VICTORIA UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF COMPUTER AND MATHEMATICAL SCIENCES

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Mean and Range Charts with the
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(34 EQRM 12)
December 1993

TECHNICAL REPORT

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A STATISTICAL COMPARISON OF MEAN AND RANGE CHARTS WITH THE METHOD OF PRE-CONTROL

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September 1993

SUMMARY

This paper provides a rationale for making a statistical comparison between the techniques of 'pre-control' and traditional $\overline{X}$ *and* R charts. Special attention is drawn to the application of both techniques to the short run manufacturing environment where, for the use of $\overline{X}$ *and* R charts, the issue of parameter estimation is an additional problem. The total discussion is given in the context of the manufacture of discrete items.

KEY WORDS Pre-Control $\overline{X}$ *and* R charts Adjusted Limits Statistical Comparison Assumed C_p value

INTRODUCTION

In 1924 Dr. Walter Shewhart first introduced the $\overline{X}$ *and* R charting technique for the statistical monitoring and control of industrial processes. Now, after many decades of use, they have become the core around which has been built a body of statistical techniques expressly designed for the purpose of controlling the quality of manufactured products.

A competing procedure, employing a different strategy and known as 'pre-control' (p.c.), was proposed by Shainin [11] in 1954 as a replacement for various special purpose plans for quality control and, in particular, as an improvement to the then 30 year old technique of $\overline{X}$ *and* R control. 'Pre-control' focuses directly on preventing non-conforming units from occurring, rather than on maintaining a process in a state of statistical control, which is the strategy underpinning the use of $\overline{X}$ *and* R charts.

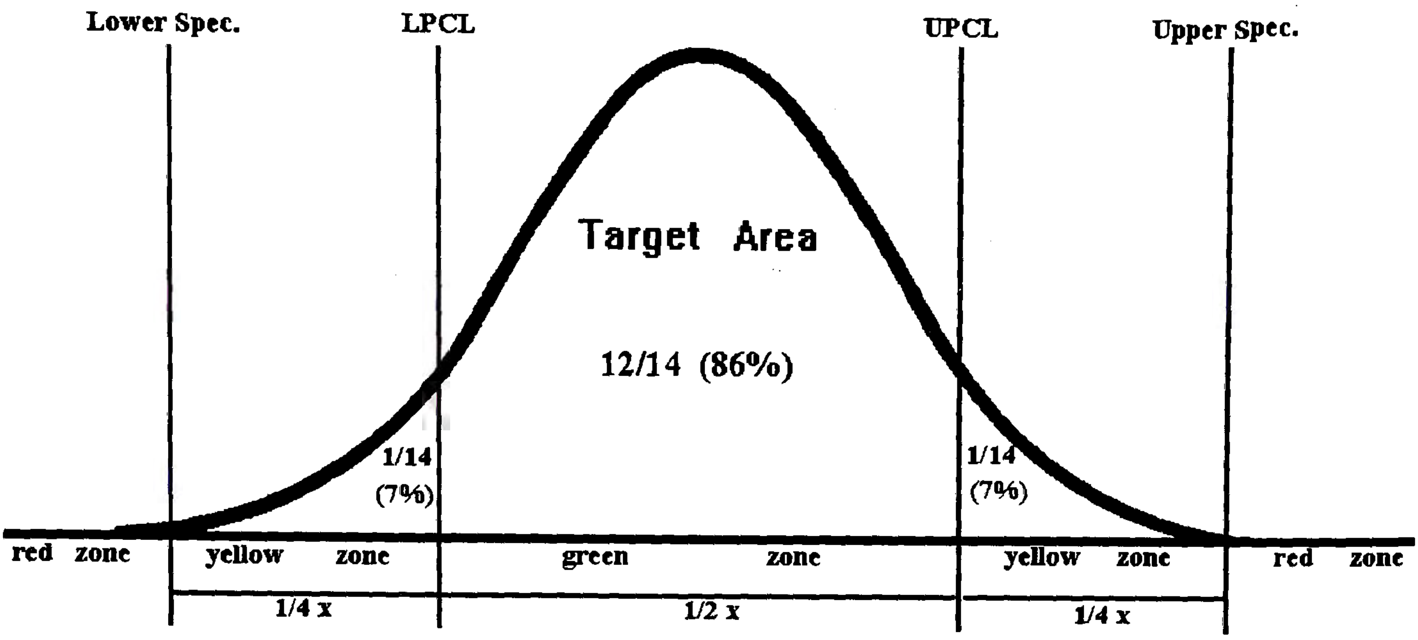
When 'weighing' the merits and disadvantages of competing industrial control procedures, issues such as statistical efficiency, cost effectiveness, extent and ease of use need all to be considered. In fact these factors, to varying degrees, play major roles in determining the overall success of quality monitoring, maintenance and improvement efforts.

After giving a brief outline of 'pre-control' and re-iterating its acclaimed practical benefits, this paper provides a rationale for making a statistical comparison between the technique and that of traditional $\bar{X}$ *and* R charts. It is assumed that the reader is familiar with the latter. Special attention is drawn to the application of both techniques to the short run manufacturing environment where, for the use of $\bar{X}$ *and* R charts, the issue of parameter estimation is an additional problem. The total discussion is given in the context of the manufacture of discrete items.

A REVIEW OF PRE-CONTROL

The basic principles underlying the 'pre-control' technique are illustrated in Figure 1.

Figure 1. Pre-Control Scheme



Suppose that the quality characteristic of interest is of the variable type such as a physical dimension. The tolerance is divided in three and the boundaries of the middle section called 'pre-control' lines. The area between these lines is described as the 'target area' or the green zone. The remaining areas between the specifications are labelled the yellow zones and those beyond the specification limits are termed the red zones. If the process is just capable of meeting the specifications, (i.e its C_p value is 1) and if centered at the nominal dimension then, by normal theory, approximately 1 in 14 times an observation will fall in either yellow zone by chance alone. It is assumed that the product characteristic under focus follows a normal distribution with a standard deviation of σ. A single observation falling in these zones is not deemed an indication of the presence of a process disruption. Two consecutive values in these zones, however, or one in the red zone, is considered adequate evidence of trouble and grounds for process adjustment.

