VICTORIA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF COMPUTER AND MATHEMATICAL SCIENCES

Factors Affecting the Choice of Target for a Process with 'Top-Up' and 'Give - Away'

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ABSTRACT

The purpose of this paper is to develop a model for the optimal selection of a process setting with a view to maximising the expected profit per manufactured item. The main focus is on a situation where product above a certain threshold is sold at a regular price, product below that threshold but above some other dimensional value can be reprocessed at a cost and sold at the regular price. All other items are unsaleable. In addition, product above the threshold implies 'give-away'. The dependencies between the process parameters and the optimal value of the process setting are graphically displayed and discussed.

Key words: Quality Selection, OptimalTarget value, Process Control.

1. INTRODUCTION

This report focuses on an extension of the work of Hunter and Kartha (1977) in which they determine the initial (and assumed static) setting of an industrial process with a view to maximising the expected profit per manufactured item. The problem revolves around the situation where product above a certain dimensional threshold attracts a fixed selling price and product below the threshold attracts a reduced yet fixed selling price. In addition, product above the threshold implies 'give-away' which diminishes the net profit per item. The essential issue is to find the most suitable process setting (the process mean) so as to effectively trade off diminished profit due to 'give-away' with diminished profit induced by producing below the stipulated threshold. Besides successfully formulating the problem, Hunter and Kartha provide a graphical method of solution. The authors consider the problem under the assumption that once the initial setting is made, no other control actions are subsequently required.

Burr (1962), Springer (1951) and Bettes (1962) all considered related problems; the latter two took economic aspects into account and determined the optimal location of the mean , which minimizes the total costs.

Golhar (1987) also addressed the issue of finding the most economic setting of a process mean, concentrating on a canning problem. He modeled a situation where the overfilled product can only be sold in the regular market and underfilled cans are emptied and refilled at the expense of a reprocessing cost. Schmidt and Pfeifer (1989) discussed the canning problem analyzed by Gohlar and explored the cost reductions achievable through a reduction in the process standard deviation.

Bisgaard, Hunter and Pallesen (1984) considered a more general problem by developing a procedure for selecting optimal values for the process mean as well as the variance. They eliminated the assumption made by Hunter and Kartha that all underfilled items can be sold for a fixed price. They considered a situation where the underfilled items are sold for a price that is proportional to the amount of ingredient in the container. The objective was to maximise profit.

Boucher and Jafari (1991) extended the above work by evaluating the problem under a sampling plan as opposed to 100% inspection.
They considered a case in which the reject criterion is based on the number of nonconforming units in a sample. Two profit functions were proposed; one for destructive testing , the other for nondestructive testing.

This current report considers a similar problem using variations of the model of Hunter and Kartha. The main focus is on a model where production between two dimensional values can be reprocessed at a cost but where items produced below the lower of these is unsaleable. As before, items initially produced above the upper threshold attract a fixed selling price but involve 'give-away' product. The problem, once again, is to obtain the optimal process setting so as to maximise the expected profit per item. The problem is formulated, the solution discussed and the nature of dependencies of the solution on the problem parameters illustrated. The existence of more sophisticated computational tools, than those available in 1977, when Hunter and Kartha published their work, removes the necessity or desirability of relying on graphical methods of solution. None-the-less, graphical displays are shown to be powerful indicators of parameter dependencies.

Process operations that involve placing fluid product into containers typically illustrates the area where problems of this nature most commonly occur. It is, therefore, in this setting that the model is framed. In this context, a production item represents an amount of product provided in a particular container.

2. GENERAL PROBLEM

Consider a process where containers are filled, with quantity requirements q, as close to L as possible. If $q \geq L$ then they are sold at a fixed selling price, with

Profit = A - Production Cost.

where A = Selling Price - Material Cost.

If, however, $L_0 \leq q < L$ the item can be 'topped-up' and sold at the same price providing

Profit = B - Production Cost,

where B =A - Additional Processing Cost.

