind of networked control systems are modelled as markovian jump systems. An observer-based FTC scheme using the delayed state information and the estimated fault value is presented to guarantee the stability of the faulty systems. An inverted pendulum example is used to illustrate the efficiency of the proposed method.

**Keywords:** Networked control systems, Markovian jump systems, input delays, fault-tolerant control

## 1 Introduction

With the rapid development of communication networks, recently, a great amount of effort has been devoting to the problems of networked control systems. NCSs are control systems in which controller and plant are connected via a communication channel. The defining feature of an NCS is that information (reference input, plant output, control input, etc.) is exchanged using a network among control system components (sensors, controller, actuators, etc.). The primary advantages of NCSs are low cost, simple installation and maintenance, increased system agility and reduced system wiring [1].

Many models have been established to represent the NCSs with its specific features, such as discrete model with delay [1], Markovian jump model [2], T-S model [3], hybrid model [4] and so on. Therefore, some method proposed for these models, such as [5]-[8], can be modified to use in NCSs. Meanwhile,

---

<sup>\*</sup>Partially supported by NNSFC Research Funding (NS2010071, NZ2010003), the National Natural Science Foundation of China (60874051) and Engineering and Physical Sciences Research Council, UK (EP/F029195).

<sup>†</sup>Tsinghua National Laboratory for Information Science and Technology, and Department of Automation, Tsinghua University, Beijing 100084, P. R. China. Email: zehuimao@nuaa.edu.cn

<sup>‡</sup>College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, P. R. China. Email: binjiang@nuaa.edu.cn (B. Jiang).

<sup>§</sup>Department of Computing and Mathematical Sciences, University of Glamorgan, Pontypridd, CF37 1DL, UK. Email: pshi@glam.ac.uk. He is also with the School of Engineering and Science, Victoria University, Melbourne 8001 Vic, Australia, and School of Mathematics and Statistics, University of South Australia, Mawson Lakes, 5095 SA, Australia.

corresponding methods [9]-[12] for these models are proposed to design and analysis NCSs. The faults in NCSs are similar to that in the conventional systems, which can lead to the performance degradation and even instability. Fault detection and diagnosis (FDD) and fault tolerant control (FTC) procedures can be designed to guarantee that the system goal is still achieved in spite of the faults (see[13]; [14] for details).

In the past years, there have existed some results about the FDD and FTC for NCSs. For some representative work on this general topic, to name a few, we refer the readers to [15, 16] and the references therein. In [17], fault detection method was proposed for a class of continuous-time Networked control systems (NCSs) with non-ideal network Quality of Service (QoS), which was described by an integrated index related to the network-induced delay, data dropout and error sequence. In [18], NCSs with with random but bounded delays were considered. Using augmented state-space method and improved V-Iteration algorithm, a class of reliable controllers were designed to make systems asymptotically stable under several stochastic disturbances and actuator failure. Among these literatures, many fruits are about fault detection and passive FTC. For passive FTC, the system is made “robust” against faults by assuming them as uncertainties [19]. In active FTC, a new control system is redesigned with (hopefully) all of the desirable properties of performance and robustness when the faults occur. Thus, it can be seen that the same controller utilized in the passive FTC systems before and after the occurrence of the fault, cannot insure the good performance of both the healthy and faulty systems. To the best of our knowledge, until now, few result has been reported about active FTC for NCSs. This motivates us to study this interesting and challenging problem, which has great potential in practical applications.

Based on our previous work [20]-[22], we present an active FTC method for NCSs with large transfer delays. We employ the multirate sampling technique to model the large random delay NCSs as Markovian jump systems with input delays. Under this model, an observer-based fault diagnosis method is proposed, which can provide accurate estimations of states and faults after faults occur. Based on the fault estimation and delayed state information, an active fault-tolerant control is designed to achieve the system stability. Finally, an inverted pendulum example is used to demonstrate the effectiveness of the theoretic results obtained.

The rest of this paper is organized as follows. Section 2 gives the system description and the definition of the design problem. An observer-based fault estimation method is derived in Section 3. Section 4 proposes the design method of the FTC with input delays. Application to an inverted pendulum example is presented in Section 5, followed by some concluding remarks in Section 6.

## 2 System description and problem formulation

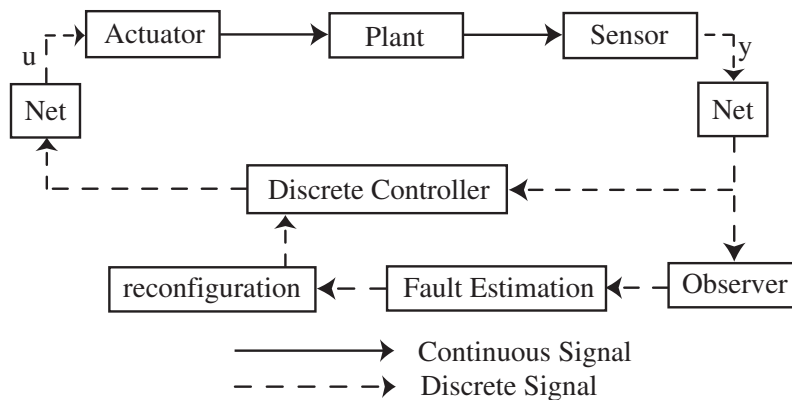


Figure 1: The block of the networked control system with FTC

Consider the NCSs as shown in Fig. 1. The continuous-time, state-space model of the linear time-invariant plant dynamics can be described by the following standard form:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ef(t) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where  $x \in R^n$  is the state vector,  $u \in R^m$  is the control input vector,  $y \in R^r$  is the measurable output vector. The matrices  $A$ ,  $B$ ,  $C$ ,  $E$  are real matrices of appropriate dimensions,  $C$  is of full row rank. The failure  $f(t) = \beta(t-T)f_0 \in R^q$  can be regarded as an additive signal, where the function  $\beta(t-T)$  is given by  $\beta(t-T) = \begin{cases} 0, & t < T \\ 1, & t \geq T \end{cases}$ . That is,  $f(t)$  is zero prior to the failure time ( $t < T$ ) and is a constant vector  $f_0 \in R^q$  after the failure occurs ( $t \geq T$ ).

