VICTORIA UNIVERSITY
MELBOURNE AUSTRALIA

A Predictor-Corrector Methods for Mixed Inverse Variational Inequalities

This is the Published version of the following publication

Shi, Chaofeng (2009) A Predictor-Corrector Methods for Mixed Inverse Variational Inequalities. Research report collection, 12 (2).

The publisher's official version can be found at

Note that access to this version may require subscription.

Downloaded from VU Research Repository https://vuir.vu.edu.au/17723/

A Predictor-Corrector Methods for Mixed Inverse Variational Inequalities

Chaofeng Shi *
School of Management, Xi'an Jiaotong University,
Xi'an, 710049, P. R. China

Abstract. In this paper, a class of mixed inverse variational inequalities is introduced and studied. We prove the existence of the solution of the auxiliary problem for mixed inverse variational inequalities, suggest a predictor-corrector method for solving the mixed inverse variational inequalities by using the auxiliary principle technique. Then it is shown that the convergence of the new method requires the partially relaxed strong monotonicity property of the operator, which is a weak condition than cocoercivity. Furthermore, the sensitive analysis of the mixed inverse variational inequality is also given. Our results can be viewed as an important extension of the previously known results for inverse variational inequalities.

Key Words and Phrases: Inverse variational inequalities, Auxiliary principle, Predictor-corrector method, Convergence.

2000 Mathematics Subject Classification: 49J40, 90C33.

1 Introduction

In recent years, variational inequalities have been generalized and extended in many different directions using novel and innovative techniques to study wider classes of unrelated problems in mechanics, physics, optimization and control, nonlinear programming, economics, regional, structural, transportation, elasticity, and applied sciences, etc. (see, for example, [2, 3, 4, 9, 10, 11] and the references therein).

An important and useful generalization of variational inequalities is called the mixed variational inequality involving the nonlinear function. It is well known that due to the presence of the nonlinear function, projection method and its variant forms including the Wiener-Hopf equations, descent methods cannot be extended to suggest iterative methods for solving the mixed variational inequalities. In particular, it has been shown that if the nonlinear function is proper, convex and lower semicontinuous, then the mixed variational inequalities are equivalent to the fixed-point problems. This equivalence has been used to suggest and analyze some iterative methods for solving the mixed variational inequalities. In this approach, one has to evaluate the resolvent of the operator, which is itself a difficult problem. To overcome these difficulties, Glowinski et al. [5] suggested another technique, which is called auxiliary principle technique. In 1999, Huang et al. [9] modified and extended the auxiliary principle technique to study the existence of a solution for a class of generalized set-valued strongly nonlinear implicit variational inequalities and suggest some general iterative algorithms. Recently, Shi et al. [11] extended the auxiliary principle technique to suggest and analyze a new predictor-corrector method for solving the generalized general mixed quasi variational inequalities.

On the other hand, He et al. [6] introduced and studied the inverse variational inequalities, which can be widely used to study the problems in economics, network and transportation, etc. (see [6, 7, 8]). The primary motivation of the research on the inverse variational inequality arises from the transportation system operation and control policies. Consider the simple network which include one origination-destination pair connected by a few routes. Let u be the imposed route cost (toll charges or subsides) and $F(u)$ be

*E-mail: shichf@163.com. Supported by NSFC (Granted No. 70525004, 60736027)

the relevant route flows. In order to avoid the traffic congestion and rationally use the road services, the administration should find a control cost u such that the relevant route flows $F(u)$ stay in a range. The equilibrium can be interpreted as an inverse variational inequality. Similar applications of inverse variational inequalities can also be found in the electrical power network management. There are peak and valley periods in the usage of electricity power. The electricity network is endangered if overloaded when the peak periods of electricity consumption occur. On the other hand, when the electricity consumption reaches the valley time, electricity could be wasted due to the difficulty of storing unused power. To ensure safety of the electricity network and to encourage off-peak usage, the electricity power company could charge different unit prices for peak time usage and valley time usage. Such a pricing scheme could lead to a good control of power usage within the reasonable range. The equilibrium of this control problem is also an inverse variational inequality.