When a container needs to be 'topped-up' it is assumed that this can be done exactly. A container such that $q < L_0$ is not 'topped-up', above all, for economic reasons, although the material does not have to be considered lost under such circumstances. The production cost p, which is the cost of filling, is assumed to be constant regardless of the amount placed in the container.

Whenever $q > L$ then there is 'give-away' product and the cost of this excess per unit measure is denoted by e. The aim is to find the mean setting of the process (assumed to be stable) so that the profit per container is maximised. Figure 1 illustrates the inter-relationships between L_0, L and T, the target dimension,

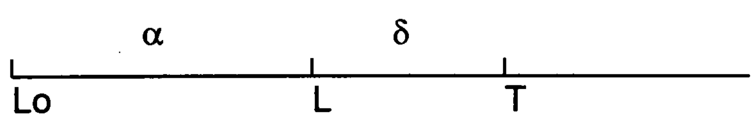


Figure 1

Shows the inter-relationships between lower limit of the process (L), the "secondary " lower limit (L_0) and the target dimension (T).

In practical applications, the Target value is ordinarily above L but it is possible for Target to be placed below L.

Unless $3\sigma > \alpha + \delta$,

where σ is the process standard deviation, then the problem is not significantly different from that considered by Hunter and Kartha. The inequality is thus assumed to hold.

In the analysis it is assumed that q is normally distributed with mean T and known variance σ^2 . In any practical situation it would be expected that the optimal value of δ, which is the focus of attention, is greater than 0, however this depends on α, the ratio between A and B, and/or on the standard deviation of the process. Again, from practical considerations, $B/A < 1$, where $A, B > 0$. The Target value, $T=L+\delta$ is called optimum if it maximises the expected profit per container.

3. THEORETICAL ANALYSIS

The profit from a single item may be written as follows:

$$P(q) = \begin{cases} (A-p) - e(q-L) & , q \geq L \\ B-p & , Lo \leq q \leq L \\ -p & , \text{otherwise.} \end{cases}$$

Thus the expected profit per item, denoted by E[P(q)], is

$$E[P(q)] = (A-p)\int_{L}^{\infty} f(q)dq - e\int_{L}^{\infty} f(q)dq + (B-p)\int_{Lo}^{L} f(q)dq - p\int_{-\infty}^{Lo} f(q)dq \qquad (1)$$

where f(q) is the p.d.f. of q i.e.

$$f(q) = (2\pi\sigma^2)^{-1/2}\exp\{-(q-T)^2/2\sigma^2\}$$

The objective is to obtain the value of δ that maximises E[P(q)] ; where $\delta = T - L$.

Let

$$\phi(x) = (2\pi^2)^{-1/2}\exp(-x^2/2)$$

and

$$\Phi(q) = \int_{-\infty}^{q} \phi(x)dx$$

Equation (1) can be simplified to

$$E[P(q)] = A\{1-\Phi(\frac{-\delta}{\sigma})\} + B\{\Phi(\frac{-\delta}{\sigma}) - \Phi(\frac{-\delta-\alpha}{\sigma})\} - e\int_{L}^{\infty}(q-L)f(q)dq - p.$$

Differentiating with respect to δ and using a result of Hunter and Kartha (1977),

$$E'[P(q)] = [(A-B)/\sigma]\phi(-\delta/\sigma) + (B/\sigma)\phi[(-\delta-\alpha)/\sigma] - e\Phi(\delta/\sigma). \qquad (2)$$

Setting equation (2) to zero, gives:

$$[\Phi(\delta/\sigma)]^{-1}\{\phi(-\delta/\sigma) + (B/(A-B))\,\phi[(-\delta-\alpha)/\sigma]\} = e\sigma / (A-B) \qquad (3)$$

The second derivative with respect to δ from (2) gives,

$$E''[P(q)] = -[(A-B)/\sigma](\delta/\sigma^2)\phi(\delta/\sigma) - (B/\sigma)[(\delta+\alpha)/\sigma^2]\phi[(\delta+\alpha)/\sigma] - (e/\sigma)\phi(\delta/\sigma).$$

If $E''[P(q)] < 0$ (with $\delta = \delta_0$) that is if

$$-\frac{\delta_0}{\sigma} - \frac{B(\delta_0+\alpha)\phi((\delta_0+\alpha)/\sigma)}{(A-B)\phi(\delta_0/\sigma)} < \frac{e\sigma}{A-B} \qquad (4)$$

then δ_0 is optimal. The solution to (3) will then give a setting for the target that will maximise the expected profit.