For this NCS, we introduce the following assumptions:

**Assumption 1:** The sampling period of the NCS is  $T$ , sensor is time-driven, controller and actuator are time-division-driven. The data sampled from the sensor are packed with the time-stamp, thus as this data arrived at the controller, the sensor-controller transfer delay can be known.

We use  $\tau_{sc} < d_1T$ ,  $d_1 \in \mathbb{Z}$  and  $\tau_{ca} < d_2T$ ,  $d_2 \in \mathbb{Z}$  to represent the sensor-controller and controller-actuator delay, respectively, and net delays  $\tau \triangleq \tau_{sc} + \tau_{ca} < (d_1 + d_2)T = dT$ .

**Assumption 2:** The sampling interval  $[kT, (k+1)T]$  is divided into  $N$ .

From Assumption 2, we can obtain that the delays  $\tau_{sc}$  belongs to the set  $\varphi = \{\bar{s}\frac{T}{N}\}$  with  $\bar{s} = 0, 1, \dots, d_1N$  and  $\tau_s$  belongs to the set  $\psi = \{s\frac{T}{N}\}$  with  $s = 0, 1, \dots, dN$ . Further, we have  $\varphi \subset \psi$ .

**Remark 1:** We choose the same division for controller and actuator here. To more general case, the divisions can be different, we can also get the set of the delays. For example, the pieces are chosen as  $p$

and  $q$  to the controller and actuator, respectively. Then the delay sets are defined as  $\{i\frac{T}{p}\}$ ,  $\{i\frac{T}{p} + j\frac{T}{q}\}$ , where  $i = 0, 1, \dots, d_1p$ ,  $j = 0, 1, \dots, d_2q$ .

Denote  $\tau_s$ ,  $\bar{s} = 0, 1, \dots, d_1N$  and  $\tau_{\bar{s}}$ ,  $s = 0, 1, \dots, dN$  as the sensor-controller and net delays, respectively. Then, under the Assumptions 1 and 2, considering the sampling period and the affect of delays, in one sampling period, the above plant's model is transformed into (the local model):

$$x(k+1) = A_c x(k) + \sum_{s=0}^m B_s u(k - \tau_s) + E_c f(k) \quad (3)$$

$$y(k) = C x(k - \tau_{\bar{s}}) \quad (4)$$

where  $x(k) = x(kT)$ ,  $y(k) = y(kT)$ ,  $f(k) = f(kT)$ ,  $A_c = e^{AT}$ ,  $B_s = \int_{T-\tau_{s-1}}^{T-\tau_s} e^{At} B dt$ ,  $E_c = \int_0^T e^{A(T-t)} E dt$ . It is easy to show that the terms  $B_s$  are variable and defined by the upper and lower bound of the integral which is about the time delays. It has been proven that the transfer delay sequences  $\tau_s$  and  $\tau_{\bar{s}}$  are Markov chains, whose probability distribution can be obtained by experiment method [15]. For the net delays  $\tau_s$ , the known one-step transition probability from state  $i$  to state  $j$ ,  $i, j \in \psi$  is given by  $p_{ij}$ , i.e.  $p_{ij} = Prob(B_s(k+1) = j | B_s(k) = i)$ . Then, the global model of the NCS can be described as

$$x(k+1) = A_c x(k) + \sum_{s=0}^m B_{is}(k) u(k - s) + E_c f(k) \quad (5)$$

$$y(k) = C x(k - \bar{s}) \quad (6)$$

where  $B_{is}$  represents the  $s$ th input delay of the  $i$ th model of the Markovian jump system,  $i = 0, 1, \dots, d(N+1) + 1$ ,  $m = dN$ .

Now, we formulate the design problem as follows:

- Under the above Markovian jump model, a fault estimation scheme is presented, which can estimate the NCSs states and faults accurately.
- The proposed active fault-tolerant controller can stabilize the NCSs using the delayed state and fault estimation information.

### 3 Observer-based fault estimation

From the analysis of above section, it follows that, due to network, the system output applied to observer/controller contains the transfer delay. that means the observer/controller only can used the delayed system output to estimate/control the system state. Therefore the following observer can only estimate the state value that is not the system current value, but some time before.

We define a transformation  $x = N^{-1}z$ , which can transform the system (5)-(6) into the following form:

$$\begin{bmatrix} z_1(k+1) \\ z_2(k+1) \\ z_3(k+1) \end{bmatrix} = \begin{bmatrix} A_{c1} \\ A_{c2} \\ A_{c3} \end{bmatrix} z(k) + \sum_{s=0}^m \begin{bmatrix} B_{is1}(k) \\ B_{is2}(k) \\ B_{is3}(k) \end{bmatrix} u(k-s) + \begin{bmatrix} E_{c1} \\ E_{c2} \\ E_{c3} \end{bmatrix} f(k) \quad (7)$$

$$y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 0 & I_{r-q} & 0 \\ 0 & 0 & I_q \end{bmatrix} z(k-\bar{s}) \quad (8)$$

where  $z_1(k) \in R^{n-r}$ ,  $z_2(k) \in R^{r-q}$ ,  $z_3(k) \in R^q$ . Therefore, only  $z_1(k)$  needs to be estimated in mean square.

**Assumption 3:**  $\text{Rank}(CE) = q$  and  $E_{c3}$  is nonsingular.

**Remark 2:** Since  $C$  is of full row rank, the first part in Assumption 1 means that the effects of the faults are independent, which is about the fault detectability. If this condition does not hold, some fault information cannot be reflected in the system outputs. Other new method should be considered, which would be our future work.

