VICTORIA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF COMPUTER AND MATHEMATICAL SCIENCES

SPC For Short Production Runs : A Review and Literature Survey

Pak F. Tang and Neil S. Barnett

(46 EQRM 13)

December, 1994

(AMS : 62N10)

TECHNICAL REPORT

VICTORIA UNIVERSITY OF TECHNOLOGY
(P O BOX 14428) MELBOURNE MAIL CENTRE
MELBOURNE, VICTORIA, 3000
AUSTRALIA

TELEPHONE (03) 688 4249 / 4492
FACSIMILE (03) 688 4050

Footscray Campus

SPC For Short Production Runs : A Review and Literature Survey

PAK F. TANG [1] and NEIL S. BARNETT [2]

Department of Computer and Mathematical Sciences,
Victoria University of Technology, Footscray Campus,
PO Box 14428, MMC Melbourne, Victoria 3000, Australia.

Key Words : Statistical Process Control, Short Production Runs, Low Volume Manufacturing, Control Charts.

ABSTRACT

The quest for control and the subsequent pursuit of continuous quality improvement in the manufacturing sector, due to increasingly keen competition, has stimulated considerable interest in statistical process control (SPC). Whilst traditional SPC techniques have proven useful as a means of monitoring and controlling the quality of manufactured products in the mass production industries, their utility in the low volume manufacturing environment has been subject to debate. Various methods and adaptations of existing long-run SPC techniques have been proposed for use with short production runs. Having presented the problems with the implementation of traditional SPC approaches to 'short-run' environments and having surveyed the literature, this paper critically reviews proposed techniques, offers some additional considerations for short production runs and outlines possible avenues for future investigation.

---

[1] **Mr. Tang is a graduate student in Statistics.**
[2] **Dr. Barnett is Associate Professor in Statistics and Head of Department.**

INTRODUCTION

Assuming sensible business and marketing practices, the success of a company will depend largely on the quality of its manufactured products and/or on the quality of the service it provides. With increasing demands on product quality, any manufacturing operation, regardless of its scale, mode of production or type of product, is compelled to monitor and control its industrial processes in some manner if it is to remain competitive. Innovative engineering and the use of statistical methods are important factors in achieving this goal.

Shewhart control charts have enjoyed considerable popularity as control and monitoring tools. These and more recent variations of them, enable production operators to detect process troubles or 'out-of-control' situations before they become critical. Appropriate corrective action can then be initiated to prevent further deterioration in process operation and so avert a negative impact on product quality. While these techniques are well suited to the mass production industries, their usefulness in low volume manufacturing environments is questionable.

In view of the limitations of traditional SPC techniques, many alternatives and adaptations of them have been devised for the express purpose of removing the barriers between SPC and short production runs.
Recently, a review of the literature on the use of SPC in batch production has been presented by Al-Salti et al. (1994). Much of their article is devoted to a particular aspect of short-run SPC, namely, the use of data transformation techniques. This paper, however, after discussing the problems of traditional control techniques in 'short-run' environments, gives a more comprehensive review of techniques proposed for short runs, provides its own contributions to the debate and offers some perspectives for the future.

PROBLEMS OF TRADITIONAL SPC FOR SHORT PRODUCTION RUNS

Besides the continuance of traditional small job shops, there has been, even in the mass production industries, an increase demand for more frequent production changes leading to a consequential proliferation of short runs. Attempts made to apply traditional control charting techniques in such environments are plagued with difficulties (see Al-Salti et al. (1991, 1992) , Bothe (1988, 1989, 1990)) . The essential problems facing those seeking to provide useful statistical tools for application in the short production run environment are those of machine 'warm up', control parameter estimation and parameter changes between product types.

Generally, process parameters are unknown and cannot be reliably determined prior to production. Under such circumstances, the current production output is commonly used to establish the trial control limits. As espoused in the quality control literature, for controlling quality characteristics on a continuous scale by means of X-Bar and R charts, typically, at least twenty five rational subgroups of four or five items each should be sampled from the process stream, in order to establish control limits. These 'temporary' limits will then be used for future on-going monitoring of the process based on similar subgroups and updated subsequently as more data become available. Using the common rule, a point plotted beyond these limits is deemed an indication of process troubles and thus calls for investigations to discover the possible assignable causes. For short production runs, however, the control limits for the X-Bar and R charts often cannot be located in the usual manner due to lack of data. Thus, one might consider estimating the control limits based on a much smaller number of subgroups, say, 5. However, it has been adequately demonstrated in the literature, that this practice is not reliable because it can increase the false alarm rate substantially. In fact, a quantitative analysis by Proschan et al. (1960) indicates, that for a sample size of four, the number of subgroups necessary to compute the conventional 3-σ control limits for an X-Bar chart, based on either average subgroup range (R-bar) or pooled variance, should not be less than seven to ensure that the chance of a false signal on each future subgroup does not exceed 1%. It was also given that an X-Bar chart based on the pooled variance requires at least nineteen subgroups each of size four to yield a probability of type I error less than 0.005. Similar information on the lack of reliability of both conventional X-Bar and R charts when the control limits are based on a small number of subgroups can be found in Hillier (1969). However, as noted in Quesenberry (1993), these authors did not address the issue of dependence among future occurrences of 'out-of-control' signals, hence their results are of limited usefulness in assessing the overall performance of the charts. Equivalently, the indicated probabilities of a false signal can be interpreted as the theoretical average values obtained from repeated applications of the same formulae for computing the control limits based on the same number of calibration subgroups and subgroup size, thus they do not reflect the actual performance of the charts. Taking the issue of dependence into consideration, Quesenberry conducted a simulation study of the properties of the run length distribution in order to evaluate the overall performance of both the conventional X-Bar and X charts with estimated control limits. His results indicate that the rate of false alarms after short runs increases, and much larger sets of calibration data than usually recommended are required to estimate the control limits so that they perform like known limits. However, these requirements can rarely be met for small batch manufacturing, particularly with frequent new designs and orders.

The problems of lack of process performance data are further aggravated by process 'warm up', which is perhaps the most important and yet least considered obstacle to meaningful and successful application of traditional control charts in small lot production. This phenomenon is a common and dominant feature of short-run processes, as instability after set-up or reset often constitutes a large proportion of production run time. Neglecting this fact and using sample data from such a period to obtain control limits will often lead to erroneous conclusions regarding past, current and future states of the process. Murray et al. (1988) demonstrated this using simulation, specifically, if process variability increases during the sampling period, an out-of-control process will often appear to be in control, as reflected by either a standard deviation or range chart with the usual decision rules imposed.

Another practical reality that characterizes short run environments, is the diversity of products made. If separate control charts are maintained for each type of product, the system becomes unwieldy.

SPC TECHNIQUES FOR SHORT PRODUCTION RUNS

Following identification of the problems discussed above, various approaches for handling the control of quality of low volume production have been presented in the literature. These approaches are built upon the principles of statistical quality control which aim for defect prevention instead of segregation of bad items from good ones. These, along with some possible methods are now critically examined and their attributes and deficiencies discussed.

1. Adjusting Control Limits Based On The Number Of Subgroups

In the event of only a small number of subgroups being available yet where early control of the process is still desirable, Hillier (1964,1967,1969) and Yang et al. (1970) proposed adjusting the control limits, both for retrospective testing and for future control in such a way that the predetermined probability of a type I error is preserved. However, it should be pointed out that, in the latter case, the limits so adjusted do not always ensure that the resulting probability of a type I error for each future subgroup is as desired, although its expected value is. This is due to the fact that, instead of treating quantities calculated from the previous in-control subgroups such as $\bar{\bar{x}}$ (grand average of the subgroup means), $\bar{r}$ (average subgroup range), $\bar{v}$ (average subgroup variance) and s (overall standard deviation) as constants, these are assumed to be variables, along with the means, ranges and variances of future subgroups for deriving the adjusted control chart factors. Furthermore, as opposed to setting the limits by the conventional method, even if the estimates of the process parameters are accurate, the resulting probability of a type I error associated with the modified limits may differ considerably from the desired value ! In the former case, although the probability of a type I error, α for each preliminary subgroup is maintained, the joint probability of false alarms from the m retrospective subgroups is not simply given by $1-(1-\alpha)^m$ due to the dependence of the plotted points. This can have a substantial impact either on the actual rate of erroneously indicating 'out-of-control' situations or on the actual ability of the chart to detect the presence of special causes while the preliminary subgroups are drawn. Other drawbacks are the number of calculations required and the likelihood of misinterpreting the information contained in the control charts, arising from changing control limits after every couple of subgroups (Ermer et al. (1989)).

A generalization of this approach to the control of the mean vector of a multivariate normal process, based on the well known Hotelling's T^2

statistic, was presented in Alt et al. (1976). Some issues of importance regarding the use of such a composite measure to monitor process stability in situations involving multivariate data were raised, for example, in Hawkins (1991, 1993).

King (1954) has also presented a similar approach for analysis of past data using X-Bar charts with control limits based on $\bar{r}$ except that the control chart factors are derived in such a way that the joint probability of a false alarm, γ is as required instead of the individual probability of a false signal for each initial subgroup. He gave a nomogram for a selected range of common subgroup sizes and numbers of subgroups from which an appropriate value of the control chart factor can be obtained for the case where $\gamma = 0.05$. The derivation of these control chart factors involves knowledge of the sampling distribution of

$$\frac{\max\left|\overline{X}_i - \overline{\overline{X}}\right|}{\overline{R}}$$

which is theoretically difficult to obtain though its approxime percentiles are attainable. Except when limits are constructed based on three or four subgroups, the given factors were obtained from random sampling experiments by ignoring the random fluctuations of $\overline{R}$. As such, the validity of the given factors is questionable. Furthermore, this method can only be useful if nomograms or tables for other values of γ, which might be preferable in practice, are widely available.