From practical considerations $\alpha > 0$ and if $\delta < 0$ then $-\delta < \alpha$ and so $\alpha + \delta > 0$, thus inequality (4) is true. If $\delta > 0$ (4) holds as $\sigma > 0$.

4. DISCUSSION

To investigate the relationships between the variables, as well as to study the effects of various model parameters on the target mean and the expected profit, several data sets were generated using *Mathematica*. The following graphs were obtained using *Mathematica* and *SPSS.*

Unless otherwise stated, each analysis is based on the example given by Hunter & Kartha .Some additional values, believed to be suitable, are also chosen by the authors. For $q \geq L$, A, the price per item is 67 and the "give-away" cost is 55. The distance between L_0 and L is 0.1 and L=1.

Discussion commences with a study of the relationship between the process standard deviation and the optimal value of δ_0. The data used is shown in Table 1 (the present model) and 1a (Hunter&Kartha model). A graphical comparison is made for the current model with that prescribed by Hunter and Kartha.

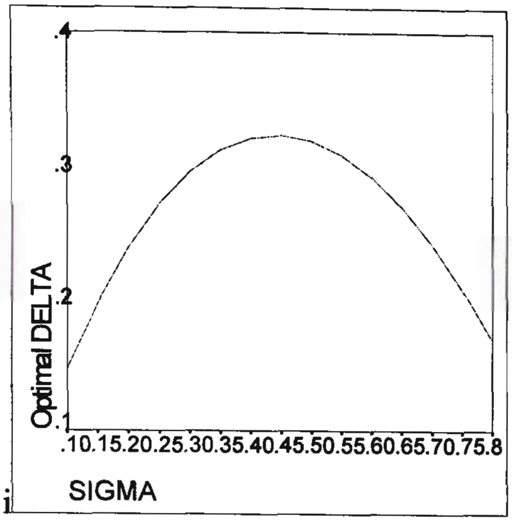
SIGMA	DELTA	PROFIT
0.10	0.15	41.16
0.15	0.20	39.98
0.20	0.24	38.78
0.25	0.27	37.57
0.30	0.30	36.39
0.35	0.31	35.27
0.40	0.32	34.22
0.45	0.32	33.26
0.50	0.32	32.41
0.55	0.31	31.69
0.60	0.29	31.12
0.65	0.27	30.71
0.70	0.24	30.47
0.75	0.21	30.42

Table 1
Shows the data generated using the present model.

SIGMA	DELTA		SIGMA	DELTA
0.01	0.03		0.25	0.18
0.03	0.06		0.27	0.17
0.05	0.09		0.29	0.16
0.07	0.11		0.31	0.16
0.09	0.13		0.33	0.15
0.11	0.15		0.35	0.13
0.13	0.16		0.37	0.12
0.15	0.17		0.39	0.10
0.17	0.17		0.41	0.08
0.19	0.18		0.43	0.06
0.21	0.18		0.45	0.04
0.23	0.18		0.47	0.02

Table 1a
Shows the data generated by using the Hunter & Kartha model.

The optimal values of δ (δ_0) plotted against different values of σ (keeping A, B, α and e constant at A=67, B=0.5A, α=0.1, e=55)) are shown in Figure 2a. Several observations are worth noting. As is clearly shown, a single optimal δ value arises from two distinct σ's. Figure 2b illustrates the same phenomena for the Hunter & Kartha model.