The time period considered here is between that the data sampled from sensor at time  $k-1$  and  $k$  have arrived in the controller side after some delays, and the data sampled at time  $k+1$  have not arrived in. By pre-multiplying

$$\begin{bmatrix} I & 0 & -E_{c1}E_{c3}^{-1} \\ 0 & I & -E_{c2}E_{c3}^{-1} \\ 0 & 0 & I \end{bmatrix} \quad (9)$$

on Eq. (7), one can obtain

$$\begin{bmatrix} z_1(k+1) - E_{c1}E_{c3}^{-1}z_3(k+1) \\ z_2(k+1) - E_{c2}E_{c3}^{-1}z_3(k+1) \\ z_3(k+1) \end{bmatrix} = \begin{bmatrix} A_{c1} - E_{c1}E_{c3}^{-1}A_{c3} \\ A_{c2} - E_{c2}E_{c3}^{-1}A_{c3} \\ A_{c3} \end{bmatrix} z(k) + \sum_{s=0}^m \begin{bmatrix} B_{is1} - E_{c1}E_{c3}^{-1}B_{is3} \\ B_{is2} - E_{c2}E_{c3}^{-1}B_{is3} \\ B_{is3} \end{bmatrix} u(k-s) + \begin{bmatrix} 0 \\ 0 \\ E_{c3} \end{bmatrix} f(k) \quad (10)$$

Note that only the state  $z_1(k)$  needs to be estimated. Define

$$\bar{A}_{c1} \triangleq A_{c1} - E_{c1}E_{c3}^{-1}A_{c3}, \quad \bar{A}_{c2} \triangleq A_{c2} - E_{c2}E_{c3}^{-1}A_{c3} \quad (11)$$

$$\bar{B}_{is1} \triangleq B_{is1} - E_{c1}E_{c3}^{-1}B_{is3}, \quad \bar{B}_{is2} \triangleq B_{is2} - E_{c2}E_{c3}^{-1}B_{is3} \quad (12)$$

$$\bar{A}_{c1} \triangleq [\bar{A}_{c11} \quad \bar{A}_{c12} \quad \bar{A}_{c13}], \quad \bar{A}_{c2} \triangleq [\bar{A}_{c21} \quad \bar{A}_{c22} \quad \bar{A}_{c23}] \quad (13)$$

with  $\bar{A}_{c11} \in R^{(n-r) \times (n-r)}$ ,  $\bar{A}_{c12} \in R^{(n-r) \times (r-q)}$ ,  $\bar{A}_{c13} \in R^{(n-r) \times q}$ ,  $\bar{A}_{c21} \in R^{(r-q) \times (n-r)}$ ,  $\bar{A}_{c22} \in R^{(r-q) \times (r-q)}$ ,  $\bar{A}_{c23} \in R^{(r-q) \times q}$ .

Then, the first and second block rows of Eq. (10) can be written as:

$$z_1(k+1) = \bar{A}_{c11}z_1(k) + \rho(k+1) \quad (14)$$

$$\lambda(k+1) = \bar{A}_{c21}z_1(k) \quad (15)$$

where  $\rho(k+1)$  and  $\lambda(k+1)$  are defined as:

$$\rho(k+1) \triangleq \bar{A}_{c12}z_2(k) + \bar{A}_{c13}z_3(k) + E_{c1}E_{c3}^{-1}z_3(k+1) + \sum_{s=0}^m \bar{B}_{is1}u(k-j) \quad (16)$$

$$\eta(k+1) \triangleq z_2(k+1) - E_{c2}E_{c3}^{-1}z_3(k+1) - \bar{A}_{c22}z_2(k) - \bar{A}_{c23}z_3(k) - \sum_{s=0}^m \bar{B}_{is2}u(k-j) \quad (17)$$

When the data sampled from the sensor are packed with the time, the sensor-controller delays are available, which can determine the current mode of the output. In this case, the observer to estimate  $z_1$  can be designed as

$$\hat{z}_1(k+1) = \bar{A}_{c11}\hat{z}_1(k) + \rho(k+1) + L_i(k)[\eta(k+1) - \bar{A}_{c21}\hat{z}_1(k)] \quad (18)$$

Let us replace the state  $z_2$  and  $z_3$  to  $y_1$  and  $y_2$  and assume  $(\bar{A}_{c11}, \bar{A}_{c21})$  is an observable pair. Then the observer gain  $L_{ij}(k)$  can be chosen such that the following matrix inequality

$$(\bar{A}_{c11} - L_i\bar{A}_{c21})^T \bar{R}_i (\bar{A}_{c11} - L_i\bar{A}_{c21}) - R_i < 0 \quad (19)$$

has positive definite solutions  $R_i > 0$ , where  $\bar{R}_i = \sum_{l=1}^m p_{ij}^l R_i$ .

From above discussion, we can let  $\hat{z} = [\hat{z}_1 \ z_2 \ z_3]^T$ , where  $[z_2 \ z_3]^T = C^{-1}[y_2 \ y_3]^T$ , where  $\hat{z}_1$  is obtained in (18). Denote  $\tilde{z}_1 = \hat{z}_1 - z_1$ , then

$$\tilde{z}_1(k+1) = (\bar{A}_{c11} - L_{ij}\bar{A}_{c21})\tilde{z}_1(k) \quad (20)$$

Choose the Lyapunov function  $V(k) = \tilde{z}_1^T(k)R_i\tilde{z}_1(k)$ , then one has

$$\begin{aligned} & E\{V(k+1)|\tilde{z}_1(k), r(k)\} - V(k) \\ &= \tilde{z}_1^T(k) \left( (\bar{A}_{c11} - L_i\bar{A}_{c21})^T \bar{R}_i (\bar{A}_{c11} - L_i\bar{A}_{c21}) - R_i \right) \tilde{z}_1(k) < 0 \end{aligned} \quad (21)$$

We can conclude that the estimation error  $\tilde{z}_1(k)$  will converge to zero in the mean square if the inequality (19) is satisfied.