Inspired and motivated by recent research going on in this fascinating and interesting field, in this paper, an important and useful generalization of inverse variational inequalities is introduced and studied, which is called the mixed inverse variational inequality involving the nonlinear function. We prove the existence of the solution of the auxiliary problem for the mixed inverse variational inequalities, suggest a predictor-corrector method for solving the mixed inverse variational inequalities by using the auxiliary principle technique. Then it is shown that the convergence of the new method requires the partially relaxed strong monotonicity property of the operator, which is a weak condition than cocoercivity. Furthermore, the sensitive analysis of the mixed inverse variational inequality is also given. Our results extend and improve the main results of He [8].

2 Preliminaries

Let H be a real Hilbert space whose inner product and norm are denoted by $\langle \cdot, \cdot \rangle$ and $\|\cdot\|$, respectively. Let $\varphi(\cdot) : H \to H$ be a nonlinear function. For a given nonlinear operator $T : H \to H$, consider the problem of finding $u \in H$ such that

$$\langle u, v - T(u) \rangle + \varphi(v) - \varphi(T(u)) \geq 0, \forall v \in H. \tag{2.1}$$

The inequality of type (2.1) is called the mixed inverse variational inequality.

If K is closed convex set in H and $\varphi(v) \equiv I_K(v)$, for all $v \in H$, where I_K is the indictor function of K defined by

$$I_K(v) = \begin{cases} 0 & \text{if } v \in K \\ +\infty & \text{otherwise} \end{cases}$$

then the problem (2.1) is reduced to finding $u \in K$, such that

$$\langle u, v - T(u) \rangle \geq 0, \forall v \in K. \tag{2.2}$$

Problem (2.2) is called inverse variational inequality, which is introduced and studied by He [6] in 2006. For the applications, formulation and numerical methods of inverse variational inequalities (2.2), we refer the reader to the survey [8].

Lemma 2.1 For all $u, v \in H$, we have

$$2\langle u, v \rangle = \|u + v\|^2 - \|u\|^2 - \|v\|^2.$$

Definition 2.1 For all $u_1, u_2, z \in H$, an operator $T(\cdot)$ is said to be partially relaxed strongly monotone, if there exists a constant $\alpha > 0$ such that

$$\langle T(u_1) - T(u_2), z - u_2 \rangle \geq -\alpha \|u_1 - z\|^2.$$

In order to obtain our results, we need the following assumption.

Assumption 2.1 The mappings $\varphi(\cdot) : H \to H$ satisfy the following conditions:

(1) $\varphi(u)$ is bounded, that is, there exists a constant $\gamma > 0$ such that $|\varphi(u)| \leq \gamma\|u\|, \forall u \in H$, and $\varphi(0) = 0$;

(2) $\varphi(\cdot)$ is lower semicontinuous and convex.

We also need the following lemma.

Lemma 2.2 [1] Let X be a nonempty closed convex subsets of Hausdorff linear topological space E, $\phi, \psi : X \times X \to R$ be mappings satisfying the following conditions:

(1) $\psi(x, y) \leq \phi(x, y), \forall x, y \in X$;

(2) for each $x \in X, \phi(x, y)$ is upper semicontinuous with respect to y ;

(3) for each $y \in X$, the set $\{x \in X : \psi(x, y) < 0\}$ is a convex set;

(4) there exist a nonempty compact set $K \subset X$ and $x_0 \in K$ such that $\psi(x_0, y) < 0$, for any $y \in X \setminus K$. Then there exists a $\overline{y} \in K$ such that $\phi(x, \overline{y}) \geq 0, \forall x \in X$.

3 The algorithm for mixed inverse variational inequalities

In this section, we give an existence theorem of a solution of the auxiliary problem for the mixed inverse variational inequality (2.1). Based on this existence theorem, we suggest and analyze a new iterative method for solving the problem (2.1).