'Pre-control' operating rules are developed around these fundamental notions. In a modified version, the well known 'first-piece inspection' procedure for approval of set-up and resumption of a corrected process is substituted with the somewhat tighter rule '...If five consecutive units are within the target area before the occurrence of a red or a yellow, the set-up is qualified and full production can begin...' The reason is, that the occurrence of this event indicates that the process is well centered and capable of producing at a satisfactory quality level with a high probability. The probabilities of approving a set-up which is centered at the nominal dimension, for various process capabilities, C_p, and using the above rule, are given in Table 1.

Table 1. Probability of Set-up Approval for Pre-Control

C_p	0.50	0.75	1.00	1.25	1.33	1.50
Prob.	0.0489	0.2210	0.4882	0.7308	0.7919	0.8838

If five consecutive greens prove difficult to obtain, then this is an indication that the process is either incorrectly centered and requires adjustment, or that the process is not capable of consistently meeting the specifications. This check rule is useful for short production runs for which the 'set-up' is a crucial factor affecting the quality of the subsequent process output.

Once the process has passed the initial set-up stage, periodically 2 consecutively produced items are examined to monitor performance. Having items in either of the yellow zones is acceptable, except when two occur consecutively. Two successive yellows on the same side of the target, signal the impending departure of the process mean from the worst acceptable situation.

If they occur on different sides of the target, the process spread has most likely increased beyond its acceptable limit. In this manner, 'pre-control' enables corrective action to be taken before unacceptable work is produced and, hopefully, avoiding repeated minor, and unnecessary corrections. In the event of getting an item in either part of the red zone, the process is stopped immediately, as it is already producing defective items! Variations in this 'pre-control' plan, applicable to less common situations are given in Shainin [11] and Putnam [8].

In order to justify his recommendation for 'pre-control', Shainin [11,12,13,14] made some efforts to discuss its statistical power. These included consideration of the long run expected proportion of nonconforming units produced resulting from the ongoing use of 'pre-control' based on a particular sampling rule. He showed that the maximum value of this quality measure, termed the average produced quality limit (APQL), does not exceed 2% for normally distributed processes if 6 inspection checks, on average, are made between typical process adjustments, [13]. Some very general discussion about the sampling frequency appeared in Satterthwaite [9], Shainin [15] and Traver [17]. Without previous knowledge of the average time between process adjustments, Shainin [14] suggested that a 20-minute sampling interval should first be used and adjusted subsequently.

As pointed out in [4], some of the expressed doubts about 'pre-control' relate to the normality assumption. In this regard, Shainin [15] stated that the effect of the process distribution is not significant to 'pre-control' as the method assists in controlling the manufacture of defects ...'by the size of the yellow zones, but not by the target area' ! Sinibaldi [16] used simulation techniques to examine the effect of non-normality on the appropriateness of 'pre-control'. In addition, he evaluated the relative performance of 'pre-control' and $\bar{X}$ *and* R control on normal and skewed distributed processes with frequently changing means. The results of the comparison indicate that $\bar{X}$ control causes less incorrect mean shift signals and has better control to target (as measured by the overall average, $\bar{\bar{X}}$ and the average distance of all items produced from the process target) than 'pre-control'. However, using the R chart to detect deterioration in the process spread results in more false alarms than using 'pre-control' for the same purpose.

Bhote [2] attempted to illustrate some 'weaknesses' in $\bar{X}$ *and* R control charting, using two case studies. Taking a more complete view, Logothetis [5] argued effectively that, despite its simplicity, 'pre-control' cannot be considered a serious alternative to SPC. He, in fact, used the same case studies as Bhote [2] (who used them to illustrate the 'weaknesses' of $\bar{X}$ *and* R control charts) to demonstrate the usefulness of SPC as a whole and the weaknesses of 'pre-control'. However, no comparison has been made between 'pre-control' and $\bar{X}$ *and* R charts on the basis of average run length (ARL). This is due to the fact that, 'pre-control' lines are derived from specification limits, causing the ARL for a given mean shift to vary according to the actual process capability (C_p).

THE PRACTICAL MERITS OF PRE-CONTROL

There is little mention of 'pre-control' in many standard text books on statistical process control, despite it having certain practical advantages over $\bar{X}$ *and* R charts. This could indicate a belief that a reasonable statistical comparison between the two techniques is not legitimate, that there is a reluctance to forsake $\bar{X}$ *and* R charting, since the method has proven useful over many years and in many industries or indicate a view aligned with that of Logothetis [5] that 'pre-control' is too limited in its perspective.

With the unique setup rule of 'pre-control', the first five consecutively manufactured units are all that is required to determine whether any process-tolerance incompatibilities exist before full production is allowed to commence. There is, of course, no definite knowledge of how many units will have to be checked before five un-interupted good ones are obtained. In comparison, when using $\bar{X}$ *and* R charts, it is necessary to have fairly long process trial runs in order to collect sufficient sample data to establish the existence of a state of statistical control, and subsequently, to estimate the process standard deviation so as to correctly locate the control lines.

Following setup approval, 'pre-control' provides for the occasional sampling of 2 consecutively produced units to monitor on-going process performance, in contrast to the routine sample size of 4 or 5 often recommended for $\bar{X}$ *and* R charts. No calculations need to be performed for 'pre-control' operation except for the extremely simple initial setting of 'pre-control' lines, whereas continual routine computations are involved with use of $\bar{X}$ *and* R charts. For these latter, it is not only necessary to calculate the control statistics for subgroups, but also necessary to estimate and revise the control limits from time to time.

For 'pre-control', measurements can be seen, compared to specification limits and easily understood by operators without the likelihood of misunderstanding or misinterpretation. Additional worker participation in decision making and problem solving may be gained through operators having a better appreciation of the techniques in use.