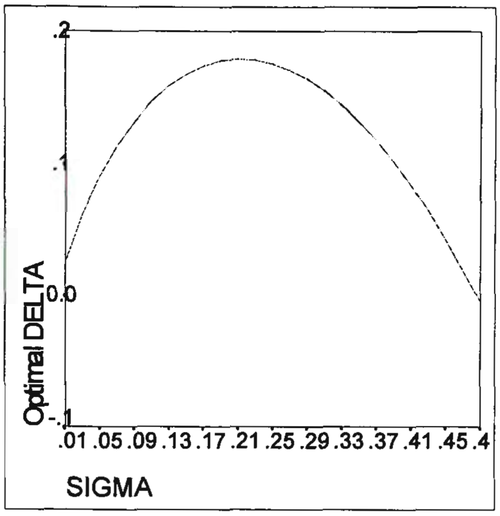


Figure 2a
Shows the optimal delta values for sigma ranging from 0.1 to 0.8. The results were obtained using present model.

Figure 2b
Shows the optimal delta values for sigma ranging from 0.01 to 0.5. The results were obtained using Hunter&Kartha model.

The second observation concerns the effect of various combinations of α, the ratio between A and B and the percentage increase in σ on the optimal value of δ. This is illustrated in Table 2, showing the percentage change in optimal δ due to a shift in σ. The chosen values for α are 0.1 and 0.3, and for B/A are 0.4, 0.5 and 0.6. Smaller shifts in σ (33% increase) cause nearly the same change in optimal δ as a shift by 66%. If the process standard deviation shifts by 100% the optimal setting of process target is not significantly affected. The bigger the ratio B/A, however, the bigger the effect of shift in standard deviation on optimal δ.

		shift in σ		
α	B/A	0.3 to 0.4 (33% increase)	0.3 to 0.5 (66% increase)	0.3 to 0.6 (100% increase)
0.1	**0.4**	8.0%	7.0%	2.5%
	0.5	8.6%	7.9%	1.1%
	0.6	12.6%	8.8%	0.2%
0.3	**0.4**	7.7%	6.9%	2.6%
	0.5	9.6%	10.0%	1.0%
	0.6	12.3%	14.2%	5.5%

Table 2

Shows the percentage change in optimal delta due to 33%, 66%, and 100% change in σ, where the distance between L and L_0 increases from 0.1 to 0.3 and the ratio between A and B is equal to 0.4, 0.5 and 0.6.

It should be pointed out that the behaviour of σ in relation to δ_0 in Figure 2a does not mean that the generated profit will be the same for the two different σ that provide the same value of δ_0. The relationship between these three variables, using the above values, is shown in Figure 3.