For given $u \in H$, consider the problem of finding a unique $z \in H$ satisfying the auxiliary mixed inverse variational inequality (denoted by P(u))

$$\langle \rho u + T(z) - T(u), v - T(z)\rangle + \rho\varphi(v) - \rho\varphi(T(z)) \geq 0, \tag{3.1}$$

for all $v \in H$, where $\rho > 0$ is a constant.

Remark 3.1 We note that if $z = u$, then clearly z is a solution of (2.1).

Theorem 3.1 If Assumption 2.1 holds, $T : H \to H$ is invertible and Lipschitz continuous, $0 < \rho\gamma < 1$, and $(1 - \rho\gamma)\|T(0)\| \leq \|T(u)\| + \rho\|u\|$, then $P(u)$ has a solution.
Proof. Define $\phi, \psi : H \times H \to H$ by

$$\phi(v, z) = \langle v, v - T(z)\rangle - \langle T(u), v - T(z)\rangle + \rho\langle u, v - T(z)\rangle - \rho\varphi(T(z)) + \rho\varphi(v)$$

and

$$\psi(v, z) = \langle T(z), v - T(z)\rangle - \langle T(u), v - T(z)\rangle + \rho\langle u, v - T(z)\rangle - \rho\varphi(T(z)) + \rho\varphi(v),$$

respectively. Now we show that the mapping ϕ, ψ satisfy all the conditions of Lemma 2.2.
Clearly, ϕ and ψ satisfy condition (1) of Lemma 2.2. It follows from Assumption 2.1(3) that $\phi(v, z)$ is upper semicontinuous with respect to z. By using Assumption 2.1 (2), it is easy to show that the set $\{v \in H|\psi(v, z) < 0\}$ is a convex set for each fixed $z \in H$ and so the conditions (2) and (3) of Lemma 2.2 hold.
Now let

$$\omega = \|T(u)\| + \rho\|u\|, K = \{z \in H : (1 - \rho\gamma)\|T(z)\| \leq \omega\}.$$

Since $T : H \to H$ is invertible, K is a weakly compact subset of H. For any fixed $z \in H \setminus K$, from Assumption 2.1, we have

$$\begin{aligned}
\psi(0,z) &= -\langle T(z), T(z)\rangle + \langle T(u), T(z)\rangle + \rho\langle u, -T(z)\rangle - \rho\varphi(T(z)) \\
&\le -\|T(z)\|^2 + \|T(u)\|\|T(z)\| + \|u\|\|T(z)\| + \rho\gamma\|T(z)\| \\
&= -\|T(z)\|(\|T(z)\| - \|T(u)\| - \rho\|u\| - \rho\gamma\|g(z)\|) \\
&< 0.
\end{aligned}$$

Therefore, the condition (4) of Lemma 2.2 holds. By Lemma 2.2, there exists a $\overline{z} \in H$ such that $\phi(v, \overline{z}) \ge 0$, for all $v \in H$, that is,

$$\langle v, v - T(\overline{z})\rangle - \langle T(u), v - T(\overline{z})\rangle + \rho\langle u, v - T(\overline{z})\rangle - \rho\varphi(T(\overline{z})) + \rho\varphi(v) \ge 0, \forall v \in H \tag{3.2}$$

For arbitrary $t \in (0,1)$ and $v \in H$, let $T(x_t) = tv + (1-t)T(\overline{z})$. Replacing v by $T(x_t)$ in (3.2), we obtain

$$\begin{aligned}
0 &\le \langle T(x_t), T(x_t) - T(\overline{z})\rangle - \langle T(u), T(x_t) - T(\overline{z})\rangle + \rho\langle u, T(x_t) - T(\overline{z})\rangle - \rho\varphi(T(\overline{z})) + \rho\varphi(T(x_t)) \\
&\le t(\langle T(x_t), v - T(\overline{z})\rangle - \langle T(u), v - T(\overline{z})\rangle) + \rho t\langle u, v - T(\overline{z})\rangle + \rho t(\varphi(v) - \varphi(T(\overline{z})).
\end{aligned}$$