Whilst, in practice, determination of the sampling interval for $\overline{X}$ *and* R charts is arbitrary, 'pre-control' provides a simple and flexible rule of 6 inspection checks per trouble indication which, on a long term basis, results in a maximum average fraction defective of less than 2% for a normally distributed process [13]. A successful application of 'pre-control' in a 'zero defects' environment has been reported by Brown [3]Regulating sampling on the basis of recent process performance, seems a more reasonable and efficient approach to adopt than sampling at fixed intervals, as it entails more frequent sampling when the process is unsatisfactory.

Such eventualities as tool wear do not cause a premature reaction from 'pre-control'. It will only issue warning signals at times when the process is soon likely to produce defective products.

Since 'pre-control' does not require exact measurements but only needs to note the zone into which the measurements fall, complex and expensive measuring equipment may be replaced by 'go/no-go' colour coded gauges. Furthermore, electronic gauging can be considerably simplified if it is only required to distinguish between a few measurement bands. As a result, there can be a reduction in capital investment and calibration costs.

Another important feature of 'pre-control' is its ready applicability to a variety of situations including the short production run manufacturing environment which has become increasingly common following the general move into Just-In-Time (JIT) production and flexible manufacturing.

Despite its many years of existence, however, and its apparent practical merits, given in brief here, 'pre-control' has not been widely adopted as a replacement for traditional $\bar{X}$ *and* R charts. Logothetis [5] extensively criticised adoption of 'pre-control' over use of $\bar{X}$ *and* R charts on a number of grounds. It is intended that the material contained in this paper will provide some additional, statistically based material, that will help facilitate a rational judgement on which of the two techniques to adopt in any given situation.

SHORT RUNS AND PRE-CONTROL

There is no universally agreed definition of a 'short run', however, the term is generally used to describe production processes with typically less than 50 items made within a single machine set-up. Short runs, therefore, at first glance, do not readily lend themselves to the use of Shewhart $\bar{X}$ *and* R charts.

The essential problem that obstructs the application of standard control charting techniques in short production run situations, is the inability to estimate the process variability, because of insufficient data. The problem is further aggravated by problems of process 'warm up'. This phenomenon is commonly a dominant feature in short-run processes, as instability after setup or reset can represent a large proportion of production time. Neglecting this fact and using data from such a period to obtain control limits can lead to erroneous conclusions regarding past, current and future states of the process. Murray et al. [7] demonstrated this using simulation.

Unlike $\overline{X}$ *and* R charts, 'pre-control' is a control technique which predetermines its control limits by reference to product specifications only, rather than requiring an accumulation of data for computation of them. It is also capable of handling the problem of process 'warm up'. It is, therefore, highly suitable for application to short production runs.

A STATISTICAL COMPARISON BETWEEN PRE-CONTROL AND X-BAR AND R CHARTS

For short production runs, when there is insufficient previous data necessary to obtain the control limits for $\overline{X}$ *and* R charts, a number of authors (see, for example, Sealy [10] and Bayer [1]) have proposed setting control limits on the assumption that the process is just capable of meeting specifications, and assuming that the mean level of the process is equal to the nominal specification. This approach is used here to provide a basis for a statistical comparison between 'pre-control' and $\overline{X}$ *and* R charts.

In the following comparison, a subgroup size of 4 is chosen for the application of $\overline{X}$ *and* R charts because this is commonly recommended. It is also assumed that the quality characteristic under consideration is normally distributed and that no supplementary run rules are used with the $\overline{X}$ chart. First, consider the probabilities of detection by the sample immediately following a process mean shift, using an $\overline{X}$ chart and a 'pre-control' chart. These probabilities are provided in Tables 2 and 3 for various combinations of process capability (C_p) and mean shifts in multiples (k) of the standard deviation (σ). In both tables, the entries are the probabilities of issuing a correct signal of the mean shift except when k = 0, in which case the values tabulated are the probabilities of a false warning. Signals from 'pre-control' that we employ here as indication of a process mean shift, are 2 consecutive items in the same yellow zone, or 1 in the red zone and the other not falling beyond the 'pre-control' line on the opposite side of the nominal value. Furthermore, it should be noted that the control limits for the $\bar{X}$ *and* R charts are set using conventional control chart factors with the additional assumptions that,

$$\mu = \frac{U+L}{2} \ and \ \sigma = \frac{U-L}{6}.$$

The entries in Table 3, other than those corresponding to $C_p = 1$, are the probabilities of detecting a mean shift of the indicated magnitudes when the C_p has been assumed to be 1 but is in fact one of the values indicated. It has been adequately demonstrated in the literature that the $\bar{X}$ chart is tardy in registering small changes in the process mean. Where the 'speedy' detection of small mean shifts are required, additional control rules or alternative charting techniques are necessary. Thus the tables provide, for comparison, probabilities for a number of realistic mean shifts; realistic in the sense that they reflect situations where $\bar{X}$ (with no additional rules) and 'pre-control' can conceivably be considered competing techniques. Besides having a lower likelihood of a false signal, the $\bar{X}$ chart possesses a higher probability of 'picking up' the mean shift irrespective of the actual process capability, except where indicated by *, when the differences between corresponding entries in the two tables are marginal. In one sense, a more reasonable comparison can be accomplished through adjusting the control limits for the $\bar{X}$ chart in such a way that the resulting probability of issuing a false signal, when $C_p = 1$, is the same as that of 'pre-control'. This involves moving the control lines nearer to the nominal value. Following such a modification, the corresponding probabilities of immediate detection are given in Table 4. Tables 2 and 4 clearly illustrate the superiority of the $\overline{X}$ chart in terms of sensitivity to process mean shifts.