It should be mentioned that the choice of  $L_i$  and  $R_i$  relates to constant matrix  $A_c$ , which implies that, for the proposed Markovian jump system, the observer gain  $L_i$  and the corresponding matrix  $R_i$  is also invariable denoted by  $L$  and  $R$ .

Define  $e_x(k) = x(k) - \hat{x}(k)$ , then we further have

$$e_x(k+1) = N^{-1}\tilde{z}(k+1) = (\bar{A}_{c11} - L_i\bar{A}_{c21})N^{-1} \begin{bmatrix} \tilde{z}_1(k) \\ 0 \\ 0 \end{bmatrix} = (\bar{A}_{c11} - L_i\bar{A}_{c21}) \begin{bmatrix} e_{x_1}(k) \\ 0 \\ 0 \end{bmatrix} \quad (22)$$

The third block row in (10) can be written as

$$y_2(k+1) = A_{c31}z_1(k) + A_{c32}y_1(k) + A_{c33}y_2(k) + \sum_{s=0}^m B_{is3}u(k-s) + E_{c3}f(k) \quad (23)$$

the system fault can be estimated as follows:

$$\hat{f}(k) = E_{c3}^{-1}[y_2(k+1) - A_{c31}\hat{z}_1(k) - A_{c32}y_1(k) - A_{c33}y_2(k) - \sum_{s=0}^m B_{is3}u(k-s)] \quad (24)$$

and the fault estimation error

$$e_f(k) = f(k) - \hat{f}(k) = -E_{c3}^{-1}A_{c31}\tilde{z}_1(k) = -E_{c3}^{-1}A_{c31}Ne_{x_1}(k) \quad (25)$$

which will converge to zero in the mean square as long as  $\tilde{z}_1(k)$  converges to zero in the mean square.

**Remark 3:** From Eq. (24), it can be seen that the faulty signal at time instant  $k$  can be estimated only after the measurements from time instant  $(k+1)$  becomes available. It means that there is a one step delay in the fault estimation, whose effect on the dynamic response can be neglected for practical application [21]. On the other hand, we can avoid such problem via setting a new vector containing the  $y_2(k+1)$ , as in [23].



## 4 FTC with input delays

In this section, the delayed estimates of states and faults provided by the observer in Section 3 are used to design the FTC law, so as to maintain the system performance. The main results are summarized in the following theorem.

**Theorem 1:** System (5) - (6) can be stabilized by the feedback control of the form

$$u(k-s) = -K_i\hat{x}(k-s) - \frac{1}{\gamma}B_{is}^+E_c\hat{f}(k) \quad (26)$$

where  $\hat{x}(k-s) = N^{-1}\hat{z}(k-s)$  is the estimation value of  $x(k-s)$ ,  $\hat{z}(k-s)$  given by (18),  $\gamma$  is the number of the non-zero  $B_{is}$  in  $i$ th submode of the Markovian jump system,  $K_i$  are obtained by the following Riccati equation,  $i = 0, 1, \dots, d(N+1) + 1$ ,  $s = 0, 1, \dots, m$ .

$$4A_{ic}^T\bar{P}_iA_{ic} - P_i + (m+1)\sum_{s=0}^m K_i^T B_{is}^T \bar{P}_i B_{is} K_i < 0 \quad (27)$$

where  $A_{ic} = A_c - B_{i0}K_i$ ,  $P_i$  are symmetric positive definite matrices, and  $\bar{P}_i = \sum_{j=1}^n p_{ij}P_j$ .

*Proof:* Applying the control (26) to (5) results in the closed-loop dynamics

$$\begin{aligned} x(k+1) &= A_{ic}x(k) - \sum_{s=0}^m B_{is}K_ix(k-s) + \sum_{s=0}^m B_{is}K_ie_x(k-s) + E_c e_f(k) \\ &= A_{ic}x(k) - \sum_{s=0}^m B_{is}K_ix(k-s) + \sum_{s=0}^m B_{is}K_ie_x(k-s) - E_c E_{c3}^{-1} A_{c31} N e_{x_1}(k) \end{aligned} \quad (28)$$

where  $e_x(k-s) = [e_{x_1}^T(k-s) \ 0 \ 0]^T$ .

Consider the Lyapunov-Krasovskii function

$$V(k) = V_1(k) + V_2(k) + V_3(k) \quad (29)$$

where

$$\begin{aligned} V_1(k) &= x^T(k)P_ix(k) + (m+1) \sum_{s=0}^m \sum_{j=k-s}^{k-1} \left( x^T(j)K_i^T B_{ij}^T \bar{P}_i B_{ij} K_i x(j) \right) \\ V_2(k) &= \sum_{s=0}^m e_x^T(k-s)H_ie_x(k-s) \\ V_3(k) &= e_f^T(k)H_ie_f(k) \end{aligned} \quad (30)$$

where  $H_i > 0$ ,  $\Gamma > 0$ ,  $P_i$  are defined by (27), and  $\bar{H}_i = \sum_{j=1}^n p_{ij}H_j$ .