Hence

$$\langle T(x_t), v - T(\overline{z})\rangle - \langle T(u), v - T(\overline{z})\rangle + \rho\langle u, v - T(\overline{z})\rangle + \rho\varphi(v)) - \rho\varphi(T(\overline{z})) \ge 0$$

and so

$$\langle T(x_t), v - T(\overline{z})\rangle \ge \langle T(u), v - T(\overline{z})\rangle - \rho\langle u, v - T(\overline{z})\rangle - \rho\varphi(v) + \rho\varphi(T(\overline{z})).$$

Letting $t \to 0$, we have

$$\langle T(\overline{z}), v - T(\overline{z})\rangle \ge \langle T(u), v - T(\overline{z})\rangle - \rho\langle u, v - T(\overline{z})\rangle - \rho\varphi(v) + \rho\varphi(T(\overline{z}))$$

Therefore, $\overline{z} \in H$ is a solution of the auxiliary problem $P(u)$. This completes the proof.

By using Theorem 3.1, we now suggest the following iterative method for solving the mixed inverse variational inequality (2.1).

Algorithm 3.1 For a given $u_0 \in H$, compute the approximate solution u_{n+1} by the iterative scheme

$$\langle \rho w_n + T(u_{n+1}) - T(w_n), v - T(u_{n+1})\rangle + \rho\varphi(v) - \rho\varphi(T(u_{n+1})) \ge 0, \forall v \in H \tag{3.3}$$

and

$$\langle \beta u_n + T(w_n) - T(u_n), v - T(w_n)\rangle + \beta\varphi(v) - \beta\varphi(T(w_n)) \ge 0, \forall v \in H, \tag{3.4}$$

where $\rho > 0$ and $\beta > 0$ are constants.

For the convergence analysis of Algorithm 3.1, we need the following result.

Lemma 3.1 Let $u \in H$ be the exact solution of (2.1) and u_{n+1} be the approximate solution obtained from Algorithm 3.1. If the operator $T(\cdot)$ is partially relaxed strongly monotone, T^{-1} is Lipschitz continuous with a constant $\sigma > 0$, and the conditions in Theorem 3.1 are satisfied, then

$$\|T(u_{n+1}) - T(u)\|^2 \le \|T(u_n) - T(u)\|^2 - (1 - 2\rho\alpha\sigma)\|T(u_{n+1}) - T(u_n)\|^2. \tag{3.5}$$

Proof. Let $u \in H$ be a solution of (2.1). Then

$$\langle \rho u, v - T(u)\rangle + \rho\varphi(v) - \rho\varphi(T(u)) \ge 0, \forall v \in H \tag{3.6}$$

and

$$\langle \beta u, v - T(u)\rangle + \beta\varphi(v) - \beta\varphi(T(u)) \geq 0, \forall v \in H, \tag{3.7}$$

where $\rho > 0$ and $\beta > 0$ are constants. Now taking $v = T(u_{n+1})$ in (3.6) and $v = T(u)$ in (3.3), we have

$$\langle \rho u, T(u_{n+1}) - T(u)\rangle + \rho\varphi(T(u_{n+1})) - \rho\varphi(T(u)) \geq 0 \tag{3.8}$$

and

$$\langle \rho w_n + T(u_{n+1}) - T(w_n), T(u) - T(u_{n+1})\rangle + \rho\varphi(T(u)) - \rho\varphi(T(u_{n+1})) \geq 0. \tag{3.9}$$

Since $T(\cdot)$ is partially relaxed strongly monotone, Adding (3.8) and (3.9), we have

$$\begin{aligned} \langle T(u_{n+1}) - T(w_n), T(u) - T(u_{n+1})\rangle &\geq \rho\langle w_n - u, T(u_{n+1}) - T(u)\rangle \\ &\geq -\rho\alpha\|w_n - u_{n+1}\|^2. \end{aligned} \tag{3.10}$$