Table 2. Power of Pre-Control-Mean Shift

C_p \ k	0	±1.0	±1.5	±2.0	±2.5
0.50	0.2488	0.5814	0.8125	0.9418	0.9879
0.75	0.0701	0.3153	0.5759	0.8072	0.9391
1.00	0.0136	0.1264	0.3166	0.5759	0.8057
1.25	0.0022	0.0412	0.1410	0.3383	0.5950
1.33	0.0012	0.0279	0.1051	0.2752	0.5223
1.50	0.0003	0.0116	0.0534	0.1685	0.3767

Table 3. Power of X-bar Chart with 3σ Limits (assumed $C_p = 1$)

C_p \ k	0	±1.0	±1.5	±2.0	±2.5
0.50	0.1336	0.6915	0.9332	0.9938	0.9998
0.75	0.0244	0.4013	0.7734	0.9599	0.9970
1.00	0.0027	0.1587	0.5000	0.8413	0.9773
1.25	0.0002	0.0401*	0.2266	0.5987	0.8944
1.33	0.0001	0.0232*	0.1611	0.5040	0.8438
1.50	0.0000	0.0062*	0.0668	0.3085	0.6915

Table 4. Power of X-bar Chart with adjusted Limits (assumed $C_p = 1$)

C_p \ k	0	±1.0	±1.5	±2.0	±2.5
0.50	0.2173	0.7782	0.9613	0.9972	0.9999
0.75	0.0642	0.5594	0.8748	0.9842	0.9992
1.00	0.0136	0.3201	0.7028	0.9373	0.9943
1.25	0.0020	0.1391	0.4664	0.8201	0.9723
1.33	0.0010	0.0985	0.3890	0.7637	0.9571
1.50	0.0002	0.0444	0.2416	0.6174	0.9030

Tables 5 and 6 provide average run-lengths for detection of a mean shift using 'pre-control' and an $\bar{X}$ chart respectively, based on the probabilities contained in Tables 2 and 3.

Table 5. A.R.L. Pre-Control

$C_p \setminus k$	0	±1.0	±1.5	±2.0	±2.5
0.50	4.02	1.72	1.23	1.06	1.01
0.75	14.27	3.17	1.74	1.24	1.06
1.00	73.53	7.91	3.16	1.74	1.24
1.25	454.55	24.27	7.09	2.96	1.68
1.33	833.33	35.84	9.51	3.63	1.91
1.50	3333.33	86.21	18.73	5.93	2.65

Table 6. A.R.L $\bar{X}$ Chart (control based on $C_p = 1$)

$C_p \setminus k$	0	±1.0	±1.5	±2.0	±2.5
0.50	7.49	1.45	1.07	1.00	1.00
0.75	40.98	2.49	1.29	1.04	1.00
1.00	370.37	6.30	2.00	1.19	1.02
1.25	5000	24.94	4.41	1.67	1.12
1.33	10000	42.92	6.21	1.98	1.19
1.50	-	161.29	14.97	3.24	1.45

Of course an ARL comparison is particularly meaningful if it is assumed that the sampling interval is common for the two methods. This further raises the matter of sampling effort, since 'pre-control' has an implied sample size of 2 and the $\bar{X}$ *and* R charts being used here for comparison, have a sample size of 4. This matter will be discussed later.

Tables 7, 8 and 9 are extensions to Table 3. where different C_p values are assumed at the outset. From these it can be seen that if C_p is taken to be 0.75 then $\bar{X}$ is superior only for those cases indicated by * in detecting a real mean shift. However, it is superior for all cases in having smaller probabilities of false alarms. When C_p is assumed to be 1.25, even if the actual value is as low as 0.5 or as high as 1.50 the $\bar{X}$ chart is superior for detecting all the given mean shifts. The probabilities of false alarms for the two methods are compatible. Similarly for the assumption of $C_p = 1.50$, except here, 'pre-control' is superior with respect to false alarms.

Table 7. Power of X-bar Chart (assumed Cp = 0.75)

C_p \ k	0	±1.0	±1.5	±2.0	±2.5
0.50	0.0455	0.5000	0.8413*	0.9773*	0.9987*
0.75	0.0027	0.1587	0.5000	0.8413*	0.9773*
1.00	0.0001	0.0228	0.1587	0.5000	0.8413*
1.25	0.0000	0.0014	0.0228	0.1587	0.5000
1.33	0.0000	0.0005	0.0102	0.0934	0.3745
1.50	0.0000	0.0000	0.0014	0.0228	0.1587

Table 8. Power of X-bar Chart (assumed Cp = 1.25)

C_p \ k	0	±1.0	±1.5	±2.0	±2.5
0.50	0.2301	0.7881	0.9641	0.9974	0.9999
0.75	0.0719	0.5793	0.8849	0.9861	0.9993
1.00	0.0164	0.3446	0.7258	0.9452	0.9953
1.25	0.0027	0.1587	0.5000	0.8413	0.9773
1.33	0.0014	0.1166	0.4239	0.7905	0.9647
1.50	0.0003	0.0548	0.2743	0.6554	0.9192

Table 9. Power of X-bar Chart (assumed Cp = 1.50)

C_p \ k	0	±1.0	±1.5	±2.0	±2.5
0.50	0.3173	0.8413	0.9773	0.9987	1.0000
0.75	0.1336	0.6915	0.9332	0.9938	0.9998
1.00	0.0455	0.5000	0.8413	0.9773	0.9987
1.25	0.0124	0.3085	0.6915	0.9332	0.9938
1.33	0.0078	0.2546	0.6331	0.9099	0.9904
1.50	0.0027	0.1587	0.5000	0.8413	0.9773

It is also of value to study the probabilistic behaviour of these two control techniques in relation to how quickly they respond to an increase in process dispersion. For 'pre-control', 2 successive measurements beyond different 'pre-control' lines constitute a warning signal that the process spread is worse than the one implicitly assumed. However, the occurrence of this event does not only depend upon the process capability, it is also affected by the deviation of the process mean from target. As reflected in Table 10, for a given level of process capability, the larger the deviation, the smaller the chance of getting such a signal. The corresponding probabilities of a signal from the R chart are given in Tables 11 and 12 for the cases where conventional and adjusted control limits are used. Control lines are adjusted in the sense that they equate the probabilities of false alarms for the two methods. As shown in these tables, an R chart clearly provides better protection against a worsening process capability. Tables 13 and 14 show the power of the conventional R chart when the assumed C_p is greater than 1.