Then, we obtain

$$\begin{aligned} & E\{V_1(K+1)|x(k), r(k)\} - V_1(k) \\ &= x^T(k+1)\bar{P}_ix(k+1) - x^T(k)P_ix(k) \\ & \quad + (m+1) \sum_{s=0}^m \left( x^T(k)K_i^T B_{ij}^T \bar{P}_i B_{ij} K_ix(k) - x^T(k-s)K_i^T B_{is}^T \bar{P}_i B_{is} K_ix(k-s) \right) \\ &= x^T(k) \left( A_{ic}^T \bar{P}_i A_{ic} - P + (m+1) \sum_{s=0}^m K_i^T B_{is}^T \bar{P}_i B_{is} K_i \right) x(k) \\ & \quad + \left( \sum_{s=0}^m x^T(k-s)K_i^T B_{is}^T \right) \bar{P}_i \left( \sum_{s=0}^m B_{is}K_ix(k-s) \right) \\ & \quad - (m+1) \left( \sum_{s=0}^m x^T(k-s)K_i^T B_{is}^T \bar{P}_i B_{is} K_ix(k-s) \right) \\ & \quad + \left( \sum_{s=0}^m e_x^T(k-s)K_i^T B_{is}^T \right) \bar{P}_i \left( \sum_{s=0}^m B_{is}K_ie_x(k-s) \right) + e_f^T(k)E_c^T \bar{P}_i E_c e_f(k) \\ & \quad - 2x^T(k)A_{ic}^T \bar{P}_i \sum_{s=1}^m B_{is}K_ix(k-s) + 2x^T(k)A_{ic}^T \bar{P}_i \sum_{s=0}^m B_{is}K_ie_x(k-s) \\ & \quad + 2x^T(k)A_{ic}^T \bar{P}_i E_c e_f(k) - 2 \left( \sum_{s=0}^m x^T(k-s)K_i^T B_{is}^T \right) \bar{P}_i \left( \sum_{s=0}^m B_{is}K_ie_x(k-s) \right) \\ & \quad - 2 \left( \sum_{s=0}^m x^T(k-s)K_i^T B_{is}^T \right) \bar{P}_i E_c e_f(k) + 2 \left( \sum_{s=0}^m e_x^T(k-s)K_i^T B_{is}^T \right) \bar{P}_i E_c e_f(k) \end{aligned} \quad (31)$$



It is also easy to show that

$$\left(\sum_{i=0}^n x_i\right)^T P \left(\sum_{i=0}^n x_i\right) \leq n \left(\sum_{i=0}^n x_i^T P x_i\right) \quad (32)$$

Then, using the Eq. (32) and definition  $e_x(k-s) = [e_{x_1}^T(k-s) \ 0 \ 0]^T$ , the following inequalities are obtained:

$$\begin{aligned} & E\{V_1(K+1)|x(k), r(k)\} - V_1(k) \\ & \leq x^T(k) \left( A_{ic}^T \bar{P}_i A_{ic} - P_i + (m+1) \sum_{s=0}^m K_i^T B_{is}^T \bar{P}_i B_{is} K_i \right) x(k) \\ & \quad + m \left( \sum_{s=0}^m x^T(k-s) K_i^T B_{is}^T \bar{P}_i B_{is} K_i x(k-s) \right) \\ & \quad - (m+1) \left( \sum_{s=0}^m x^T(k-s) K_i^T B_{is}^T \bar{P}_i B_{is} K_i x(k-s) \right) \\ & \quad + (m+1) \left( \sum_{s=0}^m e_{x_1}^T(k-s) I_e^T K_i^T B_{is}^T \bar{P}_i B_{is} K_i I_e e_{x_1}(k-s) \right) + e_f^T(k) E_c^T \bar{P}_i E_c e_f(k) \\ & \quad - 2x^T(k) A_{ic}^T \bar{P}_i \sum_{s=1}^m B_{is} K_i x(k-s) + 2x^T(k) A_{ic}^T \bar{P}_i \sum_{s=0}^m B_{is} K_i I_e e_{x_1}(k-s) \\ & \quad + 2x^T(k) A_{ic}^T \bar{P}_i E_c e_f(k) - 2 \left( \sum_{s=0}^m x^T(k-s) K_i^T B_{is}^T \right) \bar{P}_i \left( \sum_{s=0}^m B_{is} K_i I_e e_{x_1}(k-s) \right) \\ & \quad - 2 \left( \sum_{s=0}^m x^T(k-s) K_i^T B_{is}^T \right) \bar{P}_i E_c e_f(k) + 2 \left( \sum_{s=0}^m e_{x_1}^T(k-s) I_e^T K_i^T B_{is}^T \right) \bar{P}_i E_c e_f(k) \end{aligned} \quad (33)$$

where  $I_e = [I^{n-r} \ 0^{r-q} \ 0^q]^T$ . Further, we obtain that

$$\begin{aligned} & E\{V_2(K+1)|e_x(k), r(k)\} - V_2(k) \\ & = \sum_{s=0}^m e_x^T(k-s+1) \bar{H}_i e_x(k-s+1) - \sum_{s=0}^m e_x^T(k-s) H_i e_x(k-s) \\ & = \sum_{s=0}^m e_{x_1}(k-s)^T \left( (\bar{A}_{c11} - L_i \bar{A}_{c21})^T \bar{H}_i (\bar{A}_{c11} - L_i \bar{A}_{c21}) - H_i \right) e_{x_1}(k-s) \end{aligned} \quad (34)$$

and

$$\begin{aligned} & E\{V_3(K+1)|e_f(k), r(k)\} - V_3(k) \\ & = e_f^T(k+1) \bar{H}_i e_f(k+1) - e_f^T(k) H_i e_f(k) \\ & = e_{x_1}^T(k+1) (E_{c3}^{-1} A_{c31} N)^T \bar{H}_i (E_{c3}^{-1} A_{c31} N) e_{x_1}(k+1) - e_{x_1}^T(k) (E_{c3}^{-1} A_{c31} N)^T \bar{H}_i (E_{c3}^{-1} A_{c31} N) e_{x_1}(k) \\ & = e_{x_1}^T(k) (E_{c3}^{-1} A_{c31} N)^T \left( (\bar{A}_{c11} - L_i \bar{A}_{c21})^T \bar{H}_i (\bar{A}_{c11} - L_i \bar{A}_{c21}) - H_i \right) (E_{c3}^{-1} A_{c31} N) e_{x_1}(k) \\ & = e_f^T(k) \left( (\bar{A}_{c11} - L_i \bar{A}_{c21})^T \bar{H}_i (\bar{A}_{c11} - L_i \bar{A}_{c21}) - H_i \right) e_f(k) \end{aligned} \quad (35)$$