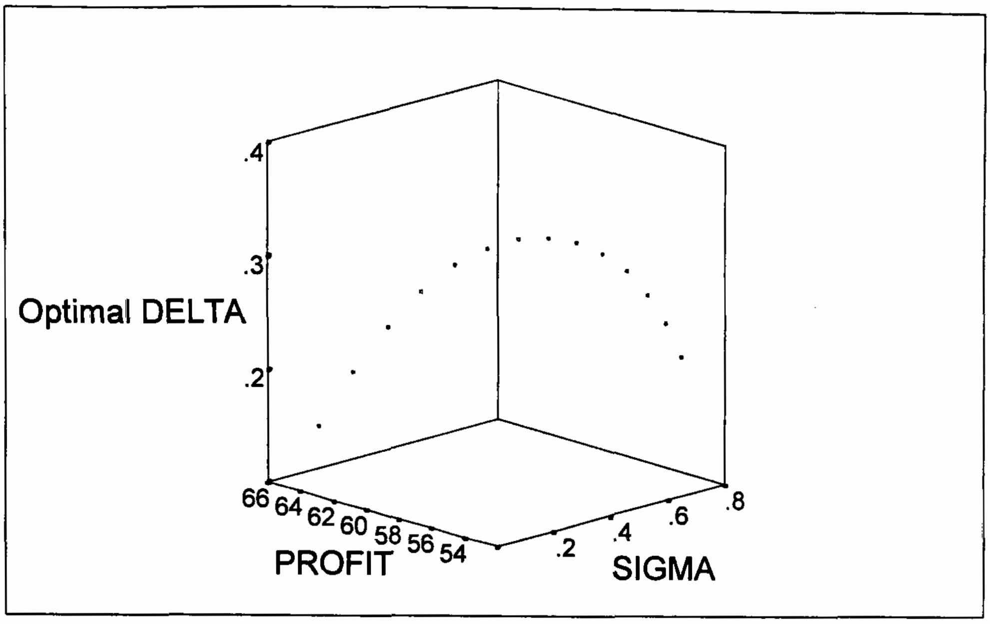


Figure 3
Shows the relationship between the optimal delta, the process standard deviation (ranging from 0.1 to 0.8) and the maximum profit.

It is to be expected that an increase in sigma leads to a decrease in profit. This is shown clearly in Figure 4. The flatness of the optimal profit curve as the standard deviation of the process gets bigger, should be noted.

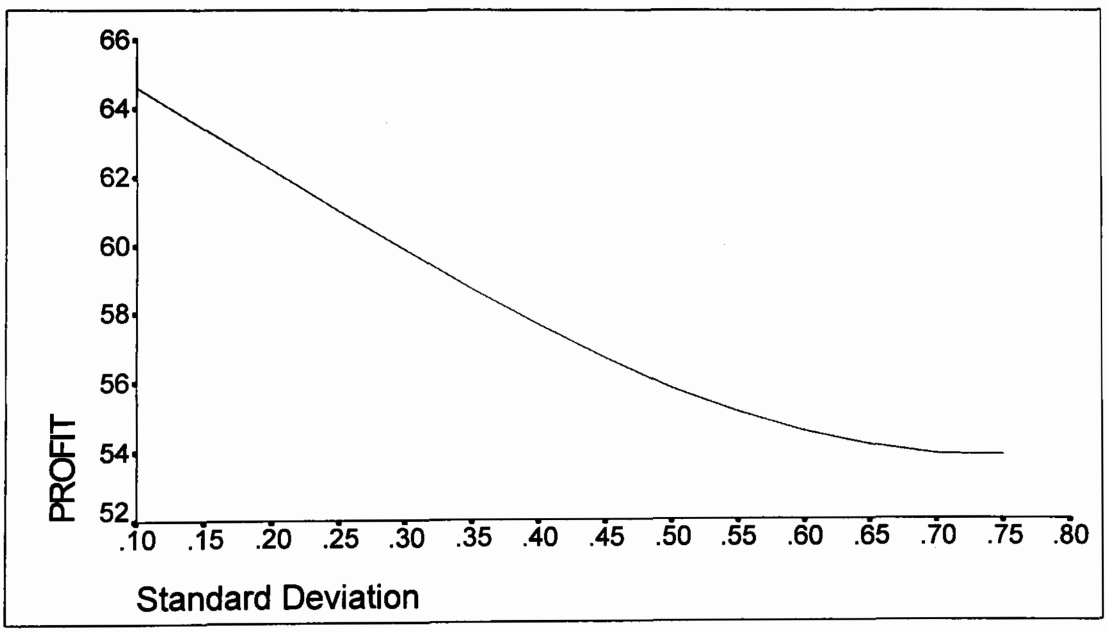


Figure 4
Shows the effect of change in process standard deviation on the maximum profit generated.

Figure 5 illustrates the effect of change in the ratio B/A on the optimal process setting. It should be observed that for small α (in this case 0.1) the optimal target setting seems to be approximately constant regardless of changes in B/A or the standard deviation of the process. Note that the result obtained from figure 2 is also clearly visible in Figure 5.