Table 10. Power of Pre-Control - Increase in Dispersion

C_p \ k	0	±1.0	±1.5	±2.0	±2.5
0.50	0.1027	0.0480	0.0189	0.0053	0.0011
0.65	0.0543	0.0246	0.0093	0.0025	0.0005
0.75	0.0340	0.0151	0.0056	0.0014	0.0003
0.85	0.0205	0.0090	0.0033	0.0008	0.0001
1.00	0.0089	0.0038	0.0014	0.0003	0.0001

Table 11. Power of R Chart-Conventional Limits (assumed $C_p = 1$)

C_p	0.50	0.65	0.75	0.85	1.00
Prob.	0.3445	0.1349	0.0613	0.0246	0.0049

Table 12. Power of R Chart-Adjusted Limits

C_p	0.50	0.65	0.75	0.85	1.00
Prob.	0.3940	0.1715	0.0850	0.0376	0.0089

Table 13. Power of R Chart (assumed Cp = 1.25)

C_p	0.50	0.65	0.75	0.85	1.00	1.25
Prob.	0.5445	0.3093	0.1904	0.1078	0.0393	0.0049

Table 14. Power of R Chart (assumed Cp = 1.50)

C_p	0.50	0.65	0.75	0.85	1.00	1.25	1.50
Prob.	0.6851	0.4745	0.3445	0.2355	0.1192	0.0289	0.0049

If the $\overline{X}$ *and* R charts are for use with short production runs, it may not make a great deal of practical sense to compare their effectiveness with 'pre-control' on the basis of average run length, as was done in Tables 5 and 6. This is the case when the total production time is less than the time taken to collect enough samples to match the ARL. As an alternative, we consider the probability of detection within 5 successive samples following a given mean shift. This probability is plotted against process mean shift in standard deviation units for 'pre-control' and $\overline{X}$ charts with both conventional and adjusted control limits in figures 2a to 2f where the $\overline{X}$ chart is constructed under the assumption that C_p is 1. As shown, there is no remarkable difference between 'pre-control' and $\overline{X}$ charts with either conventional or adjusted control limits if $C_p = 0.5$ *or* 0.75 . However, considerable differences exist between these techniques if the process is more than capable, especially for mean shifts ranging from 1σ *to* 2σ.

Figure 2a.

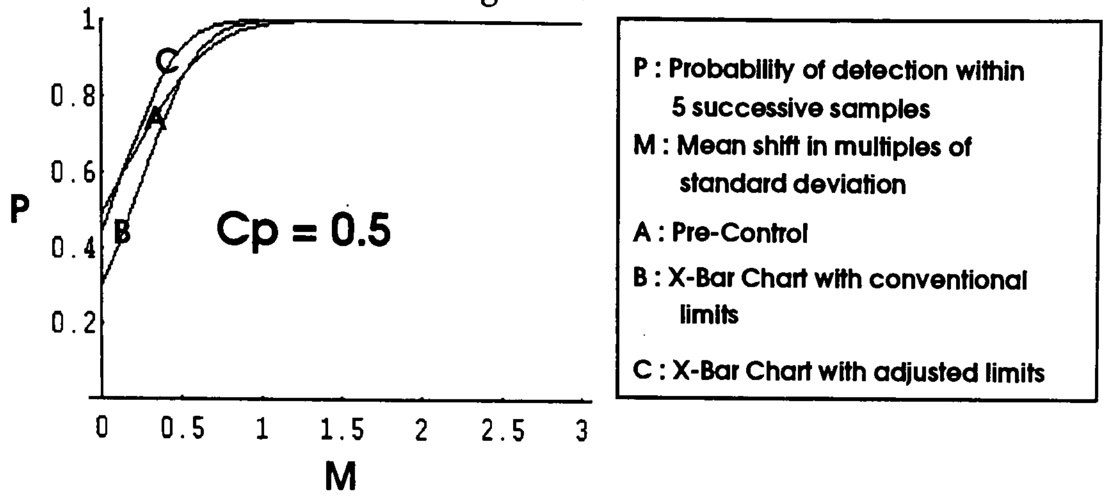


Figure 2b.

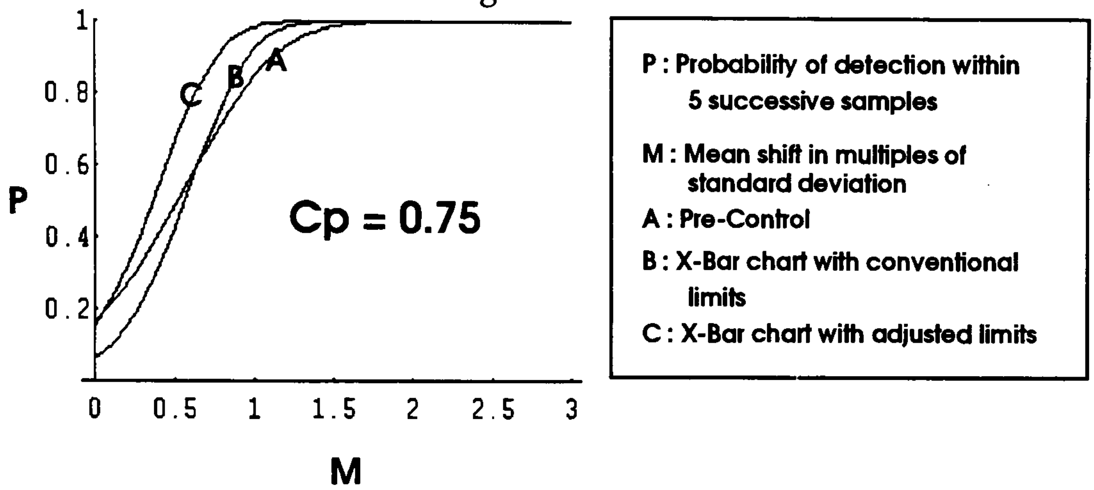


Figure 2c.