Define  $\xi(k) = [x(k) \ x(k-1) \ \cdots \ x(k-dN) \ e_{x_1}(k-1) \ \cdots \ e_{x_1}(k-dN) \ e_f(k)]^T$ ,  $K_d = \text{diag}\{K_i\}$ ,  $\bar{P}_d = \text{diag}\{\bar{P}_i\}$ ,  $I_d = \text{diag}\{I_e\}$ ,  $\tilde{B}_d = [B_{i0} \ B_{i1} \ \cdots \ B_{i(dN)}]$ ,  $B_d = \text{diag}\{B_{i0}, B_{i1}, \dots, B_{i(dN)}\}$ . Considering (29), (33), (34) and (35), one can further obtain that

$$\begin{aligned}
& E\{V(K+1)|x(k), e_x(k), e_f(k), r(k)\} - V(k) \\
&= E\{V_1(K+1)|x(k), r(k)\} - V_1(k) + E\{V_2(K+1)|e_x(k), r(k)\} - V_2(k) \\
&+ E\{V_3(K+1)|e_f(k), r(k)\} - V_3(k) \\
&\leq \xi^T(k) \begin{bmatrix} \Gamma_{11} & -A_{ic}^T \bar{P}_i \tilde{B}_d K_i & A_{ic}^T \bar{P}_i \tilde{B}_d K_i I_e & A_{ic}^T \bar{P}_i E_c \\ * & -K_d^T B_d^T \bar{P}_d B_d K_d & K_d^T B_d^T \bar{P}_d B_d K_d I_d & -K_i^T \tilde{B}_d^T \bar{P}_i E_c \\ * & * & \Gamma_{33} & I_e^T K_i^T \tilde{B}_d^T \bar{P}_i E_c \\ * & * & * & \Gamma_{44} \end{bmatrix} \xi(k) \\
&\triangleq \xi^T(k) M \xi(k)
\end{aligned}$$

where

$$\begin{aligned}
\Gamma_{11} &= A_{ic}^T \bar{P}_i A_{ic} - P_i + (m+1) \sum_{s=0}^m K_i^T B_{is}^T \bar{P}_i B_{is} K_i \\
\Gamma_{33} &= (m+1) I_d^T K_d^T B_d^T \bar{P}_d B_d K_d I_d + \text{diag}\{(\bar{A}_{c11} - L_i \bar{A}_{c21})^T \bar{H}_i (\bar{A}_{c11} - L_i \bar{A}_{c21}) - H_i\} \\
\Gamma_{44} &= (\bar{A}_{c11} - L_i \bar{A}_{c21})^T \bar{H}_i (\bar{A}_{c11} - L_i \bar{A}_{c21}) - H_i + E_c^T \bar{P}_i E_c
\end{aligned}$$

If  $M < 0$ , then  $E\{V(K+1)|x(k), r(k)\} - V(k) < 0$ , which means the system (28) is stable. It is well known that for any  $\alpha > 0$  and real vectors  $a$  and  $b$  that  $2a^T b \leq \alpha a^T a + \frac{1}{\alpha} b^T b$ . Setting  $\alpha = 1$ , we have the following inequality

$$\begin{aligned}
& E\{V(K+1)|x(k), e_x(k), e_f(k), r(k)\} - V(k) \\
&\leq c_1 |x(k)|^2 + \sum_{s=0}^m c_{2s} |x(k-s)|^2 + \sum_{s=0}^m c_{3s} |e_{x_1}(k-s)|^2 + c_4 |e_f(k)|^2 \\
&- \left[ \sqrt{3} A_{ic} \bar{P}_i^{-1/2} x(k) + \sum_{s=0}^m \frac{1}{\sqrt{3}} B_{is} K_i \bar{P}_i^{-1/2} x(k-s) \right]^T \\
&\quad \times \left[ \sqrt{3} A_{ic} \bar{P}_i^{-1/2} x(k) + \sum_{s=0}^m \frac{1}{\sqrt{3}} B_{is} K_i \bar{P}_i^{-1/2} x(k-s) \right] \\
&- \left[ \sqrt{3} E_c \bar{P}_i^{-1/2} e_f(k) + \sum_{s=0}^m \frac{1}{\sqrt{3}} B_{is} K_i \bar{P}_i^{-1/2} x(k-s) \right]^T \\
&\quad \times \left[ \sqrt{3} E_c \bar{P}_i^{-1/2} e_f(k) + \sum_{s=0}^m \frac{1}{\sqrt{3}} B_{is} K_i \bar{P}_i^{-1/2} x(k-s) \right] \tag{36}
\end{aligned}$$

where

$$\begin{aligned}
c_1 &= \lambda_{max}(G_1), \quad G_1 = 4A_{ic}^T \bar{P}_i A_{ic} - P_i + (m+1) \sum_{s=0}^m K_i^T B_{is}^T \bar{P}_i B_{is} K_i + \left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right) A_{ic}^T \bar{P}_i A_{ic}, \\
c_{2s} &= \lambda_{max}(G_{2s}), \quad G_{2s} = \left(-\frac{1}{3} + \frac{1}{\mu_3}\right) K_i^T B_{is}^T \bar{P}_i B_{is} K_i, \\
c_{3s} &= \lambda_{max}(G_{3s}), \quad G_{3s} = \left(m+1 + \mu_1 + \mu_2 + \frac{1}{\mu_4}\right) K_i^T B_{is}^T \bar{P}_i B_{is} K_i + \bar{A}_{Li}^T \bar{H}_i \bar{A}_{Li} - H_i \\
c_4 &= \lambda_{max}(G_4), \quad G_4 = \bar{A}_{Li}^T \bar{H}_i \bar{A}_{Li} - H_i + (\mu_3 + \mu_4 + 4) E_c^T \bar{P}_i E_c, \\
\bar{A}_{Li} &= \bar{A}_{c11} - L_i \bar{A}_{c21}.
\end{aligned} \tag{37}$$

The constants  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$  and  $\mu_4$  are appropriately chosen such that  $G_1 < 0$ ,  $G_2 < 0$ ,  $G_3 < 0$  and  $G_4 < 0$ . From (31), if  $e_x = 0$  and  $e_f = 0$ , according to Theorem 1, it is easily obtained that  $E\{V_1(K+1)|x(k), r(k)\} - V_1(k) < 0$ . Since  $e_x$  and  $e_f$  are uniformly bounded,  $x(k)$  in (28) is uniformly bounded as well. Thus the system (28) is input-state stable with considering  $e_x$  and  $e_f$  as inputs.  $\square$

**Remark 4:** The considered fault in this paper is time-invariant, thus as soon as the fault estimation scheme detects and estimates the fault using the delayed output and estimation value of state, the fault estimation can be used to design the controller without considering the effect of the delays.