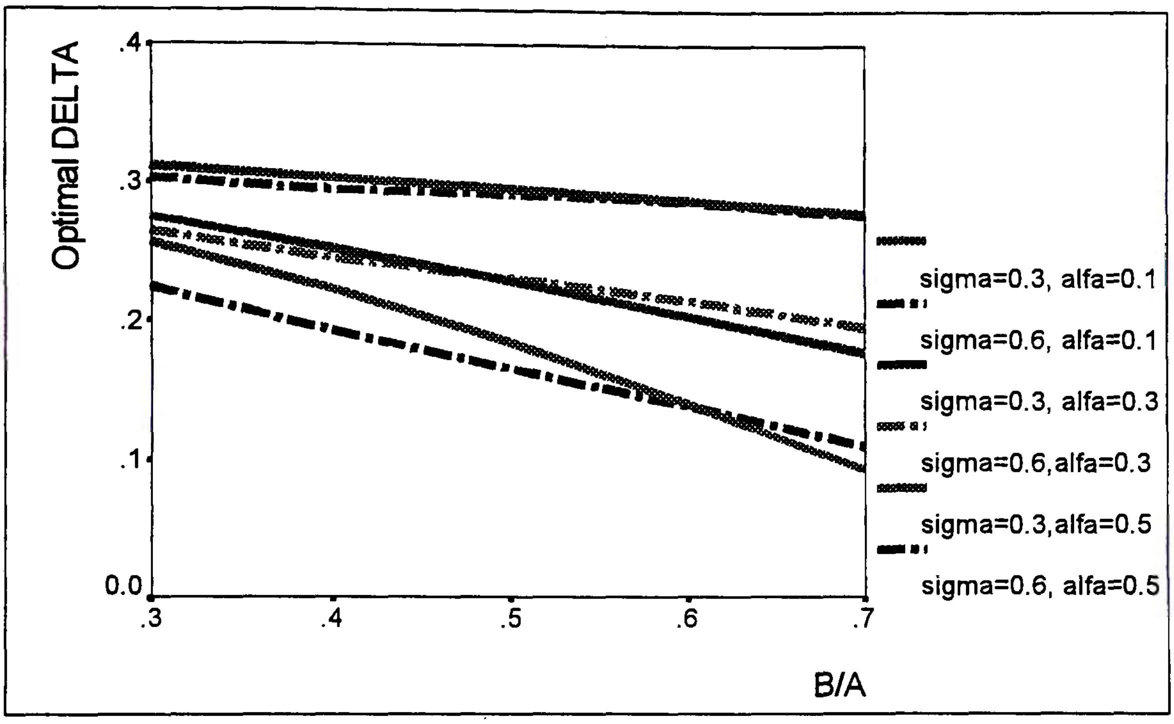


Figure 5

Shows 3 pairs of curves. Each pair has assigned the same two values for standard deviation (0.3 and 0.6) but different alfa values (0.1, 0.3 and 0.5); the same for both curves within each pair. The graph shows the effect of change in the ratio between A and B on the optimal target setting

More precise analysis of the relation between the optimal target value of the process and B/A is shown in Table 3, which illustrates the percentage change in δ_0 due to change in pricing policy. It can be observed is that for relatively small α, if σ increases by 100% then even a large increase in the ratio between A and B has a minor effect on optimal δ.

Change in B/A

α	σ	from 0.4 to 0.6	from 0.3 to 0.7
0.1	**0.3**	5.1%	10.8%
	0.6	2.8%	8.7%
0.3	**0.3**	19.2%	35.5%
	0.6	12.5%	26.1%
0.5	**0.3**	37.0%	63.8%
	0.6	28.2%	51.2%

Table 3
Shows the procentage change in optimal process setting due to change in the ratio B to A. The distance between L and L_0 varies from 0.1 to 0.5, the process standard deviation shifts from 0.3 to 0.6 and B/A changes from 0.4 to 0.6 and from 0.3 to 0.7.

The further L_0 is from L (i.e. the larger the α) the bigger the effect of B/A on optimal δ. These changes will be larger for smaller σ.

Figure 6 illustrates the result of relaxing or tightening the distance between L and L_0 on the optimal solution. An increase in α reduces the value of the optimal δ i.e. brings it closer to L_0. This effect is more significant for small values of α as well as bigger ratios of B/A. As α increases this effect diminishes.