Setting $u = T(u) - T(u_{n+1})$ and $v = T(u_{n+1}) - T(w_n)$ in (2.2), we obtain

$$\begin{aligned} \langle T(u_{n+1}) - T(w_n), T(u) - T(u_{n+1})\rangle &= (1/2)\{\|T(u) - T(w_n)\|^2 - \|T(u_{n+1}) - T(w_n)\|^2 \\ &\quad - \|T(u) - T(u_{n+1})\|^2\}. \end{aligned} \tag{3.11}$$

It follows from (3.10) and (3.11) that

$$\|T(u_{n+1}) - T(u)\|^2 \leq \|T(w_n) - T(u)\|^2 - (1 - 2\rho\alpha\sigma)\|T(u_{n+1}) - T(w_n)\|^2, \tag{3.12}$$

Similarly, we have

$$\begin{aligned} \|T(u) - T(w_n)\|^2 &\leq \|T(u_n) - T(u)\|^2 - (1 - 2\beta\alpha\sigma)\|T(u_n) - T(w_n)\|^2 \\ &\leq \|T(u_n) - T(u)\|^2, 0 < \beta < 1/(2\alpha\sigma). \end{aligned} \tag{3.13}$$

and

$$\begin{aligned} \|T(u_{n+1}) - T(w_n)\|^2 &= \|T(u_{n+1}) - T(u_n) + T(u_n) - T(w_n)\|^2 \\ &= \|T(u_{n+1}) - T(u_n)\|^2 + \|T(u_n) - T(w_n)\|^2 \\ &\quad + 2\langle T(u_{n+1}) - T(u_n), T(u_n) - T(w_n)\rangle. \end{aligned} \tag{3.14}$$

Combining (3.12)-(3.14), we get

$$\|T(u_{n+1}) - T(u)\|^2 \leq \|T(u_n) - T(u)\|^2 - (1 - 2\rho\alpha\sigma)\|T(u_{n+1}) - T(u_n)\|^2.$$

The required result.

Theorem 3.2 Let H be finite dimensional, $T : H \to H$ be invertible, T^{-1} is Lipschitz continuous and $0 < \rho < 1/(2\alpha\sigma)$. Let $\{u_n\}$ be the sequences obtained from Algorithm 3.1, $u \in H$ be the exact solution of (2.1) and the conditions in Lemma 3.1 are satisfied, then $\{u_n\}$ strongly converge to a solution of (2.1).

Proof. Let $u \in H$ be a solution of (2.1). Since $0 < \rho < 1/(2\alpha\sigma)$, it follows from (3.5) that the sequence $\{\|T(u) - T(u_n)\|\}$ is nonincreasing and consequently $\{u_n\}$ is bounded. Furthermore, we have

$$\sum (1 - 2\rho\alpha\sigma)\|T(u_{n+1}) - T(u_n)\|^2 \leq \|T(u_0) - T(u)\|^2,$$

which implies that

$$\lim_{n\to\infty} \|T(u_{n+1}) - T(u_n)\| = 0. \tag{3.15}$$

Let $\hat{u}$ be the cluster point of $\{u_n\}$ and the subsequence $\{u_{n_j}\}$ of the sequence $\{u_n\}$ converge to $\hat{u}$. Replacing w_n by u_{n_j} in (3.3) and (3.4), the limit $n_j \to \infty$ and using (3.14), we have

$$\langle \hat{u}, v - T(\hat{u})\rangle + \varphi(v) - \varphi(T(\hat{u})) \geq 0, \forall v \in H,$$

which implies that $\hat{u} \in H$ is a solution of (2.1), and

$$\|T(u_{n+1}) - T(u)\|^2 \leq \|T(u_n) - T(u)\|^2. \tag{3.16}$$

Thus it follows from the above inequality that the sequence$\{u_n\}$ has exactly one cluster point $\hat{u}$ and $\lim_{n\to\infty} T(u_n) = T(\hat{u})$. Since T is invertible and T^{-1} is Lipschitz continuous, $\lim_{n\to\infty} u_n = \hat{u}$. The required result.