2. Control Charts Based On Individual Measurements

If the problem is one of short production runs and lack of data, a possible solution might be use of individual readings in place of averages.

The use of individual readings is natural anyway if there is no natural subgrouping of the observed data. Two basic types of chart employing individual measurements are:

(i) Individual values and Moving Range (I-MR) charts or X-MR charts.

(ii) Target Individual-Moving Range (Target I-MR) charts or the ΔX-MR charts.

These charts attempt to maximise the information obtained from the limited amount of available data. They are suitable for situations where :

(a) processes have limited output within a single set-up,

(b) processing time per unit item is long or the data accumulation rate is slow,

(c) testing or measurement is expensive or time consuming,

(d) testing is destructive.

The first of these two charting methods has been around for many years and is well documented (see for eg., Burr (1976) and Grant et al. (1980)). Originally, it was designed primarily for use with batch-type chemical processes (see for eg., Bicking (1962)). Burr (1954) suggested this as one of the possible methods which can cater for short production runs. In his proposal, he advocates use of 2-σ control limits rather than the conventional 3-σ limits for the individual values chart to compensate for its lack of sensitivity to mean shifts. The method has also been considered by Nugent (1990) for use in short run manufacturing environments.

The second, as its name implies, differs slightly from the first in that the target or the nominal specification is subtracted from the measurements before they are plotted (see Nugent (1990) and Ermer et al. (1989)). In his paper, Ermer et al. (1989) highlighted some practical merits of this charting method in comparison to other existing 'short run' techniques. However, this method will not work in circumstances where no target is available, as frequently occurs with products having a one-sided tolerance.

For both types of chart, the control limits are determined based on successive moving ranges of size 2. Several other estimators of process spread that can be used with individual values charts were given and compared (in terms of relative efficiency under the stable in-control assumption) by Roes et al. (1993). In the same paper, the authors also proposed the approach as reviewed under (1) above for setting the control limits both for monitoring of future process performance based on individual observations and retrospective testing of the possible presence of out-of-control conditions while the calibration sample is drawn with a slight change to the latter. Although it is well known that unbiased estimation based on standard deviation is of greatest efficiency for a stable normal process, the average moving range ($\overline{mr}$) is used to provide an estimate of the inherent process variability because it is not only computationally simple but it can also safeguard against the likely events of trends, cycles or other irregular patterns in the calibration data (i.e it minimizes the inflationary effects from these conditions and hence the inherent process variability will not be overestimated). Thus, situations are avoided where 'out-of-control' processes appear to be in-control. However, besides re-emphasizing the fact that displaying a moving range chart will only cause confusion due to correlation between consecutive moving ranges, Roes et al. (1993) substantiated Nelson's (1982) view with a probabilistic justification, that doing so has no real added value because the chart of individual values contains almost all the information available. If the measurements generated by the process are known to be independently normally distributed and a state of statistical control has been achieved, as demonstrated by the retrospective use of I-MR or Target I-MR charts, Cryer et al. (1990) suggested that, for future process monitoring, control limits for the individual values charts should be estimated based on the sample standard deviation instead of the average moving range, due to its relatively superior efficiency. The control limits for MR charts can of course be determined in a similar manner though subsequent monitoring of the process variability is based on moving ranges.

As for $\overline{X}$ charts, run rules such as that of Nelson (1984) can be effectively applied to the individual values charts to identify non-random variations or systematic process changes, hence providing better protection against potential process problems. Of course, there is an increased false alarm rate associated with these additional control rules but if power of the charts is of paramount importance and outweighs the costs of searching needlessly for non-existing assignable causes, the use of these rules can be useful.

The major practical benefits that can be gained by using individual readings instead of averages are :

(a) Measurements can be seen, compared to specification limits and easily understood.

(b) Substantial savings in time and cost may be accrued as a result of less sampling, testing or measurement.

(c) Improved employee involvement in decision making and problem solving which could be catalytic in bringing about quality and productivity improvements, as a result of operators having better appreciation of the techniques in use.

Individual values charts with conventional control limits should, however, be considered with reference to statistical efficiency. First, and most importantly, their sensitivity to substantial shifts in process average is less than that of the usual $\overline{X}$ charts. In addition, Shainin (1954) has made comparisons between various control plans including the use of individual values charts and $\overline{X}$ charts, in terms of average fraction defective produced, for equal 'hunting' and for the same expected number of pieces inspected. In particular, he has shown that, if the tolerance equals 6.4σ, on average, $\overline{X}$ charts with a subgroup size of four result in a lower percentage of defectives than individual values charts for a process mean shift of 1σ and 2σ. Although greater sensitivity may be gained by the use of narrower limits or additional warning lines, such sensitivity is gained at the expense of increasing the chance of false lack of control indications.

The second problem with individual values charts centres around the normality assumption of the process distribution. If the underlying distribution is not normal, this will tend to distort the interpretation of the control limits. On the other hand, $\overline{X}$ values will tend to normality fairly rapidly by virtue of the Central Limit Theorem, provided the underlying distribution isn't too 'bazaar'.

3. Mixing Production Lots and Normalizing Process Output Data

Recent developments in the use of statistical process control in multi-component and low-volume manufacturing environments have focussed mainly on studying and monitoring the process irrespective of the type of parts or products being manufactured. The basic idea with this approach is that values for different products or components being assessed on the basis of the same quality characteristic but with different design specifications, (i.e belonging to the same product family) can be plotted together on the same chart, provided that they are the output of a homogeneous process. Homogeneity means that the components should, technically, be machined under similar conditions, for example, in terms of cutting tools, tool holders, component holding methods and setting-up methods etc (Al-Salti et al. (1991)). The same principle applies to chemical manufacturing processes where similar chemicals are produced in small batches on an, 'as needed' basis. Emphasis on process homogeneity is important in order to reduce or eliminate the effect of variability due to extraneous sources, thus ensuring only inherent variation exists. This avoids erroneous appraisal of the process and any process irregularities can be more readily detected. In addition, the measurement process should be adequate enough, in terms of accuracy, and carried out in a consistent manner for every measured component.

Several papers (Al-Salti et al. (1991,1992), Armitage et al. (1988), Bothe (1988,1989,1990a,1990b), Burr (1989), Crichton (1988), Dovich (1989), Montgomery (1991), Nugent (1990), Thompson (1989)) have given accounts of this approach. It is accomplished by means of data transformations which effectively eliminate the differences between the types of products or components. A list of possible transformation techniques that cover a claimed majority of manufacturing situations, along with conditions of use, is presented by Al-Salti et al. (1992). A comprehensive comparison of control charts based on the given techniques for a set of industrial data is also provided in the same paper. Other techniques involving data transformations can be found in Crichton (1988), Armitage et al. (1988), Burr (1989), Thompson (1989), Al-Salti et al. (1991) and Farnum (1992).

Of all the existing control charting methods using transformation techniques, the most extensively discussed have been the commonly called Nominal and Standardized charts (see for eg., Bothe (1988,1989,1990a,1990b) and Robinson (1991)). The common feature characterising these methods is that process data for different parts are rescaled with appropriate factors, so that they all fit on to a single chart, thus avoiding the need for an excessive number of separate charts.

For a Nominal chart, the difference between subgroup average and its corresponding nominal, $\overline{X}_i - N_i$ is plotted for detecting whether or not any assignable causes of variation affecting the process average are present. For the purpose of dispersion control, the use of the traditional Range chart is recommended as adequate. In situations where the process tolerance is unilateral (hence no nominal specification is available) or the process mean for the component differs significantly from its nominal size, the design target or historical average may be used instead (Dovich (1988), Al-Salti et al. (1992)).

As long as the subgroup size remains unchanged, this kind of chart will have constant control limits regardless of the type of parts being machined.The method also allows the control limits to be calculated sooner through the combination of data from successive lots. As such, control of the process under surveillance can be achieved earlier.

Nominal XBAR & R charts may be used as a useful diagnostic tool for process problems provided the following conditions are strictly adhered to :

(a) the process is homogeneous,
(b) the variance is constant across different components,
(c) the process is not changed or modified, which would reduce the process dispersion.

Quite often, different components exhibit different amounts of variation, even though they are produced by the same machine. If the fundamental assumption of constant standard deviation is violated, the Nominal chart is no longer valid. In this instance, Standardized or Short Run XBAR-R charts, which are a straightforward adaptation of the Nominal charts, are recommended by Bothe (1989,1990b). These are simply a further extension of the previous idea where process dispersion is normalized for the traditional Shewhart chart along with the mean bias to enable components with different averages, as well as different dispersions, to be plotted on the same chart (Ermer at al. (1989)).

The problem with this method lies in the establishment and subsequent updating of the two scaling factors, namely $Tar(\bar{\bar{x}}_i)$ and $Tar(\bar{R}_i)$ for each part. Several ways to obtain these values were suggested by Bothe (1990b). After eliminating outliers, historical data in the form of prior charts, records from final inspection or audit results of prior runs may be used to calculate these values. In the case of new components having no historical data, reasonable values can be determined from data for similar components provided this is guided by the experience, knowledge and judgement of the personnel involved with the process under consideration. As another alternative, these values may even be derived from the design specifications if no other better choices are available.

As long as the appropriate scaling factors are available, out-of-control points can be detected right from the beginning of production.

In addition to the portrayal of observations from various part numbers, Bothe (1989) stated that this method permits the plotting of observations on different product characteristics of the same part, on the same chart. In this manner, a single chart with data from all operations plotted on it can be used to keep track of the material for a particular lot.

This approach of mixing production lots and normalizing process output data has been highlighted by many authors as an essential part of the quality assurance and improvement strategy in low volume, wide variety manufacturing for two main reasons :

(a) Reduction of the number of control charts and charting effort required which results in time and cost savings as well as higher productivity.