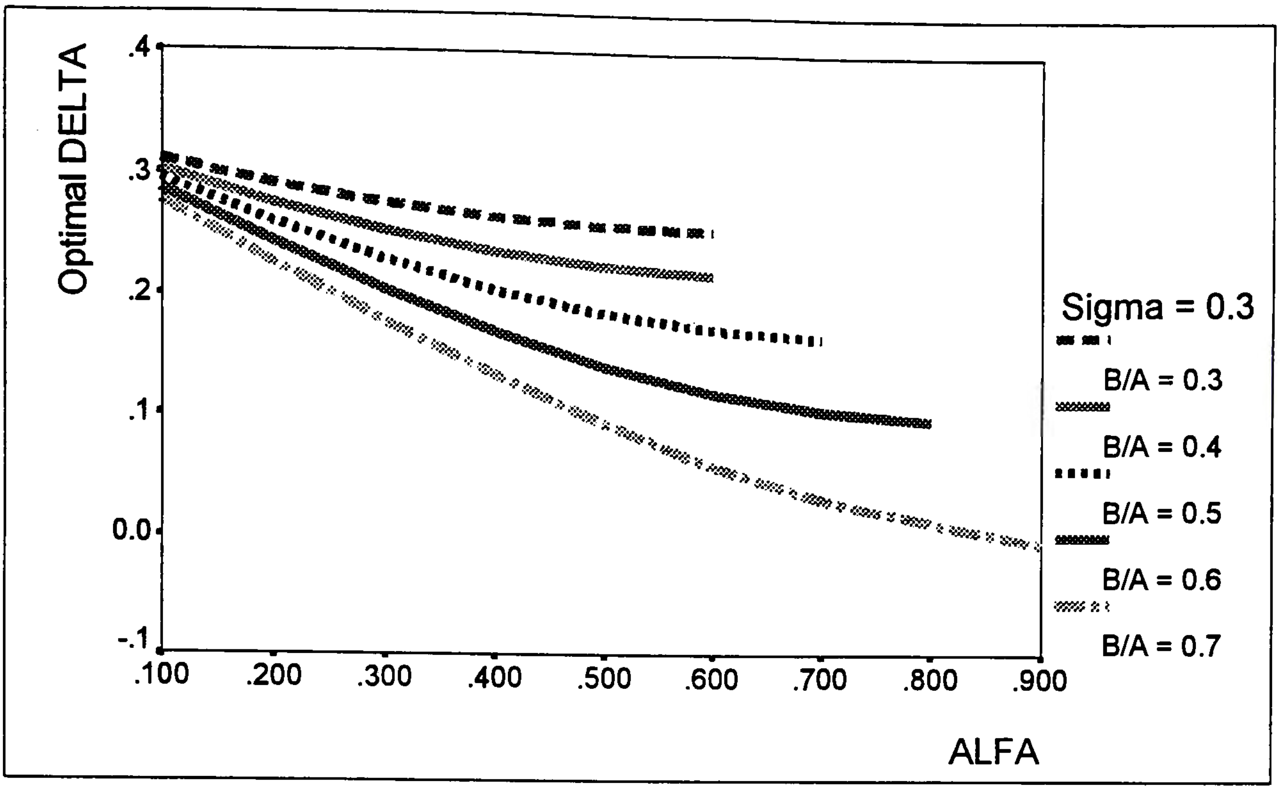


Figure 6
Shows curves of optimal delta values against values of alfa for sigma = 0.3 and B/A from 0.3 to 0.7.

5. CONCLUDING REMARKS

In this report the problem of selecting an optimal process mean in the presence of 'top-up' and 'give-away' has been defined and analysed.
The dependencies between the process parameters and the optimal value have been described.

The results would seem to indicate that even if the process variance deteriorates there is little gain in adjusting the mean *per se* but more appropriate strategy would be to concentrate on reducing variability as this increase will diminish profit per item. Furthermore, if there is an increase in α, which would likely be accompanied by a decrease in B (i.e. the ratio B/A will decrease) then there is again little advantage in adjusting the process setting.

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