4 Parametric mixed inverse variational inequalities

In this section, we give an existence theorem of a solution of the parametric problem for the mixed inverse variational inequality (2.1).

For given $\lambda \in H$, consider the problem of finding a unique $u \in H$ satisfying the parametric mixed inverse variational inequality

$$\langle u, v - T_\lambda(z)\rangle + \varphi(v) - \varphi(T_\lambda(z)) \geq 0, \tag{4.1}$$

for all $v \in H$, where T_λ is dependent on the parameter $\lambda \in H$.

Theorem 4.1 If Assumption 2.1 holds, for any $\lambda \in H$, $T_\lambda : H \to H$ is invertible and Lipschitz continuous, T_λ^{-1} is strongly montone with a constant $\alpha > 0$, and $T_\lambda(0) = 0$, then the parametric mixed inverse variational inequality has a solution.

Proof. Define $\phi, \psi : H \times H \to H$ by

$$\phi(v,u) = \langle v, v - T_\lambda(u)\rangle - \langle T_\lambda(u), v - T_\lambda(u)\rangle + \langle u, v - T_\lambda(u)\rangle - \varphi(T_\lambda(u)) + \varphi(v)$$

and

$$\psi(v,u) = \langle T_\lambda(u), v - T_\lambda(u)\rangle - \langle T_\lambda(u), v - T_\lambda(u)\rangle + \langle u, v - T_\lambda(u)\rangle - \varphi(T_\lambda(u)) + \varphi(v),$$

respectively. Now we show that the mapping ϕ, ψ satisfy all the conditions of Lemma 2.2.
Clearly, ϕ and ψ satisfy condition (1) of Lemma 2.2. It follows from Assumption 2.1(2) that $\phi(v, u)$ is upper semicontinuous with respect to u. By using Assumption 2.1 (2), it is easy to show that the set $\{v \in H | \psi(v, u) < 0\}$ is a convex set for each fixed $u \in H$ and so the conditions (2) and (3) of Lemma 2.2 hold.
Since T_λ^{-1} is strongly monotone,

$$\langle u, T_\lambda(u)\rangle \geq \alpha \|T_\lambda(u)\|^2. \tag{4.2}$$

Now let

$$K = \{u \in H : \|T_\lambda(u)\| \leq \gamma/\alpha\}.$$

For any fixed $u \in H \setminus K$, from (4.2), we have

$$\gamma \|T_\lambda(u)\| < \alpha \|T_\lambda(u)\|^2. \tag{4.3}$$

Combining (4.2) and (4.3),we have

$$\langle u, T_\lambda(u)\rangle - \gamma \|T_\lambda(u)\| > 0. \tag{4.4}$$

From (4.4) and Assumption (2.1), we have

$$\langle u, T_\lambda(u)\rangle + \varphi(T_\lambda(u)) > 0. \tag{4.5}$$

Since $T_\lambda : H \to H$ is invertible, K is a weakly compact subset of H. From Assumption 2.1, we have

$$\begin{aligned} \psi(0,u) &= -\langle T_\lambda(u), T_\lambda(u)\rangle + \langle T_\lambda(u), T_\lambda(u)\rangle + \langle u, -T_\lambda(u)\rangle - \varphi(T_\lambda(u)) \\ &= -(\langle u, T_\lambda(u)\rangle + \varphi(T_\lambda(u))) \\ &< 0. \end{aligned}$$

Therefore, the condition (4) of Lemma 2.2 holds. By Lemma 2.2, there exists a $u \in H$ such that $\phi(v, u) \geq 0$, for all $v \in H$, that is,

$$\langle v, v - T_\lambda(u)\rangle - \langle T_\lambda(u), v - T_\lambda(u)\rangle + \langle u, v - T_\lambda(u)\rangle - \varphi(T_\lambda(u)) + \varphi(v) \geq 0, \forall v \in H \tag{4.6}$$