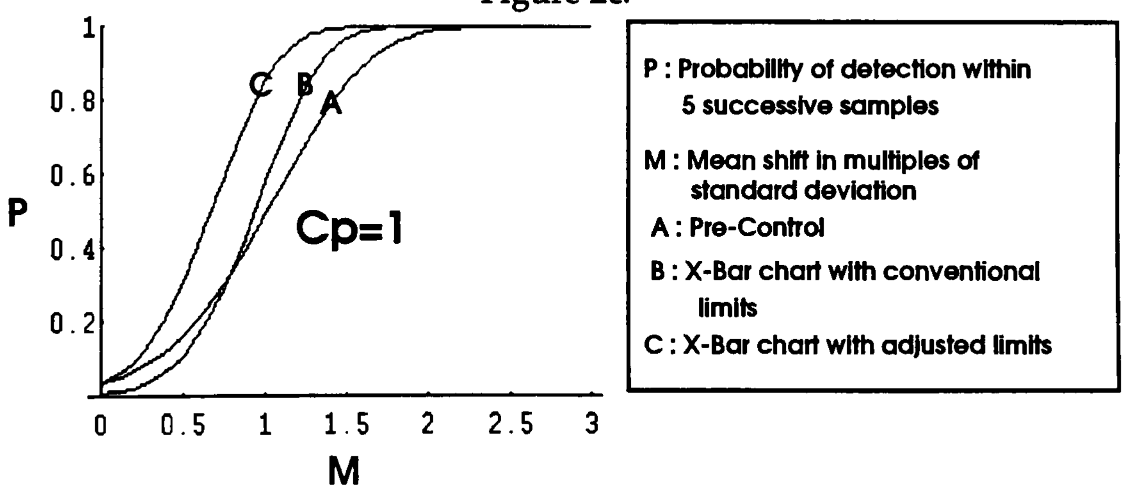


Figure 2d.

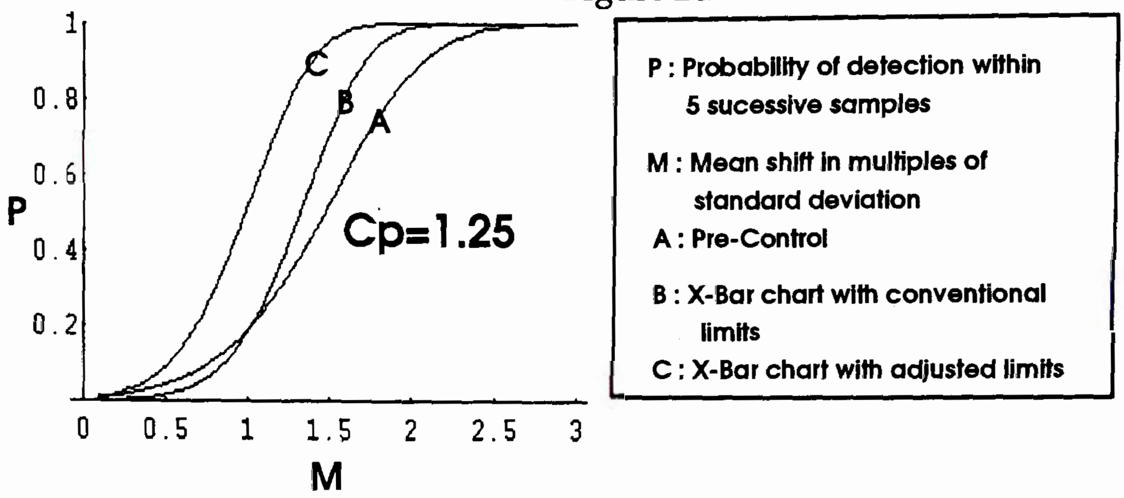


Figure 2e.

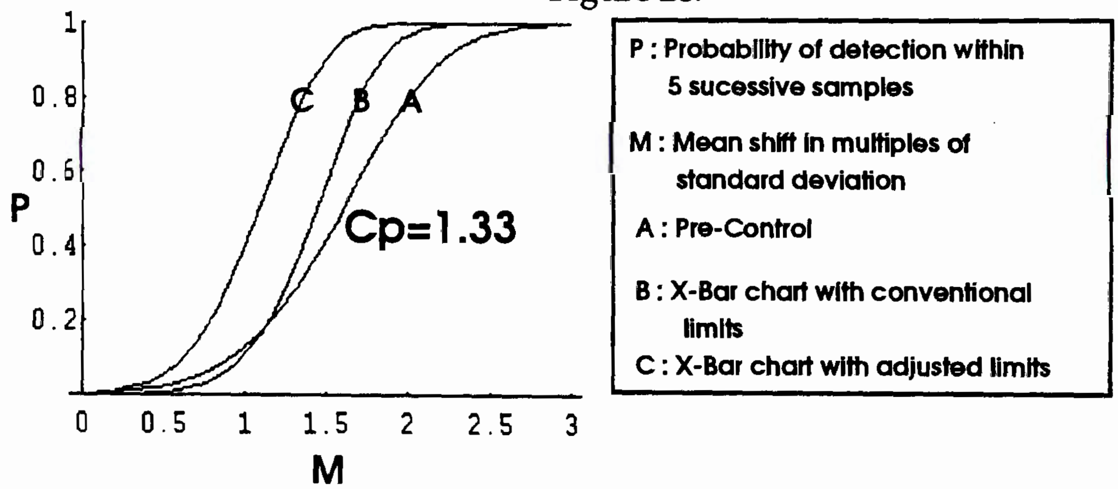
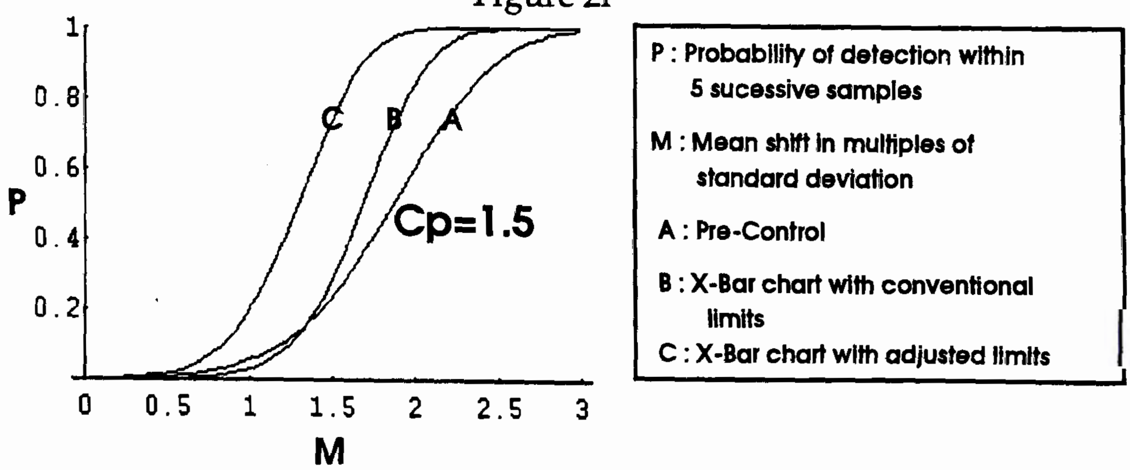