(b) Time-related process changes such as runs, trends and cycles can be more readily detected since pertinent process data is not scattered over separate charts.

While it is true that the first point may justify its use, some authors have expressed doubts concerning the appropriateness of combining data from different components to maximize information about the process behaviour. Ermer et al. (1989) argued convincingly that the desire for process improvement will unlikely be realized with this idea. As noted in their paper, different parts produced by the same machine are not necessarily sufficiently similar in terms of dispersion to justify plotting them together on the same Nominal chart. This statement is true because other operating factors, such as type of material, may have a significant effect on the variability of the parts. Recognising this fact, a study to determine whether any of the influential factors has a systematic effect on the variability is necessary prior to the implementation of this charting method. As demonstrated by Koons et al. (1991a,1991b)), data collected for this purpose can be examined by multiple regression and ANOVA techniques.

In addition, even if the parts do not differ in variability, the special causes of variation affecting one of them may not be the same as those influencing others, and it would be difficult to separate out the special causes of variation for each part in order to determine if an out of control signal is truly present on the chart. This invalidates the use of sequence rules with Nominal charts, across product changes, to identify non-random patterns which suggest the presence of assignable causes.

Ermer et al. (1989) also stated that there is a contradiction between the Standardized charting method and the philosophy of monitoring a process which is truly homogeneous. The reason provided for this is that as process dispersions are quite different, the processes obviously differ not only in terms of special causes of variation but also in the common causes of variation affecting them, these processes, therefore, should not be evaluated together. Unless they are monitored with separate charts, the use of sequence rules to provide useful diagnostic information will not be appropriate. Furthermore, identification and subsequent removal of common causes of variation in the individual processes will be made complicated. This, they maintain, definitely obstructs continuous process improvement.

As pointed out by Ermer et al. (1989), the normalizing approach also loses its appeal as a quality control procedure due to the following practical problems :

(a) Although the calculations for standardized charts are not difficult, they are more involved than traditional Shewhart charts.

(b) The coded data do not appear to have any physical meaning to production operators who usually have little background in statistics.

(c) Since the control limits for this chart never change, process improvements that are being implemented from time to time will not be reflected by the chart. In other words, no visual impression is available as to how much improvement has been made to the process.

Additionally, considerable effort and time have to be spent to obtain, review and revise the scaling factors if necessary. Furthermore, care must be taken to ensure that proper scaling factors are being used in each calculation to avoid misinterpretation regarding the stability of the controlled process.

In any kind of machine set-up, whether manual or automatic, there exists a certain amount of natural variability in the setting regardless of how well it is performed. This issue must not be overlooked and set-up acceptance should be based on sound statistical principles. On a long term basis, for the same machine or production process, the set-up error, which is defined as the deviation of average output from the desired value, tends to fluctuate in a random manner around its expected value which is usually assumed to be zero. In every set-up, therefore, as long as the set-up error is within its natural spread, no adjustment is necessary and the production should be allowed to run to avoid over control. The set-up method can of course be improved to reduce this source of variation provided it is economically feasible to do so.

In the presence of significant set-up variation, the use of Nominal and Short Run charts is not appropriate, Robinson (1991) and Bothe (1990b) demonstrated this. In particular, out of control conditions are indicated by the charts although the process is well in statistical control. These warning signals are actually caused by the set-up variation rather than out of control states. To cope with this situation, Bothe (1990b) suggested that two sets of scaling factors, i.e 'set-up' and 'run' factors should be used for each part number, he further discussed how they can be derived. Similarly, Robinson (1991) proposed that separate charts should be drawn for monitoring setup-to-setup variation and piece-to-piece variation in isolation. He and Robinson et al. (1993) illustrated this idea and exemplified how this can be achieved. For monitoring the first source of variation, he suggested charting the average value of the first five pieces (or the so called 'first offs') relative to nominal specification and the control limits for the resulting chart are derived by treating this as an individual values chart. As for control of piece-to-piece variation (or drift within a batch or a set-up), he suggested use of the traditional Range chart or a chart for the difference in average departure from nominal between the first five pieces and the last five pieces (or the 'last offs') for each short production run. He also mentioned use of the more practical 'Modified' control limits on the 'Deviation' chart to ensure that most individual items produced will conform to specifications. However, as he pointed out, this approach is not suitable if the required tolerance varies between runs.

When piece-to-piece variation is confounded with set-up variation, the Nominal and Short run charts do not provide adequate means of reflecting the actual status of the process. By contrast, separate monitoring of these components of variation does not only give a real picture of the process but also provides some guidance as to what might need to be fixed when an out-of-control signal is present.

In a low volume manufacturing environment, efficient use of the limited amount of data and the need for increased sensitivity to process changes are very important. In the light of these, Al-Salti et al. (1991) investigated the suitability of the moving average, moving range and cusum control charting techniques using transformed individual readings as a potential answer. They arrived at the conclusion that problems encountered during the implementation of a traditional SPC approach to small batch manufacturing can be avoided. However, the problem here again centers around the validity of evaluating different types of component together as if they are manufactured by the same process. Furthermore, historical data, which are required for estimating the scaling factors used in the transformations, may not be readily available. It was also pointed out, in the same paper, that cyclical behaviour and trends are expected on the moving average and moving range charts though nothing has changed. This is likely to cause difficulty in interpretation.

In statistical process control, measurement error should be given due consideration as this constitutes part of the inherent variation of a stable cause system. More simply stated, the natural variability observed in measured values of the quality characteristic of any industrial product is due in part to the variability of the product and in part to the variability in the method of measurement. This latter is sometimes negligible, but at other times cannot be ignored without risk. If the standard deviation of measurement error varies in some systematic manner, a Nominal chart will give misleading signals, especially when the measurement error is relatively significant.

In his paper, Farnum (1992) incorporated this component of variation into some process models having made certain reasonable assumptions. He then developed a charting procedure for one particular model. This assumes nonconstant process and measurement error, more specifically the short run process has constant coefficient of variation coupled with a measurement system whose error variability is proportional to the true reading. The resulting procedure seeks to remove the differences in average and dispersion between various components to enable them to be monitored with the use of a single chart. As long as subgroup size does not change, this charting method yields a common set of control limits for every component irrespective of their design specifications. These limits can be established early by utilizing data from different production lots in a predetermined manner. As with any charting method which plots different components on the same chart, caution should be exercised when sequence rules are applied.

Although efforts were made to justify the proposed models, a problem lies in identification of the model that is most appropriate in any given instance. Particularly is the problem acute if the decision as to which model to use is made by those without relevant knowledge, experience and proper training. It is also worth noting that, when an out-of-control point is present, the special causes responsible for this observation might not be easily identified, i.e one cannot tell whether the process has really changed, or whether the observed change is due to an inconsistent method of measurement or to the measurement process not being carried out using the stipulated procedure.

4. 'Self-Starting' Procedure Based on 'Running' Estimates of the Process Parameters

A series of articles by Quesenberry (1991a,1991b,1991c) presented an innovative approach to control, particularly pertinent to short-run processes where the total output is low and processes are in the start-up phase where early control is desired. The first paper considered independently identically distributed normal processes whereas the following two are devoted to monitoring processes with attribute data, namely, Binomial and Poisson processes, under various assumptions about the process parameters. Unlike the preceding approach, a non-linear transformation technique, specifically the 'Probability Integral Transformation' technique, in conjunction with the usual linear transformation, was used to develop new charting procedures. These procedures enable production operators to begin charting essentially with the first units or samples of production whether or not prior knowledge of the process parameters is available. Consequently, the task of identifying and removing assignable causes, and thereby bringing the process into control, can begin at an earlier stage. For the case where no relevant data is available in advance of a production run, the control parameters are 'estimated' and 'updated' sequentially from the current data stream. These 'running' estimates, together with the immediately succeeding observations are in turn used to test whether the process remains stable (see Hawkins (1993), pp.258). These dynamic estimates also provide information about the process capability thus allowing production personnel to determine if the process can be expected to meet the specifications consistently. However, for the case of monitoring quality characteristics on a continuous scale, Castillo et al. (1994) cautioned that the normality assumption of the measurements on which the relevant 'Q' charts are based should be checked in practice because, '.... for a small number of parts skewed distributions may characterize the process better'.

As the resulting plotted statistics are either standard normal variables with independent observations or approximately so, the proposed 'Q' charts are all plotted on standard normal scales. Although this permits the plotting of components with significantly different averages and dispersions, and even different statistics, on the same chart, thus simplifying charting administration, caution should be exercised and different symbols should be used to avoid misinterpretation. When subgroups or sampling inspection units vary in size, it is well understood that this situation is difficult to handle by classical methods. By contrast, the control limits and interpretation of point patterns for 'Q' charts are not affected by a varying sample size.

The behaviour of these charts for particular situations were studied using simulated data. The results show that 'Q' charts based on known and unknown parameters, are in close agreement with each other after the first few points, for in-control processes. As for processes with sustained shift in a parameter, however, the points on 'Q' charts which update the parameter estimates progressively from the data sequence will eventually settle into a pattern indicative of an in-control process. This problem was also highlighted by Castillo et al. (1994). They showed that the strength of the signal from 'Q' charts (for the case with unknown mean but with known standard deviation) when a persistent step change in mean occurs depends on both the number of samples *before* and *after* the shift. It was demonstrated in the same paper that, as a consequence of this, the average run length (ARL) performance of the 'Q' charts is poor in some cases. In order to enhance the sensitivity of 'Q' charts (for the case with unknown standard deviation) to changes in process mean, Castillo et al. suggested different ways for estimating the unknown standard deviation.

Due to the discrete nature of Binomial and Poisson processes, comparisons were made between the proposed 'Q' charts, standard normalizing charts and charts using other transformation techniques in the goodness of their normal approximations. Generally, it is found that charts based on 'Q' transformations are superior.