**Remark 5:** Compare with some existing results on this issue, the good futures of the results obtained in this paper are in two aspects: 1) It proposes an active fault-tolerant control design method for networked control systems, which contains a fault estimation scheme. 2) The NCSs are modelled as Markovian jump systems, so this paper also provides a new active FTC method for Markovian jump systems.

## 5 An illustrative example

In this section, a well known inverted pendulum system is used to illustrate a potential application field of our approach. The continues plant model is described as:

$$\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & -4.978 & -0.7187 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 3.7335 & 7.8959 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0.9756 \\ 0.6423 \\ -0.7317 \end{bmatrix} u(t) \\
y(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t)
\end{aligned}$$

We choose the sampling period  $T = 0.01s$ , the division of the sampling interval  $N = 2$ , the max delay  $\tau_{max} < 2T$ , then it is easily obtained the delays set  $\varphi = \{0, 0.005, 0.01, 0.015\}$ . The fault distribution matrix in Eq. (1) and (2) is defined as

$$E = -B, \quad f(t) = \begin{cases} 0; & t < 4(sec) \\ 0.4; & 4 \leq t \leq 30(sec) \end{cases}$$

According to the modelling method in Section 2, we obtain 7 submodes of the Markovian jump system. Due to the large number of the submodes, only two of them are illustrated here.

Mode 0 ( $\tau = 0$ ):

$$\dot{x}(t) = \begin{bmatrix} 1 & 0.0010 & 0.0003 & 0.01 \\ 0 & 0.9514 & -0.007012 & 0 \\ 0 & 0.0002 & 1 & 0.01 \\ 0 & 0.03643 & 0.07884 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0.01 \\ 0.0095 \\ 0.0064 \\ -0.0069 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t)$$

Mode 3 ( $\tau = 0.01, 0.005$ ):

$$\dot{x}(t) = \begin{bmatrix} 1 & 0.0010 & 0.0003 & 0.01 \\ 0 & 0.9514 & -0.007012 & 0 \\ 0 & 0.0002 & 1 & 0.01 \\ 0 & 0.03643 & 0.07884 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0.005012 \\ 0.004812 \\ 0.003203 \\ -0.00355 \end{bmatrix} u(t-2) + \begin{bmatrix} 0.005012 \\ 0.004812 \\ 0.003203 \\ -0.00355 \end{bmatrix} u(t-1)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t-s_1)$$

$$E = \begin{bmatrix} -0.01 & -0.0095 & -0.0064 & 0.0069 \end{bmatrix}^T$$

In fact,  $rank(CE) = 1$ . According to the fault estimation and fault-tolerant control method proposed in this paper, the state estimation is given by (18), then the actuator faults are estimated by (24) and compensated by Theorem 1. In Fig. 2, the actuator fault estimation with satisfactory accuracy is shown. Fig. 3 depicts the output trajectories of the closed-loop system. It can be seen that the dynamic system outputs (states) converge to zero.

## 6 Conclusion

In this paper, we have investigated the problem of observer-based FTC for a class of NCSs with large delays. The considered NCSs are modelled as Markovian jump systems. Based on this model, an active fault-tolerant controller is designed, which can guarantee that the system state converges to zero in the mean square. Simulation results of an inverted pendulum system are included to verify the effectiveness of the proposed method.

Further research work includes two directions: 1) consideration of time-varying faults; 2) extension of the proposed approach to nonlinear NCSs.

## References



- [1] W. Zhang, M. S. Branicky and S. M. Philips, Stability of networked control systems, *IEEE Control System Magazine*, 21(1), 84-99, 2001.
- [2] Y. Q. Xia, Z. Zhu and M. S. Mahmoud,  $H_2$  control for networked control systems with markovian data losses and delays, *ICIC Express Letters*, 3(3)(A), 271-276, 2009.
- [3] Y. Zheng, H. J. Fang and H. O. Wang, Takagi-Sugeno fuzzy-model-based fault detection for networked control systems with markov delays, *IEEE Trans. on Systems, Man, and Cybernetics - Part B: Cybernetics*, 36(4), 924-929, 2006.
- [4] D. Nešić and A. R. Teel, Input-to-state stability of networked control systems, *Automatica*, 40(12), 2121-2128, 2004.
- [5] M. S. Mahmoud and P. Shi, Robust Kalman filtering for continuous time-lag systems with Markovian jump parameters, *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 50(1), 98-105, 2003.
- [6] M. S. Mahmoud and P. Shi, Robust stability, stabilization and  $H_\infty$  control of time-delay systems with Markovian jump parameters, *International Journal of Robust and Nonlinear Control*, 13(8), 755-784, 2003.
- [7] M. S. Mahmoud, P. Shi and Y. Shi, Output feedback stabilization and disturbance attenuation of time-delay jumping systems, *IMA J. Mathematical Control and Information*, 20(2), 179-199, 2003.
- [8] M. S. Mahmoud, Uncertain jumping systems with strong and weak functional delays, *Automatica*, 40(3), 501-510, 2004.
- [9] L. Sheng and Y. J. Pan, Stochastic stabilization of sampled-data networked control systems, *Int. J. of Innovative Computing, Information and Control*, 6(4), 1949-1972, 2010.