Figure 2f



EQUATING SAMPLING EFFORT

In the discussion so far, the total sampling effort hasn't been taken into consideration; the assumption being made that the time and cost of sampling, measurement or testing are not significant. This may, however, be unrealistic in certain circumstances. If it is, no useful comparison can be made unless the relative sampling frequency of 'pre-control' and $\overline{X}$ and R control is first set in such a way that both methods involve the same sampling effort.

To compensate for the smaller sample size of 2 for 'pre-control', we assume that process checks are made twice as often as $\overline{X}$ and R control with a sample size of 4. This being the case, we focus on the average number of items required to detect a change in the process mean or process dispersion, using the two methods.

In Tables 15 and 16, the average number of units sampled before 'picking up' various mean shifts under different process capability levels are given for 'pre-control' and $\overline{X}$ chart control (control lines fixed on the basis that $C_p = 1$). To facilitate the comparison, we have computed the following index :

$$I_{MS} = \left\{ \frac{ANII_{MS}(PC)}{ANII(\overline{X})} \quad if \quad k = 0 \right.$$

$$I_{MS} = \left\{ \frac{ANII(\overline{X})}{ANII_{MS}(PC)} \quad if \quad k \neq 0 \right.$$

where

$ANII_{MS}(PC)$ is the average number of items inspected before detecting the mean shift using 'pre-control' except when k=0, in which case it is the average number of items inspected prior to the occurrence of a false signal.

$ANII(\overline{X})$ is the average number of items inspected before detecting the mean shift using a conventional $\overline{X}$ chart (control based on $C_p = 1$) except when k=0, in which case it is the average number of items inspected prior to the occurrence of a false signal.

If $I_{MS} > 1$ when $k \neq 0$ then 'pre-control' performs better than an $\overline{X}$ chart in the sense that, on average, it 'picks up' the mean shift with fewer sampled items. Similarly, when k = 0 and $I_{MS} > 1$ then 'pre-control' takes longer, on average, before issuing a false signal. The values of this index for various combinations of mean shift (in multiples of σ) and actual process capability are tabulated in Table 17. As shown, whilst being superior in detecting mean shifts of all the given magnitudes, irrespective of the actual process capability, 'pre-control' is far worse than $\overline{X}$ control with regard to false alarms, a factor alluded to by Logothetis [5].

Table 15. $ANII_{MS}(PC)$

C_p \ k	0	±0.5	±1.0	±1.5	±2.0	±2.5	±3.0
0.50	8.04	6.30	3.44	2.46	2.12	2.02	2.00
0.75	28.53	15.95	6.34	3.47	2.48	2.13	2.03
1.00	147.01	56.60	15.83	6.32	3.47	2.48	2.13
1.25	917.36	243.97	48.53	14.19	5.91	3.36	2.45
1.33	1686.06	400.16	71.69	19.03	7.27	3.83	2.63
1.50	6407.56	1200.90	172.74	37.42	11.87	5.31	3.18

Table 16. ANII($\overline{X}$) (sample size 4, Control based on Cp = 1)

C_p \ k	0	±0.5	±1.0	±1.5	±2.0	±2.5	±3.0
0.50	30	13.0	5.79	4.29	4.03	4.00	4.00
0.75	164	37.9	9.97	5.17	4.17	4.01	4.00
1.00	1481	175.8	25.21	8.00	4.75	4.09	4.00
1.25	22611	1342.4	99.85	17.65	6.68	4.47	4.05
1.33	60458	2867.5	171.71	24.83	7.94	4.74	4.09
1.50	588674	17189.8	644.16	59.87	12.96	5.78	4.29

Table 17. I_{MS}

C_p \ k	0	±0.5	±1.0	±1.5	±2.0	±2.5	±3.0
0.50	0.2685	2.0580	1.6815	1.7414	1.8955	1.9763	1.9967
0.75	0.1744	2.3736	1.5717	1.4894	1.6818	1.8838	1.9744
1.00	0.0992	3.1066	1.5931	1.2665	1.3691	1.6490	1.8778
1.25	0.0406	5.5024	2.0575	1.2442	1.1300	1.3305	1.6511
1.33	0.0279	7.1660	2.3952	1.3048	1.0921	1.2382	1.5558
1.50	0.0109	14.3105	3.7290	1.6001	1.0921	1.0896	1.3467

A similar comparison between 'pre-control' and use of the R chart (control based on $C_p = 1$) with respect to detection of increase in process variance can also be made. Let

$$I_{VI} = \frac{ANII_{VI}(PC)}{ANII(R)} \quad if \quad C_p = 1$$
$$I_{VI} = \frac{ANII(R)}{ANII_{VI}(PC)} \quad if \quad C_p < 1$$

where

$ANII_{VI}(PC)$ is the average number of items inspected before detection of the increase in process spread using 'pre-control' except when $C_p = 1$, in which case it is the average number of items inspected prior to the occurrence of a false signal.

$ANII(R)$ is the average number of items inspected before detection of the increase in process spread using a conventional R chart (control based on $C_p = 1$) except when $C_p = 1$, in which case it is the average number of items inspected prior to the occurrence of a false signal.

The values of $ANII_{VI}(PC)$, $ANII(R)$ and I_{VI} are provided in Tables 18, 19 and 20 respectively. The R chart can be seen to be more sensitive to increases in the process spread except where marked by *.