Despite its mathematical elegance, this 'Q' approach suffers from a number of drawbacks. Since no simple recursive formula is available and highly sophisticated computations are involved, implementation requires the use of complex mathematical algorithms. Reduction of this sophisticated concept to simple to use practical tools remains an issue.

Like any other methods involving transformation, the resulting plotted points on 'Q' charts do not appear to have any physical meaning to production operators.

Another disadvantage of this approach is the failure of 'Q' charts to reflect both the process-tolerance incompatibilities and severe off-target conditions which occur right from the beginning of production runs, except for the case where process parameters are known in advance or can be reliably estimated from the available historical data. As a consequence of this, unless close examination of the raw data is carried out, timely corrective actions will likely not be initiated until a considerable number of defects have been produced. This problem arises because such charts are designed to ensure that the process under surveilance is in a state of statistical control and hence they are unlikely to indicate any process trouble if no change in the process parameters takes place, even though the process is incapable or substantially off-target immediately after set-up.

It is perhaps worth noting that the two expressions (13) in Quesenberry (1991a) are in error. The correct versions of the formulae are :

$$w_i = \sqrt{\frac{n_i(n_1+\cdots+n_{i-1})}{n_1+\cdots+n_i}}\left(\frac{\overline{X}_i-\overline{\overline{X}}_{i-1}}{S_{p,i}}\right)$$

and

$$Q_i(\overline{X}_i) = \Phi^{-1}[G_{n_1+\cdots+n_i-i}(w_i)]$$
$$i = 2, 3, \cdots .$$

where the notation used here is as defined in the original paper. In fact, the former is pointed out in Corrigenda (October 1991 issue of JQT), but the latter is left out. It should also be noted that the number of degrees of freedom of the t-distribution associated with the argument w_i for the case where all the subgroups are of the same size (i.e $n_1 = n_2 = \ldots = n_i = n$, say) as given in equation (25) of Castillo et al. (1994) are in error and should be replaced by $i(n-1)$. As such, the validity of their simulation results on the run length performance of this 'Q' chart is questionable.

A significant contribution to this approach comes from Dawkins (1987) who proposed two cusum procedures based on transformed individual readings for checking the constancy of process average and variability obtained at process start-up, along with some implementation details and illustrative examples. These were proposed as substitutes for the standard cusum procedures which generally assume known parameters. The need for such an approach and the reasons for the inappropriateness of traditional cusum procedures were explained by means of an example on bias and precision control for chemical assays. The proposed method provides another useful alternative for controlling, particularly, short run processes as it does not require knowledge of process parameters in advance of production runs and eliminates the need for a separate preliminary study.

In order to effectively apply the cusum procedure, it is well understood that successive values for which the sum is accumulated should be independent and identically distributed. For this reason, the following transformation formula (attributable to Wallace (1959)) was suggested to obtain a sequence of independent and approximately standard normal variables, U_j's, :-

$$U_j = \left(\frac{8j-15}{8j-13}\right)\left[(j-2)\ln\left(1+\frac{T_j^2}{j-2}\right)\right]^{\frac{1}{2}}$$

where

$$T_j = \sqrt{\frac{(j-1)}{j}}\left(\frac{X_j - \overline{X}_{j-1}}{S_{j-1}}\right)$$

X_j : j^{th} individual reading

$\overline{X}_j$: mean of the first j readings

S_j : standard deviation of the first j readings

By maintaining a cusum of successive U_j's, starting from the 3rd observation, and using the established control rule, process mean stability can thus be monitored progressively without having to wait until adequate process performance data has built up. However, this method should not be used indiscriminately. Careful examination of the above transformation formula reveals that the resulting U_j's are always positive and can be regarded as 'folded' standard normal variables which can assume positive values only. In fact, this normal approximation formula was originally considered by Wallace (1959) for converting upper tail values of the student-*t* distribution to corresponding standard normal deviates. Hence, the need for a modification to the formula is indicated. The minor change necessary is simply the addition of a negative sign to the transformed value U_j if T_j is less than zero. For purposes of controlling process dispersion, Hawkins suggested using the scale cusum given in his previous paper (Hawkins (1981)) which involves cumulative summing of the following quantities :-

$$V_j = \frac{\left(\sqrt{|U_j|} - 0.822\right)}{0.349}$$

He made some efforts to justify his recommendation for a 'self-starting' cusum over the adoption of a cusum procedure based on some start-up calibration data. These included consideration of the average run length properties of the two methods. His simulation results indicate that 'self-starting' cusum procedures are superior to those obtained with some 25 special start-up values, not to mention the short run situations where usually much less than this is available for initiating conventional cusum charts.

A final concern about this approach is its likely lack of robustness to both non-normality and to the presence of outliers in the underlying distribution of process measurements. Without previous data, there is often no assurance that the process output will conform reasonably to a normal distribution. The question arises, therefore, as to what effect departures from the normality assumption will have upon performance. Hawkins (1987) argued, by quoting others' results, that his method works for non-normal heavy-tailed data with little loss in ARL performance. However, his argument is unconvincing because evaluating the efficiency of the proposed method should involve consideration of the distribution resulting from the given transformation and the possible correlations between successive transformed values, and not solely the applicability of the *t*-test based on the studentized deviate, T_j. This is a question that warrants further investigation, perhaps by some simulation studies.

The latter issue is particularly pertinent as a sequence of measurements (which might include occasional outliers) is used simultaneously both for process control and to refine parameter estimates. Apparently, incorporating unknowingly occasional valid extreme observations into the estimates of the process parameters will cause inflation or deflation of them. This can have a substantial negative impact on the performance of the control method. To cope with this as well as to protect the cusum method (which is intended primarily for 'picking up' sustained mean shifts of small magnitude) from signals generated solely by isolated outliers, Hawkins (1993) suggested a robustification approach using 'winsorisation'. 'Winsorizing' the measurements means that any measurement beyond a preset threshold will be set equal to it and used in subsequent calculations. In this manner, 'winsorisation' reduces or limits the effect of outliers on the parameter estimates and the properties of the control charts. This idea was further discussed by Hawkins (1993) who also examined the relationship between the 'winsorizing' constants for 'self-starting' cusums and cusums based on a large process performance study. He stated that this method provides good protection against outliers with little additional cost in computational effort. In the same paper, he also showed that, 'winsorizing' causes little loss to cusum procedures in responsiveness to actual mean shifts for various sets of clean, contaminated and clean-contaminated data.

5. Control Based on Exponentially Weighted Moving Averages

In the event of the historical process mean, μ_0 being available but not the standard deviation, Castillo et al. (1994) proposed two alternative methods as improvements to the 'Q'-type mean control technique based on individual measurements. These methods are based on some *exponentially weighted moving average* (EWMA) type control statistics. The first method results from a straightforward adaptation of the standard EWMA control algorithm with the smoothing factor, λ chosen to be 0.1, the initial value of the mean equated to μ_0 and the unknown standard deviation, σ estimated sequentially in some suggested manner. The resulting EWMA statistic is plotted on a control chart with limits

$$\mu_0 \pm L\sqrt{\tfrac{\lambda}{2-\lambda}}\hat{\sigma}_t$$

where $$\hat{\sigma}_t = \frac{S_t}{c_4}$$

with $S_1 = X_1 - \mu_0,$ $$S_t = \sqrt{\tfrac{1}{t-1}\sum_{i=1}^{t}(X_i - \overline{X}_t)^2}, \quad t = 2,3,\ldots$$

c_4 is a control chart factor depending on t that can be found from most of the standard text books on SPC and L denotes the number of standard deviations for the width of the control limits.

The other control algorithm was derived from the well known *Kalman* model (see for eg., Crowder(1989)) upon noting that the assumed i.i.d in-control model for the process measurements can be represented by a special case of the Kalman model. The resulting control statistic is given by the following recursive expression :-

$$\hat{\mu}_t = (1-\lambda_t)\hat{\mu}_{t-1} + \lambda_t X_t; \qquad \hat{\mu}_0 = \mu_0$$

This latter was referred to as the *adaptive Kalman filtering* control method since the smoothing factors or Kalman weights, λ_t's change adaptively according to

$$\lambda_t = \frac{q_{t-1}}{q_{t-1} + \sigma_t^2}$$

The variance components of the model, including the process variance, σ^2 and the *posterior* variance of the current process mean, q_t, are estimated and updated from the data sequence as follows :-

$$\hat{\sigma}_1^2 = S_1^2 \quad , \qquad \hat{\sigma}_t^2 = S_t^2 \qquad t = 2,3,\ldots$$

$$\hat{q}_t = \hat{q}_{t-1}(1-\lambda_t).$$

The initial value $\hat{q}_0$ was found to have practically no effect on the ARL performance of the method provided it is greater than zero. Following Crowder (1989), the control limits for this method were given as

$$\mu_0 \pm L\sqrt{\hat{q}_t}$$

It was demonstrated using simulation, that for a particular choice of the design parameters; λ, $\hat{q}_0$ and L, both control methods have better ARL performance than the corresponding 'Q' chart for 'picking up' sustained mean shift, especially when the shift size is small. It was also found that these methods (with the particular choice of the design parameters) are superior to the classical Shewhart chart (classical in the sense that the process parameters are assumed known) for small magnitudes of mean shift. Furthermore, it was observed that the first control method has a comparable performance to the classical EWMA chart. Despite these, the choice of the design parameters are arbitrary and no consideration is made of the optimality of the design for this control technique.