For arbitrary $t \in (0,1)$ and $v \in H$, let $T_\lambda(x_t) = tv + (1-t)T_\lambda(u)$. Replacing v by $T_\lambda(x_t)$ in (4.6), we obtain

$$\begin{aligned} 0 &\leq \langle T_\lambda(x_t), T_\lambda(x_t) - T_\lambda(u)\rangle - \langle T_\lambda(u), T_\lambda(x_t) - T_\lambda(u)\rangle + \langle u, T_\lambda(x_t) - T_\lambda(u)\rangle - \varphi(T_\lambda(u)) + \varphi(T_\lambda(x_t)) \\ &\leq t(\langle T_\lambda(x_t), v - T_\lambda(u)\rangle - \langle T_\lambda(u), v - T_\lambda(u)\rangle) + t\langle u, v - T_\lambda(u)\rangle + t(\varphi(v) - \varphi(T_\lambda(u)). \end{aligned}$$

Hence

$$\langle T_\lambda(x_t), v - T_\lambda(u)\rangle - \langle T(u), v - T_\lambda(u)\rangle + \langle u, v - T_\lambda(u)\rangle + \varphi(v)) - \varphi(T_\lambda(u)) \geq 0$$

and so

$$\langle T_\lambda(x_t), v - T_\lambda(u)\rangle \geq \langle T(u), v - T_\lambda(u)\rangle - \langle u, v - T_\lambda(u)\rangle - \varphi(v) + \varphi(T_\lambda(u)).$$

Letting $t \to 0$, we have

$$\langle T_\lambda(u), v - T_\lambda(u)\rangle \geq \langle T_\lambda(u), v - T_\lambda(u)\rangle - \langle u, v - T_\lambda(u)\rangle - \varphi(v) + \varphi(T_\lambda(u))$$

Therefore, $u \in H$ is a solution of the parametric problem (4.1). This completes the proof.

References

[1] S. S. Chang, *Variational Inequalities and Complementarity Theory with Application*, Shanghai Sci. Technol., Shanghai, 1991.

[2] F. Facchinei and J. S. Pang, *Finite-Dimensional Variational Inequalities and Complementarity Problems*, Springer Series in Operations Research, Springer-Verlag, 2003.

[3] Y.P. Fang and N.J. Huang, A new system of variational inclusions with (H,η)-monotone operators in Hilbert spaces, *Computers Math. Appl.* **49** (2005), 365-374.

[4] Y.P. Fang and N.J. Huang, H-monotone operator and resolvent operator technique for variational inclusions, *Appl. Math. Comput.* **145**(2003), 795-803.

[5] R. Glowinski, J. L. Lions and R. Tremolieres, *Numerical Analysis of Variational Inequalities*, North-Holland, Amesterdam, 1981.

[6] B. S. He, Henry X. Liu, X. L. Fu and X. Z. He, Solving a class of constrained 'black-box' inverse variational inequalities, Sept. 2006, Sciencepaper Online (http://www.paper.edu.cn)

[7] B. S. He, Henry X. Liu, M. Li and X. Z. He, PPA-based methods for monotone inverse variational inequalities, June 2006, Sciencepaper Online (http://www.paper.edu.cn)

[8] B. S. He, X. H. Liu, The inverse variational inequalities in economics, Dec. 2006, Sciencepaper Online (http://www.paper.edu.cn)

[9] N. J. Huang, Y. P. Liu, Y. Y. Tang And M. R. Bai, On the generalized set-valued strongly nonlinear implicit variational inequalities, *Comput. Math. Appl.*, 37 (1999), 29-36.

[10] C.F. Shi and S.Y. Liu, Generalized set-valued variational inclusion in q-uniformly smooth Banach spaces, *J. Math. Anal. Appl.* **296**(2004), 553-562.

[11] C. F. Shi, S. Y. Liu and J. L. Lian, Predictor-Corrector methods for generalized general multivalued mixed quasi variational inequalities, *J. Inequal. Pure and Appl. Math.*, 4(2003), Art. 38.