Table 18. $ANII_{VI}(PC)$

C_p \ k	0	±0.5	±1.0	±1.5	±2.0	±2.5	±3.0
0.50	19.47	23.59	41.70	105.77	375.23	1805.10	11445
0.65	36.83	44.94	81.25	214.38	805.58	4180.40	29010
0.75	58.90	72.19	132.25	357.20	1389.97	7557.50	55637
0.85	97.73	120.23	222.93	615.09	2471.39	14040.90	109485
1.00	224.05	277.05	521.94	1481.50	6215.00	37483.00	315409

Table 19. ANII(R) (sample size 4, control based on Cp=1)

C_p	0.50	0.65	0.75	0.85	1.00
ANII(R)	11.61	29.68	65.39	162.94	811.69

Table 20. I_{VI}

C_p \ k	0	±0.5	±1.0	±1.5	±2.0	±2.5	±3.0
0.50	0.5964	0.4923	0.2785	0.1098	0.0309	0.0064	0.0010
0.65	0.8060	0.6605	0.3653	0.1385	0.0369	0.0071	0.0010
0.75	1.1102*	0.9059	0.4945	0.1831	0.0471	0.0087	0.0010
0.85	1.6673*	1.3552*	0.7309	0.2649	0.0659	0.0116	0.0010
1.00	0.2760	0.3413	0.6430	1.8252	7.6569	46.1790	388.5830

In many applications, the cost, effort or time expended to investigate a trouble-free process only to conclude subsequently that no change has occurred, is high. Under such circumstances, it seems appropriate to evaluate the relative effectiveness of competing control procedures having equated, for the two methods, the average number tested to produce a false signal. For this reason, the control limits of the $\overline{X}$ *and* R charts were adjusted so that both lead to the same average time elapsed or average number of items inspected prior to the occurrence of a false signal as 'pre-control', when $C_p = 1$. The resulting $ANII(\overline{X})$ *and* $ANII(R)$ values are shown in Tables 21 and 22 respectively. As illustrated in Tables 15 and 21, if the process capability is correctly assumed (i.e $C_p = 1$) or underestimated (i.e $C_p > 1$), the adjusted $\overline{X}$ chart requires a much smaller number of units, on average, to 'pick up' the given mean shift except when C_p=1 and k=2, in which case the difference is marginal. For C_p= 0.5 or C_p= 0.75, it is found that in most cases (marked with *), 'pre-control' is marginally better than the adjusted $\overline{X}$ chart. It can also be seen that false signals from the adjusted $\overline{X}$ chart tend to occur after a longer period of time when the process is incapable. However, if the process is more than capable, the adjusted $\overline{X}$ chart will tend to issue a false signal sooner than 'pre-control' although, due to the large magnitudes, this is of little practical consequence. It should be noted that we have deliberately omitted those cases where k=2.5 and k=3.0 in Table 21 because mean shifts as large as these are likely to be detected early anyway irrespective of method. The R chart is found to be far superior to 'pre-control' in reacting to worsening process capability (refer to Tables 18 and 22). This is especially true when the process is not on target.

Table 21. ANII($\bar{X}$) (sample size 4, adjusted limits, control based on Cp=1)

C_p \ k	0	±0.5	±1.0	±1.5	±2.0
0.50	14.84	8.72*	4.91*	4.12*	4.01*
0.75	40.96	15.64	6.30	4.39*	4.04*
1.00	147.01	35.27	9.58	5.09	4.15*
1.25	693.33†	102.17	17.90	6.73	4.48
1.33	1208.12†	151.78	22.95	7.62	4.67
1.50	4329.05†	385.80	42.27	10.60	5.30

* these are slightly larger than the corresponding figures for 'pre-control' in Table 15.
† these are smaller than the corresponding figures for 'pre-control' in Table 15.

Table 22. ANII(R) (sample size 4, adjusted limits, control based on Cp=1)

C_p	0.50	0.65	0.75	0.85	1.00
ANII(Adjusted R)	8.72	17.69	32.28	65.13	224.05

CONCLUSIONS

On practical considerations and from the perspective of monitoring and control, proponents of 'pre-control' state the method to be superior to $\bar{X}$ *and* R charts. Its simplicity and versatility make it a useful tool for a large variety of applications. Nevertheless, as shown in the previous sections, $\bar{X}$ *and* R control charting still have merits when basing a comparison on statistical grounds.

It is clear, that if sampling effort is of little importance, C_p is known and provides a value of 1, 1.25 or 1.50, then the $\bar{X}$ chart is superior in picking up mean shifts greater than 1σ . When σ is not known, as is frequently the case in short production runs, and therefore its value has to be estimated or assumed for use of Shewhart charts, in many instances $\bar{X}$ control is seen to still be superior. Specifically, if the C_p is assumed to be 1.25 but is actually between 0.5 and 1.5 then $\bar{X}$ control is superior to 'pre-control' in 'picking up' shifts in the process mean. If C_p is assumed to be 1, yet actually has a value somewhere between 0.5 and 1.50, then an $\bar{X}$ chart is as good as or significantly better for detecting mean shifts than 'pre-control'. For detection of a deterioration in the process capability we have observed the standard R chart to be more sensitive than 'pre-control'. It would seem, therefore, that the advocacy of Maxwell [3] and others is sound, that in the absence of knowledge of σ we can use, for construction of standard $\bar{X}$ *and* R charts, an assumed C_p value of 1. Certainly this is so in regard to providing a more sensitive instrument for process control than 'pre-control.'

When sampling effort is important a comparison that fairly compares the two techniques when the sampling effort is the same, reveals that for capable processes, $\bar{X}$ *and* R charts are superior to 'pre-control'.

The material presented in this paper has taken a perspective that focuses narrowly on monitoring and control. Broadening the perspective and perceiving charting techniques as merely a part in the effort of continuous improvement underscores further the value of traditional Shewhart charts.

We have sought to create some common ground for $\bar{X}$ *and* R control and 'pre-control' in order to examine their perfomance for monitoring and control on a statistical basis. Hopefully, the material contained herein will provide more complete grounds on which to base a comparison between the two techniques and thus facilitate more rational judgements on which of the two to use in any given situation.

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