A control method based on the adaptive Kalman filtering similar to the above, coupled with a *tracking signal* feature, has also been proposed as an improvement to the 'Q' chart for the case where both μ and σ are unknown. In order to use this method, some prior estimates of μ (denoted by $\hat{\mu}_0$) and σ (denoted by $\hat{\sigma}_0$) are required. In the short-run or low volume manufacturing environment, reasonably accurate estimates may not be available. However, it was shown using simulation, that this control technique can provide better ARL performance than the 'Q' chart for large shift sizes (more than 2 standard deviations) if the estimate of μ differs from the true value by less than 1 standard deviation and the σ is under or over-estimated by less than 50%. The method involves the computation of the so-called *smoothed error statistic* and the *smoothed mean absolute deviation*, defined respectively as :-

$$Q(t) = \alpha e_1(t) + (1-\alpha)Q(t-1); \qquad Q(0) = 0$$

$$\Delta(t) = \alpha|e_1(t)| + (1-\alpha)\Delta(t-1); \qquad \Delta(0) = 0.8\hat{\sigma}_0\sqrt{2/(2-\lambda_t)}$$

where the one-step-ahead forecast error,

$$e_1(t) = X_t - \hat{\mu}_{t-1}$$

and the variance of the process is updated sequentially by

$$\hat{\sigma}_t^2 = \alpha\hat{\sigma}_{t-1}^2 + (1-\alpha)(X_t - \overline{X}_{t-1})^2$$

For this control technique, a signal is triggered when

$$\left|\frac{Q(t)}{\Delta(t)}\right| > L$$

where the control limit L is between 0 and 1.

As the computational effort involved is substantial, implementation of the above control algorithms requires computerisation. Other issues of practical importance include ease of use and understanding of the methods by shop-floor personnel. Furthermore, no guidelines about the choice of the design parameters to achieve desired operating performance are available.

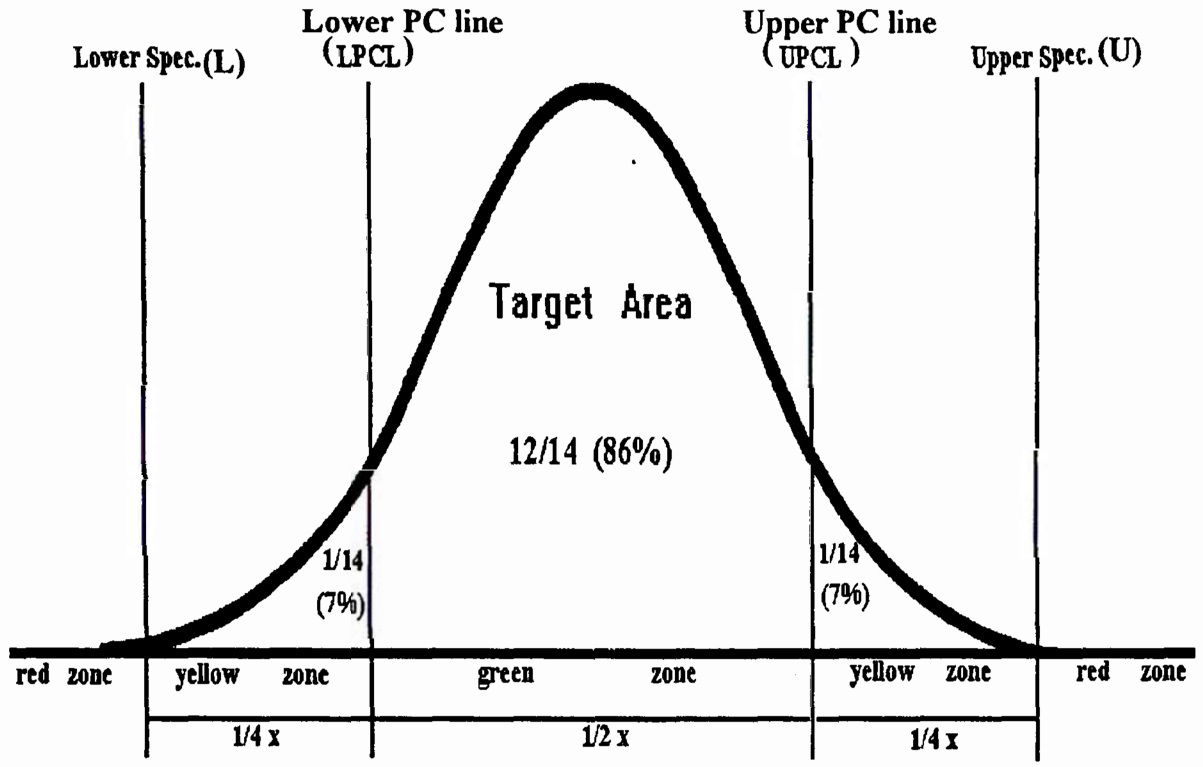
All the methods discussed thus far either ignore or give inadequate consideration to the problem of process 'warm up', when the process is invariably unstable. On the other hand, the following approach, which determines the 'control' limits based on given specifications, appears to be capable of handling this problem effectively.

6. Deriving 'Control' Limits From Specifications

Without the data necessary to set up conventional control charts, compounded with the problem of process 'warm up', it makes some sense to use product specifications to provide control information. A control technique that pre-determines its 'control' limits by reference only to the specifications rather than requiring an accumulation of data for computation of control limits, is known as Pre-Control (P.C).

P.C was first proposed by Shainin (1954) as a replacement for various on-line quality control methods and, in particular, as an improvement to $\overline{X}$ and R charts. It provides a simple, flexible and effective tool for process monitoring as well as set-up approval, particularly in the low volume manufacturing environment. P.C is conceptually different from traditional charting techniques in that it focusses directly on preventing non-conforming units from occurring rather than on maintaining a process in a state of statistical control. The basic principles underlying the P.C technique are illustrated in Figure 1.

Figure 1. Pre-Control Scheme



Suppose that the quality characteristic of interest is a variable on a continuous scale such as a physical dimension. The tolerance width is divided by 4. The boundaries of the middle half of the tolerance then become the Pre-Control lines. The area between these P.C lines is called the Target Area (T.A) or green zone. The remaining areas inside the tolerance are labelled yellow zones and those beyond the specification limits are called red zones. Assuming that the measurements on the quality characteristic under focus are normally distributed, correctly centered and that the process is just capable of meeting the specifications (i.e its natural spread, 6σ, equals the tolerance width), then approximately 1 in 14 times an observation will fall in either yellow zone by chance alone. Two consecutive values in these zones or one in the red zone is deemed adequate grounds for stopping the process and making adjustments.

Pre-Control operating rules are developed around these fundamental notions. In P.C, the decision for approval of set-up and resumption of a corrected process is based on the following rule :- '...If five consecutive units are within the target area before the occurrence of a red or two consecutive yellows, the set-up is qualified and full production can commence...' The reason is that this occurrence signifies that the process is well centered and highly likely to be producing at a satisfactory quality level. In fact, Bhote (1980) stated, without proof, that, with a slightly different rule, a minimum C_{pk} of 1.33 will automatically result ! However, this statement is subject to debate. As a counter example, it can be shown that, there is as large as 70% chance that a process with C_p of 1.33, but with a mean level of 0.5 standard deviation units away from the target (i.e a process with C_{pk} of 1.17) will be approved for running ! As the quality level resulting from such a process may not be satisfactory, as in manufacturing situations where critical or safety components are produced, such a definite unqualified statement is dangerous. Unlike $\overline{X}$ and R charts, whenever any process-tolerance incompatibilities exist, this rule ensures that appropriate study and action is initiated as soon as production is attempted. It is this process check rule that is the source of the largest claimed dollar savings and quality improvements (Satterthwaite (1973)).

Following set-up approval, P.C necessitates occasional sampling of just two consecutive units, to monitor the process performance, in contrast with the usual sample size of four or five for traditional control charting methods. Having items in either of the yellow zones is acceptable except when two occur consecutively because this is taken as indicative of the imminent production of defective items. Two successive yellows on the same side of the target signal the departure of the process mean from the target value or nominal specification, whereas if they occur on different sides, the process spread has most likely increased beyond its acceptable limit. In this manner, P.C enables corrective action to be taken usually before unacceptable work is produced and, hopefully, avoids repeated minor, and unnecessary corrections. In the event of getting an item in either part of the red zone, the process is stopped immediately, as it is already producing defective items ! Variations in this basic P.C plan applicable to less common situations are given in Shainin (1954) and Putnam (1962).

In order to justify his recommendation for 'pre-control', Shainin (1954, 1965, 1984a, 1984b) made some effort to discuss its statistical power. These included consideration of the expected proportion of non-conforming items produced resulting from the ongoing use of 'pre-control' based on a particular sampling rule. He showed that, using the currently recommended sampling rule of six inspection checks, on average, per trouble indication, a maximum average fraction defective (termed the Average Produced Quality Limit (APQL)) of less than 2% results for a normally distributed process, on a long term basis. In view of the implicit assumption from which the APQL is derived, that no change in process setting or spread occurs after set-up, the validity of this measure is questionable. However, a successful application of this technique in a 'zero defects' environment has been reported by Brown (1966). Some general guidelines about sampling frequency appeared in Satterthwaite (1973), Shainin (1988) and Traver (1985). Without previous knowledge of the average time between process adjustments, Shainin (1984a,1984b) suggested that a 20-minute sampling interval should first be used and adjusted subsequently.

As pointed out in Cook (1989), apart from the necessary frequency of periodic sampling, most of the doubts about 'pre-control' relate to the normality assumption. The general consensus amongst practitioners, is that even for stable processes it is doubtful that the fit to normality in the distribution tails is particularly close. By means of simulation, Sinibaldi (1985) examined the effect of non-normality on the appropriateness of 'pre-control'. His results show that 'pre-control' works for some skewed distributed processes but the total process spread, 6σ, has to be three quarters of the tolerance or less for the ability to hold a process target to markedly decrease. In addition, he evaluated the relative performance of 'pre-control' and $\overline{X}$ *and* R control on normal and skewed distributed processes with frequently changing means. The results of the comparison indicate that 'pre-control' causes more incorrect mean shift signals and has less effective control to target (as measured by the overall average, $\overline{\overline{X}}$ and the average distance of all items produced from the process target) than $\overline{X}$ control. However, using 'pre-control' to detect deterioration in the process spread results in fewer false alarms than using its counterpart, the R chart, for the same purpose.