- [10] V. Vesely and T. N. Quang, Robust output networked control system design, *ICIC Express Letters*, 4(4), 1399-1404, 2010.
- [11] X. He, Z. D. Wang, and D. H. Zhou, Robust fault detection for networked systems with communication delay and data missing, *Automatica*, 45(11), 2634-2639, 2009.
- [12] F. Yang and H. J. Fang, Control structure design of networked control systems based on maximum allowable delay bounds, *J. of the Franklin Institute*, 346(6), 626-635, 2009.
- [13] B. Jiang and F. N. Chowdhury, Parameter fault detection and estimation of a class of nonlinear systems using observers, *J. of the Franklin Institute*, 342(7), 725-736, 2005.
- [14] H. J. Yang, Y. Q. Xia and P. Shi, Observer-based sliding mode control for a class of discrete systems via delta operator approach, *J. of the Franklin Institute*, 347(7), 1199-1213, 2010.
- [15] X. He, Z. D. Wang, D. H. Zhou Robust  $H_\infty$  filtering for networked systems with multiple state delays, *Int. J. of Control*, 80(8), 1217-1232, 2007.
- [16] Z. H. Mao, B. Jiang and P. Shi, Observer based fault-tolerant control for a class of nonlinear networked control systems, *J. of the Franklin Institute*, 347(6), 940-956, 2010.
- [17] Z. Gu, D. B. Wang and D. Yue, Fault detection for continuous-time networked control systems with non-ideal QoS, *Int. J. of Innovative Computing, Information and Control*, 6(8), 3631-3640, 2010.
- [18] C. X. Yang, Z. H. Guan and J. Huang, Stochastic fault tolerant control of networked control systems, *J. of the Franklin Institute*, 346(10), 1006-1020, 2009.
- [19] J. Chen and R. J. Patton, *Robust model-based fault diagnosis for dynamics systems*, Boston: Kluwer Academic Publishers, 1999.
- [20] Z. H. Mao, B. Jiang and P. Shi,  $H_\infty$  fault detection filter design for networked control systems modelled by discrete Markovian jump systems, *IET Control Theory and Application*, 1(5), 1336-1343, 2007.
- [21] B. Jiang and F. N. Chowdhury, Fault estimation and accommodation for linear MIMO discrete-time systems, *IEEE Trans. on Control System Technology*, 13(3), 493-499, 2005.
- [22] P. Shi, Boukas, E. K. and Agarwal, R. K., Control of Markovian jump discrete-time systems with norm bounded uncertainty and unknown delay, *IEEE Trans. on Autom. Control*, 54(11), 2139-2144, 1999.
- [23] B. Jiang, J. L. Wang and Y. C. Soh, Robust fault diagnosis for a class of bilinear systems with uncertainty, *Proc. of the 38th IEEE Conference on Decision and Control*, Phoenix, USA, 5, 4499-4504, 1999.

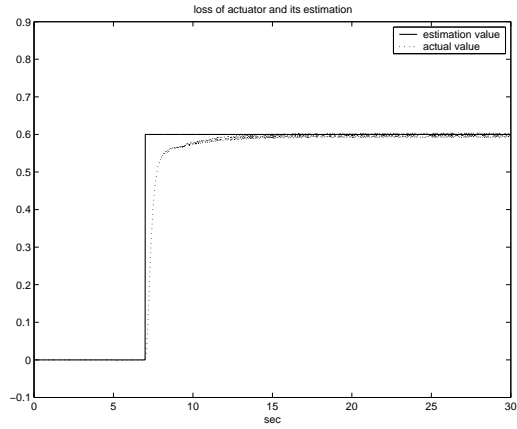
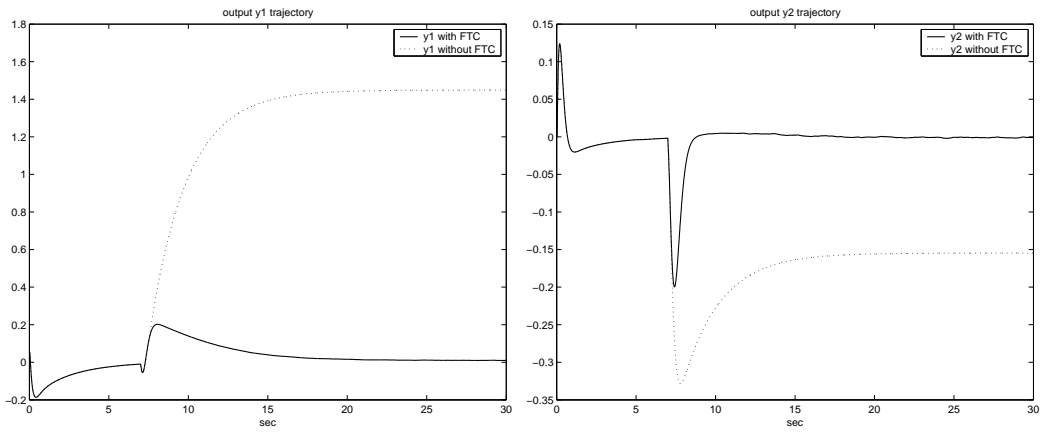
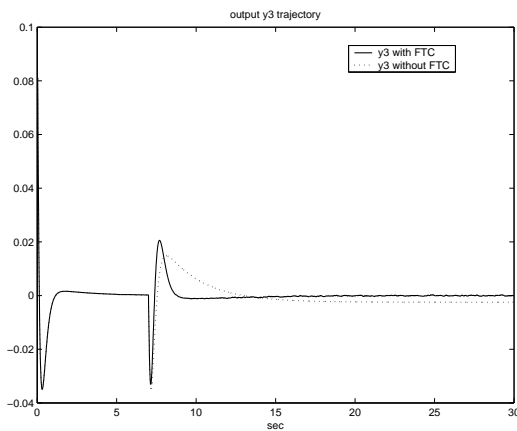


Figure 2: Estimation of actuator faults



(a) output  $y_1$

(b) output  $y_2$



(c) output  $y_3$

Figure 3: Output trajectories