In an attempt to illustrate some 'weaknesses' in $\overline{X}$ *and* R control, Bhote (1980) highlighted some touted attributes of 'pre-control' which include the following :-

(a) No calculations need to be performed for 'pre-control' operation except the extremely simple initial setting of 'pre-control' lines.

(b) Measurements can be observed and compared to specification limits in a way that is easily understood by operators without much likelihood of misinterpretation.

(c) Such eventualities as tool wear do not cause a premature reaction from 'pre-control'. It will only issue warning signals at times when the process is soon likely to produce defective products.

Taking a more complete view, Logothetis (1990) argued effectively that, despite its simplicity, 'pre-control' cannot be considered a serious technique of statistical process control. He, in fact, used the same case studies as Bhote (1980) (who used them to illustrate the 'weaknesses' of $\overline{X}$ *and* R control charts) to demonstrate the usefulness of statistical process control as a whole and the weaknesses of 'pre-control'.

The most productive use of Pre-Control is achieved when the process capability and tolerance are closely matched. In the event that the process capability is more than adequate, Hopper (1969) stated that, although it can be safely used, the basic form of P.C would result in premature stoppages for resetting. In such circumstances, he suggests use of 'modified' Pre-Control lines to make the most efficient use of this desirable state of affairs. However, this has the added difficulty of estimating the process standard deviation. In addition, P.C in this modified form does not provide adequate protection against worsening process capability, though it is unlikely to happen during production for short run processes.

When the process is not capable of economically producing within tolerance, P.C would lead to unnecessary tampering with the process as the warning signals are, in fact, due to random variation. Such actions are bound to increase the amount of defective work produced. In this case, both Satterthwaite (1973) and Hopper (1969) agree with the relaxation of Pre-Control lines. The use of process capability to determine P.C lines as mentioned by the latter would result in statistical process control in the true Shewhart sense, i.e identification and removal of assignable causes of variation in the production system. Again, the problem here lies in the establishment of the required process capability. Given an estimate of the process spread, $\overline{X}$ & R charts with conventional control limits, a subgroup size of four and usual decision rules imposed will be more sensitive to substantial shifts in process average and increases in standard deviation. Apart from having a smaller chance of false signals, it can be shown that these customary charting techniques are superior in detecting a mean shift of more than 0.68σ and an increase in standard deviation by more than a quarter respectively. As such, the use of P.C in this modified form in situations where such process changes are more likely to occur may not be justified.

While P.C has many useful features and practical advantages, Montgomery (1991) warned, by giving several drawbacks, that it should not be used indiscriminately. From the standpoint of its intended purpose, namely, capability control, however, only two points he made are relevant. First, since no chart is usually constructed, all the sensitizing rules and pattern-recognition procedures associated with the control chart cannot be used. Thus, the diagnostic information about the process contained in point patterns, along with the record keeping aspect of the chart is lost. Second, the assumption of adequate process capability is crucial because this will otherwise result in poor performance of P.C operation as mentioned above.

The simplicity and versatility of P.C make it an attractive tool for a large variety of applications. However, despite its many years of existence, P.C remains a largely under-used technique.

Although for P.C not essential, charting provides objective control evidence in the form of a graphical record to customers. Furthermore, it enables diagnosis of potential process problems by analysing the point patterns displayed on the chart, even though there is no trouble indication from P.C. In view of these benefits, Lewis (1991) presented a chart which exploits the concept of Pre-Control in conjunction with run techniques producing the so called, Preliminary Control Chart.

A new type of control chart which originates from the idea of Pre-Control, named the 'Balance' chart (B.C) has been introduced by Thomas (1990). It offers solutions to a number of different types of control problems.

B.C can operate in several different modes. With most types of control chart, prior estimation of the process mean and standard deviation is necessary. However, when B.C is used in Pre-Control mode, it eliminates the need for estimating these parameters but instead, derives 'pseudolimits' ($\pm$pL) from the specified tolerance. As such, this chart makes it possible to start monitoring a process without any information about its performance. For this reason, it provides another useful alternative for controlling short run processes.

Thomas (1991) suggested that the use of B.C in Pre-Control mode requires continuous monitoring. Successive measurements from the process on a certain quality characteristic are classified as -1, 0 or 1 according to their values relative to ±pL and specification boundaries. Cumulative recording and plotting of these data about a target line give information both on the process 'accuracy' and 'precision'. A mathematical derivation of the control limits which define the maximum deviation of the plot from the target line, and the maximum number of positive and negative changes from the start of the run is given in Thomas (1990). In addition, several rules governing the maximum number of changes in a given run length were developed to indicate the possible presence of process troubles. With manual charting, however, too many supplementary rules will complicate the interpretation of the Balance chart.

Besides data scoring, B.C has the unique feature that the operating rules and control limits are common to every application of the chart. Thus, it has great potential for computerisation.

Like 'pre-control', this technique does not require exact measurements, but only needs to know into which 'band' the measurements fall. In order to justify his recommendation, the author who proposed this charting method also provides comparison of the Balance chart and the X-BAR chart operating characteristics for a mean shift of 1 standard deviation, along with some illustrative examples which clearly show that B.C possesses higher sensitivity.

The last two charting methods falling in this category were presented by Bayer (1957) (see also Sealy (1954)) and Maxwell (1953). They were developed for quality control applications in job shops with diversified product and many set-ups and short-runs. Both methods are essentially the same, as they are adaptations of the Nominal X-BAR & R charts where limits are derived on the assumption that the process is just capable of meeting the required tolerance. The only difference between them is that the latter expresses the coded measurements and 'control' limits in terms of 'cells'.

Representing the full tolerance width by 10 cells, the resulting 'cell' chart has constant control limits regardless of the tolerance or the actual process capability, provided the sample size remains unchanged. Thus, it is possible to have just one chart per machine on which all parts and all tolerances processed can be controlled. However, these methods of control charting cannot handle unitolerance situations.

A comparison between XBar and R control and 'pre-control', based on certain statistical grounds and assuming a barely capable process has been presented by Tang and Barnett (1994). The results of the comparison indicate that the former are superior in many circumstances.

7. Adjusting Set-up Continuously Based On Process Output

As a substitute for conventional SPC for low volume production, an entirely different approach was proposed in Lill et al. (1991), 'Statistical Setup Adjustment' (SSA). This represents a form of 'feedback control' where the deviation from the desired dimension or error of the measured output characteristic, is used to calculate the best possible adjustment to be made in a machine set-up, starting with the first piece produced. As such, it is not a set-up approval method but one which provides an algorithm as to how much adjustment should be made as each of the successive observations arises. Methods are also presented to minimize the number of adjustments, to avoid early false signals and to anticipate the effects of a known trend such as tool wear.

As discussed earlier, in the presence of significant set-up variation, Robinson (1991) and Bothe (1990b) proposed separate monitoring of the set-up processes and their subsequent runs. If the set-up varies from the desired setting but is within predictable limits, no machine adjustment is necessary. This is due to the fact that such corrective action is not only uneconomical, but would probably result in a greater percentage of defects. SSA differs from this method in that it does not accept the risk of inaccurate set-up as a consequence of natural set-up variation which inevitably exists, but is constantly 'forcing' the set-up value to the desired dimension. This approach is, therefore, in line with Taguchi's idea of quality loss, i.e emphasis is placed on the uniformity of product quality characteristic about its target value rather than mere conformance to specifications.

In Statistical Setup Adjustment, both the machine variations and set-up errors are modelled with conceptual normal populations. From available information and experience, a 'maximum likelihood' estimator of the set-up error can be obtained and hence the correct adjustment derived. However, determination of the standard deviation of set-up variability based on subjective judgement, as suggested, is somewhat arbitrary and hence its reliability is doubted. In fact, it is possible to obtain such an estimate directly from the available data.

In this work, the implicit assumption is made that set-up is the critical or dominant 'system' that largely determines quality of the output. In other words, defects are the direct result of the accuracy of tools or precision of adjustment of the set-up. Therefore, theoretically, SSA does not provide protection against mean shift or increase in process spread due to some special causes during the production run.

If set-up is the dominant cause system, this method works provided the effects of adjustments are manifested instantaneously and in full. The realization of this, however, requires dynamic machine control with automatic inspection feedback and measurable means of adjustment.

8. Monitoring Process Input Parameters

By monitoring the process output, traditional SPC and the approaches discussed above, at best, indicate only when production is not free of troubles. In many instances, when an out-of-control condition is indicated, numerous corrective measures are possible and the correct course of action is not always obvious. As such, delay in preventing waste is inevitable. For small lot production, this can be regarded as the same shortcoming as 'post mortem' inspection !

In view of this limitation, recent research into the area of applying SPC in low volume manufacturing environments has given up trying to monitor the process output but instead has concentrated on the process inputs (see for eg., Foster (1988), Thompson (1989)). Foster (1988) presented this idea for controlling highly technical or time consuming processes where corrective measures for unacceptable work are often uncertain or even unknown. The implementation strategy for the suggested approach involves the creation of a 'true' process by compiling a 'Master Process Requirements List' from all specifications used for a particular process, selection of the vital few critical input parameters to be monitored and process capability evaluation.

9. Economically Optimal Control Procedures

While, traditionally, the development of SPC techniques has been mainly concerned with statistical efficiency, the ultimate objective of any process control strategy is cost reduction as a result of reduced scrap, rework and rejects, improved product quality and increased productivity. This objective may be accomplished by having an economically optimum policy governing the process monitoring, adjustment and maintenance activities. In the light of this, over the last four decades, a considerable amount of study has been devoted to the design of process control methods with respect to economic criteria. Various process models and cost structures have been proposed and the corresponding optimal control strategies derived. However, much of the theoretical work on incorporating cost considerations into the design of process control procedures has been undertaken implicitly in the context of long production runs.

The economic decision models currently available for on-line quality control can be broadly classified into two types. These are the economic-process-control models and the economic models for traditional SPC. In their paper, Adams et al. (1989) distinguished between these two models and hightlighted some similarities and differences between them. A thorough review of the literature on the latter was provided by Montgomery (1980). Recently, Ho et al. (1994) supplemented this work by presenting more detailed and complete discussions of different models and aspects of economic design for traditional SPC, and by summarizing the published work on economic designs of control charts covering the period from 1981 to 1991. For typical examples of the former, the reader is referred to Box et al. (1963, 1974), Bather (1963) and Taguchi (1981).

Crowder (1992) considered a short run economic-process-control model in which observations on a certain measured quality characteristic of the product are assumed to be generated by an integrated moving average (IMA(1,1)) process and the costs involved consist of the usual quadratic loss of process mean being off-target and the fixed cost for each adjustment. He also made the assumptions that any adjustment made to the process has a known effect (i.e no adjustment error) and that an adjustment changes the process mean instantaneously or before the next sample measurement is taken (i.e no process dynamics or inertia). Sampling cost and sampling interval were not formally considered. Furthermore, deterministic drift and step or cyclical changes were not taken into consideration.

The proposed model seeks to find the sequence of adjustments, $a_t's$, which minimizes the total expected loss, $L(n)$, incurred throughout the production run as given by the following expression :-

$$L(n) = E\left\{\sum_{t=1}^{n}\left(c_1\mu_t^2 + c_2\delta(a_{t-1})\right)\right\}$$

where c_1 is the cost parameter associated with any squared deviation of process mean, μ_t from target (assumed, without loss of generality, to be 0), c_2 represents the cost of adjustment irrespective of magnitude, n is the terminating sample number and

$$\delta(a) = 1 \quad if \; a \neq 0$$
$$= 0 \quad if \; a = 0$$

Using dynamic programming or the backwards induction technique, the author derived an algorithm which enables the optimal control or adjustment strategy (i.e the optimal sequence of adjustments, $a_t's$) to be obtained numerically. An approximation formula was also given for the case where the total number of inspections, $n \leq 10$ and the cost ratio, $c = \frac{c_2}{c_1} > 200$. In general, his results can be stated as follows. The resulting decision procedure as to when and how much adjustment should be made is based on the Bayes (or Posterior) estimate of the current process mean. It was also found that the 'control' or adjustment limits are changing with time and becoming wider towards the end of a production run, in contrast to the fixed limits proposed by some for the asymptotic case. This solution, he stressed, is consistent with the philosophy of traditional SPC in that it calls for adjustments only when the process mean is substantially off-target. In addition, it is found to be intuitively reasonable as the 'widening' action limits will decrease the likelihood of performing economically unjustified adjustments or maintenance near the end of a production run. In the same paper, Crowder demonstrated, by an example, that using the infinite-run (fixed) limits for the short-run problem with relatively large adjustment costs can significantly increase the total expected cost.

Woodward et al. (1993) also presented an approach to short run SPC which takes economic factors into consideration. In this work, they assumed a normal linear model in which three components of variation are involved. These are the set-up, adjustment (or resetting) and inherent process variabilities. The model also implies that there is no delay for any adjustment to take effect, no occurrence of parameter changes within a machine set-up or production run and that the process standard deviation is constant irrespective of product types. In comparison to that of Crowder (1992), these authors proposed a more realistic cost structure which includes the following components :

(i) inspection cost
(ii) rework cost
(iii) scrapping cost
(iv) cost associated with adjustment
(v) quadratic loss of being off-target

They considered a sequential scheme with three possible control actions at each decision point and attempted to derive a control rule using Bayesian methods such that the decision made at any stage of the sequential procedure minimizes the expected loss over all possible future decisions based on a given cost function. However, the solution of this optimal control problem is not straightforward and requires the use of techniques such as backwards induction. As they stated, in practice, it is impossible to find an optimal rule for this control plan because of the complexity introduced by the three-way decision structure. In view of this, a simplification in which a control decision is only made at two stages was considered. Even with this simplified scheme, determination of the decision boundaries remains complicated and requires a great deal of numerical computation.

For both the proposed economic process control models, some prior knowledge of the process parameters such as the variance terms or availability of some relevant historical data for their estimation is assumed. Woodward et al. (1993) described a method to quantify the historical information pertinent to their model. In practice, this could be a problem because historical data for this purpose is rarely sufficient in the short-run environment.

Another practical problem with these SPC approaches is the difficulty in specifying the cost parameters. This is due to the fact that some of the cost factors are intangible. For example, it is difficult to figure out the value of the cost parameters associated with the quadratic loss of being off-target and the loss due to process downtime (as a consequence of process adjustment), even by someone who has substantial knowledge of production and of the cost involved. As a first step to the implementation of these economic decision models, it is advisable to carry out a sensitivity analysis of the models to identify the critical parameters and subsequently exercise greater caution in their determination. However, this is a time-consuming exercise.

CONCLUDING REMARKS AND FUTURE PERSPECTIVES

The problems of using SPC in the short run environment are substantial but not entirely insummountable, as the work to date has exemplified. An important factor to consider in all SPC developments is the need to find easy to use practical tools. Even methods developed using sophisticated techniques need, if they are to be adopted as practical instruments, to be able to be synthesised into methods that are easy to use and easy to understand.

Computer power being as it is, there is a tendency to feel that there is no longer the need to provide compact analytic solutions or methods that appeal because of their simplicity. However, idealised, simple solutions are often preferred practically to more exact complex methods even with the ready availability of computer technology.

Taking these comments 'on board', there is undoubtedly scope to develop more meaningful economic designs for the short run environment. Similarly, if complexity can be contained, the area of multivariate investigations is wide open for future development.

REFERENCES

ADAMS, B. M. and WOODALL, W. H. (1989). "An Analysis of Taguchi's On-Line Process-Control Procedure Under a Random-Walk Model". *Technometrics* 13, pp. 401-413.

AL-SALTI, M. and ASPINWALL, E. M. (1991). "Moving Average Moving Range and CUSUM Modelling in Small Batch Manufacturing". *Proceedings of 3rd Conference of Asia Pacific Quality Control Organisation.*

AL-SALTI, M., ASPINWALL, E. M. and STATHAM, A. (1992). "Implementing SPC in a Low-volume Manufacturing Environment". *Quality Forum* 18, pp. 125-132.

AL-SALTI, M., and STATHAM, A. (1994). "A Review of the Literature on the Use of SPC in Batch Production". *Quality and Reliability Engineering International* 10, pp. 49-61.

ALT, F. B., GOODE, J. J. and WADSWORTH, H. M. (1976). "Small Sample Probability Limits For The Mean of a Multivariate Normal Process". *ASQC Technical Conference Transactions - Toronto*, pp. 170-176.

ARMITAGE, S. J. and WILHARM, M. T. (1988). "Techniques For Expanding The Sphere of Statistical Process Control". *Tappi Journal*, July, pp. 71-77.

BATHER, J. A. (1963). "Control Charts and Minimization of Costs". *Journal of Royal Statistical Society, Series B* 25, pp. 49-80.

BAYER, H. S. (1957). "Quality Control Applied to a Job Shop". *ASQC National Convention Transactions*, pp. 131-137.

BHOTE, K. E. (1980). *World Class Quality*, AMA Management Briefing.

BICKING, C. A. (1962). "Quality Control in the Chemical Process Industries". in *Quality Control Handbook*, 2nd. ed., Eds. J.M. Juran, L.A. Seder & F.M. Gryna, Jr., McGraw-Hill Inc., New York, NY.

BOTHE, D. R. (1988). "SPC for Short Production Runs". *Quality*, December, pp. 58-59.

BOTHE, D. R. (1989). "A Powerful New Control Chart for Job Shops". *ASQC 43rd Annual Quality Congress Transactions*, pp. 265-270.

BOTHE, D. R. (1990a). "A Control Chart for Short Production Runs". *Quality Australia* 7, pp. 19-20.

BOTHE, D. R. (1990b). "SPC For Short Production Runs". *Quality Australia* 7, pp. 53-56.

BOX, G. E. P. and JENKINS, G. M. (1963). "Further Contributions to Adaptive Quality Control : Simultaneous Estimation of Dynamics : Non-Zero Costs". *Bulletin of the International Statistical Institute* 34, pp. 943-974.

BOX, G. E. P., JENKINS, G. M. and MacGREGOR, J. F. (1974). "Some Recent Advances in Forecasting and Control, Part II". *Applied Statistics* 23, pp.158-179.

BROWN, N. R. (1966). "Zero Defects the Easy Way with Target Area Control". *Modern Machine Shop*, July, pp. 96-100.

BURR, I. W. (1954). "Short Runs". *Industrial Quality Control*, September, pp. 17-22.

BURR, I. W. (1976). *Statistical Quality Control Methods*. Marcel Dekker Inc., Milwaukee, Wisconsin.

BURR, J. T. (1989). "SPC in The Short Run". *ASQC 43rd Annual Quality Congress Transactions*, pp. 776-780.

CASTILLO, E. D. and MONTGOMERY, D. C. (1994). "Short-Run Statistical Process Control : Q-Chart Enhancements and Alternative Methods". *Quality and Reliability Engineering International* 10, pp. 87-97.

COOK, H. M. Jr. (1989). "Some Statistical Control Techniques for Job Shops". *43rd Annual Quality Congress Transactions*, pp. 638-642.

CRICHTON, J. R. (1988). "Guidelines and Techniques for Applying Control Charts in a Batch Operation". *Tappi Journal*, December, pp. 91-95.

CROWDER, S. V. (1989). "An Application of Adaptive Kalman Filtering to Statistical Process Control". in *Statistical Process Control in Automated Manufacturing*, Eds. J. B. Keats & N.F. Hubele, Marcel Dekker, New York, NY.

CROWDER, S. V. (1992). "An SPC Model For Short Production Runs : Minimizing Expected Cost". *Technometrics* 34, pp. 64-73.

CRYER, J. D. and RYAN, T. P. (1990). "The Estimation of Sigma For an X Chart : $\overline{MR}/d_2$ or S/c_4 ?". *Journal of Quality Technology* 22, pp. 187-192.

DOVICH, R. A. (1988). "Small Lot SPC...Really !". *Machine and Tool Blue Book*, December, pp. 12-14.

ERMER, D. S. and BORN, D. D. (1989). "Statistical Process Control for Short Production Runs : ΔX-MR Control Charts". *Quality Improvement Techniques For Manufacturing, Products And Services*, Eds. A.H. Abdelmonem, AT & T Bell Laboratories, pp. 43-87.

FARNUM, N. R. (1992). "Control Charts for Short Runs: Nonconstant Process and Measurement Error". *Journal of Quality Technology* 24, pp. 138-144.

FOSTER, G. K. (1988). "Implementing SPC in Low Volume Manufacturing". *ASQC Annual Quality Congress Transactions*, pp. 261-267.

GRANT, E. L. and LEAVENWORTH, R. S. (1980). *Statistical Quality Control*, 5th ed., McGraw-Hill, New York, NY.

HAWKINS, D. M. (1981). "A CUSUM for a Scale Parameter". *Journal of Quality Technology* 13, pp. 228-231.

HAWKINS, D. M. (1987). "Self-Starting Cusum Charts for Location and Scale". *The Statistician* 36, pp. 299-315.

HAWKINS, D. M. (1991). "Multivariate Quality Control Based on Regression-Adjusted Variables". *Technometrics* 33, pp. 61-75.

HAWKINS, D. M. (1993). "Regression Adjustment for Variables in Multivariate Quality Control". *Journal of Quality Technology* 25, pp. 170-182.

HAWKINS, D. M. (1993). "Robustification of Cumulative Sum Charts by Winsorization". *Journal of Quality Technology* 25, pp. 248-261.

HILLIER, F. S. (1964). "$\overline{X}$ Chart Control Limits Based on A Small Number of Subgroups". *Industrial Quality Control*, February, pp. 24-29.

HILLIER, F. S. (1967). "Small Sample Probability Limits for the Range Chart". *American Statistical Association Journal*, December, pp. 1488-1493.

HILLIER, F.S. (1969). "$\overline{X}$ and R Chart Control Limits Based On a Small Number of Subgroups". *Journal of Quality Technology* 1, pp. 17-26.

HO, C. and CASE, K. E. (1994). "Economic Design of Control Charts : A Literature Review for 1981-1991". *Journal of Quality Technology* 26, pp. 39-53.

HOPPER, A. G. (1969). *Basic Statistical Quality Control.* McGraw-Hill, New York, NY.

KING, E. P. (1954). "Probability Limits for The Average Charts When Process Standards Are Unspecified". *Industrial Quality Control*, May, pp. 62-64.

KOONS, G. F. and LUNER, J. J. (1991a). "SPC : Use in Low Volume Manufacturing Environment". in *Statistical Process Control In Manufacturing*, Eds. J.B. Keats & D.C. Montgomery, Marcel Dekker, New York, NY & ASQC Quality Press, Milwaukee, Wisconsin.

KOONS, G. F. and LUNER, J. J. (1991b). "SPC in Low Volume Manufacturing : A Case Study". *Journal Of Quality Technology* 23, pp. 287-295.

LEWIS, E. (1991). "The Preliminary Control Chart". *Quality Forum* 17, pp. 106-110.

LILL, M. H., YEN CHU and KEN CHUNG. (1991). "Statistical Setup Adjustment for Low Volume Manufacturing". in *Statistical Process Control in Manufacturing*, Eds. J.B. Keats & D.C. Montgomery, Marcel Dekker, New York, NY & ASQC Quality Press, Milwaukee, Wisconsin.

LOGOTHETIS, N. (1990). "The Theory of Pre-Control : A Serious Method or A Colourful Naivity ?". *Total Quality Management* 1, pp. 207-220.

MAXWELL, H. E. (1953). "A Simplified Control Chart for the Job Shop". *Industrial Quality Control*, November, pp. 34-37.

MONTGOMERY, D. C. (1980). "The Economic Design of Control Charts : A Review and Literature Survey". *Journal of Quality Technology* 12, pp. 75-87.

MONTGOMERY, D. C. (1991). *Introduction To Statistical Quality Control*, 2nd ed., John Wiley & Sons, New York, NY.

MURRAY, I. and OAKLAND, J. S. (1988). "Detecting Lack of Control in a New, Untried Process". *Quality And Reliability Engineering International* 4, pp. 331-338.

NELSON, L. S. (1982). "Control Charts for Individual Measurements". *Journal of Quality Technology* 14, pp. 172-173.

NELSON, L. S. (1984). "The Shewhart Control Chart Tests for Special Causes". *Journal of Quality Technology* 16, pp. 237-239.

NUGENT, B. A. (1990). "SPC in Short Run Processes". *Proceedings of The IEEE 1990 National Aerospace and Electronics Conference*, May 21-25, Dayton, OH, USA. pp. 1304-1307.

PUTNAM, A. O. (1962). "Pre-Control". *Quality Control Handbook*, 2nd ed., Eds. J.M. Juran, L.A. Seder & F.M. Gryna, Jr., McGraw-Hill, New York, NY.

PROSCHAN, F. and SALVAGE, I. R. (1960). "Starting a Control Chart". *Industrial Quality Control*, September, pp. 12-13.

QUESENBERRY, C. P. (1991a). "SPC Q Charts for Start-up Processes and Short or Long Runs". *Journal of Quality Technology* 23, pp. 213-224.

QUESENBERRY, C. P. (1991b). "SPC Q Charts for a Binomial Parameter p: Short or Long Runs". *Journal of Quality Technology* 23, pp. 239-246.

QUESENBERRY, C. P. (1991c). "SPC Q Charts for a Poisson Parameter : Short or Long Runs". *Journal of Quality Technology* 23, pp. 296-303.

QUESENBERRY, C. P. (1993). "The Effect of Sample Size on Estimated Limits for $\overline{X}$ and X Control Charts". *Journal of Quality Technology* 25, pp. 237-247.

ROBINSON, G. (1991). "SPC for Short Production Runs: An Example". *Quality Australia* 8, pp. 55-57.

ROBINSON, G., TOUW, J. V. D. and VEEVERS, A. (1993). "Control Processes, Not Products". *The Quality Magazine*, June, pp. 88.

ROES, K. C. B., DOES, R. J. M. M. and SCHURINK, Y. (1993). "Shewhart-Type Control Charts for Individual Observations". *Journal of Quality Technology* 25, pp. 188-198.

SATTERTHWAITE, F. E. (1973). "Pre-control for Supervisors". *Quality Progress*, February, pp. 26-28.

SEALY, E. H. (1954). *A First Guide To Quality Control For Engineers.* Her Majesty's Stationery Office, UK.

SHAININ, D. (1954). "Chart Control Without Charts -- Simple, Effective J. & L. Quality Pre-control". *ASQC Quality Control Convention Paper*, pp. 405-417.

SHAININ, D. (1965). "Techniques for Maintaining a Zero Defects Program". *AMA Management Bulletin* 71, pp. 16-21.

SHAININ, D. (1984a). "How to Improve Upon the Benefits of the Good, Old $\overline{X}$ and R Control Charts". *World Quality Congress '84 Transactions*, pp. 48-56.

SHAININ, D. (1984b). "Better Than Good Old $\overline{X}$ & R Charts Asked By Vendees". *ASQC 38th Quality Congress Transactions - Chicago*, pp. 302-307.

SHAININ, D. (1988). "Pre-control". *Quality Control Handbook*, 4th ed., J.M. Juran, McGraw-Hill, New York, NY.

SINIBALDI, F. J. (1985). "Pre-control, Does It Really Work with Non-normality". *ASQC Quality Congress Transactions - Baltimore*, pp. 428-433.

TAGUCHI, G. (1981). *On-Line Quality Control Control During Production*. Tokyo, Japanese Standards Association.

TANG, P. F. and BARNETT, N. (1994). "A Comparison of Mean and Range Charts with Pre-Control Having Particular Reference to Short Run Production". To appear in Nov-Dec issue *of Quality and Reliability Engineering International*.

THOMAS, D. E. (1990). "The Balance Chart : A New SPC Concept". *Quality And Reliability Engineering International* 6, pp. 357-371.

THOMAS, D. E. (1991). "The Balance of Probability : A New Concept of Statistical Process Control". *Quality Forum* 17, pp. 129-140.

THOMPSON, L. A. Jr. (1989). "SPC and The Job Shop". *ASQC 43rd Annual Quality Congress Transactions*, pp. 896-901.

TRAVER, R. W. (1985). "A Good Alternative to $\overline{X}$-R Charts". *Quality Progress*, September, pp. 11-14.

WALLACE, D. L. (1959). "Bounds on Normal Approximations to student's and the Chi-square Distributions". *Annals of Mathematical Statistics* 30, pp. 1121-1130.

WOODWARD, P. W. and NAYLOR, J. C. (1993). "An Application of Bayesian Methods in SPC". *The Statistician* 42, pp. 461-469.

YANG, C. H. and HILLIER, F. S. (1970). "Mean and Variance Control Chart Limits Based On a Small Number of Subgroups". *Journal of quality Technology* 2, pp. 9-16.