23rd Australasian Conference on the Mechanics of Structures and Materials

2014

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Publication details

Parker, A & Tran, D 2014, 'Application of independent component analysis in vibration', in ST Smith (ed.), *23rd Australasian Conference on the Mechanics of Structures and Materials (ACMSM23)*, vol. II, Byron Bay, NSW, 9-12 December, Southern Cross University, Lismore, NSW, pp. 1115-1120. ISBN: 9780994152008.

APPLICATION OF INDEPENDENT COMPONENT ANALYSIS IN VIBRATION

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ABSTRACT

This paper investigates the application of Independent Component Analysis (ICA) in vibration of structures to infer excitation forces from vibration response signals only. First, fundamental notions of ICA, statistical independence (SI) of variables and an ICA algorithm for real number, *fastICA*, were reviewed. The performance of ICA was assessed using harmonic force excitation as source signals and mixture signals are displacement responses from finite element simulation of vibration of a cantilever beam. It was found that *fastICA* could extract source signals that share most dynamic characteristics of the excitation source signals. Further studies are proposed to tailor ICA better to vibration signals.

KEYWORDS

Independent component analysis, statistical independence, fastICA, vibration.

INTRODUCTION

Independent Component Analysis (ICA) is fundamentally a blind source separation method that seeks to separate underlying components from available data whether the data are in the form of sounds, images, vibration responses or financial share prices. Since 1990s, ICA has been of great interest to researchers in diversified areas of statistics, medical imaging, and structural damage detection (Zang et al 2004). ICA relies on response data collected by sensors, called *mixture signals*, and the assumption that the independent component sources, called *source signals*, are statistically independent, to extract the unknown source signals. Most of the studies require that there are as many sensors as there are independent components and that the system behaves linearly, but non-linear behaviour and both under and over-determined cases have also been solved. ICA assumes that there is a relationship between **S**, the vector representing source signals, or underlying components and **X**, the vector representing mixture or response signals of the system to the source signals. In the simplest form the relationship is linear and can be expressed as: **X** = **AS, where X** is available from sensors output, while both **A** and **S** have to be determined, **A** is the mixing matrix. ICA seeks the optimum solution out of all possible matrices **A** such that the statistical independence of **S** is maximized. It is basically an optimization problem based on a bold assumption of statistical independence of source signals.

Statistical Independence

From the point of view of statistics, two scalar variables of exchangeable data **X** and **Y** are statistically independent if and only if their joint probability density function (jpdf), p(x,y), is a product of their individual probability density function (pdf), p(x) and p(y):

$$p(x,y) = p(x) \cdot p(y) \tag{1}$$

Eq. 1 is universally accepted but in practice, when only values of variables **X** and **Y** are given, only p(x) and p(y) can be readily computed but p(x,y) cannot be, unless SI had been ensured or assumed.

Most authors (Hyvärinen et al 2001, Stone 2004) used simple measures of kurtosis and negentropy as simple measures of SI, based on the fact that the mixtures, as a consequence of Central Limit Theorem, would be more Gaussian than the sources. An equivalent assumption, albeit heuristic, is that the sources would be more non-Gaussian, leading to accepting the objective function of optimization as seeking sources of maximum non-Gaussianity, which can be measured by kurtosis and negentropy.

Kurtosis is defined as $\mathrm{kurt}(x) = E\{x^4\} - 3(E\{x^2\})$, where the symbol E stands for expected value. Thus kurt (x) is a normalized version of fourth moment of (statistical) distribution of **X**. Although simple to calculate, kurtosis is sensitive to outliers. Negentropy borrows the concept of entropy from Thermodynamics which represents the degree of being unstructured, unorganized, unpredictability, is also popular in Theory of Information. For a distribution Y, entropy of a variable is defined in terms of pdf as $H(y) = -\int p(y) \log p(y) dy$. Negentropy J is then defined as $J(y) = H(y_{Gauss}) - H(y)$, where y_{Gauss} is a Gaussian random variable of the same covariance matrix as y, which is shown by Information Theory of having the largest entropy among all random variables of equal variance. Thus negentropy is always non- negative. It is more involved to compute negentropy than kurtosis, and like kurtosis, it refers to only one variable, whereas the concept of SI is inherently concerned with two or more variables. In this aspect, the concept of mutual information in Information Theory defined as:

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$$I(X,Y) = \sum_{y \in Y} \sum_{x \in X} p(x,y) \log\left(\frac{p(x,y)}{p(x)\,p(y)}\right) \tag{2}$$

is more directly related to SI, but it still requires the availability of jpdf. Mutual information can however be approximately computed (Comon and Jutten 2010). On the other hand, the equality of the two sides of Eq.1 should be understood in a statistical sense and this highlights the basic concept of statistical hypothesis testing to ensure not to commit Type I and Type II error: a test must have hypotheses, the null and alternative hypothesis, a corresponding statistic and a measure of the reliability of the test.

In Eq.1, cumulative distribution functions can replace the respective pdf, as so do expected values of absolutely integrable functions of variables, including positive powers of x and y:

$$E\{g(x).h(y)\} = E\{g(x)\}.\ E\{h(y)\} \tag{3}$$

$$E\{x^p\ y^q\} = E\{x^p\}\ .\ E\{y^q)\} \tag{4}$$

It follows from Eq. 4 that SI is more stringent requirement than un-correlatedness, as un-correlatedness requires only $E\{x.y\} = E\{x\}.\ E\{y\}$, i.e only for the case that both p and q equal 1. Thus statistical independence implies un-correlatedness but the reverse is not true. At the same time, variables describing physically independent phenomena are intuitively thought to be statistically independent but it is not generally true (Stone 2004).

Although the first research paper on SI appeared in 1935 (Wilks 1935), a simple and rigorous testing procedure from the point of view of statistics was only available from 2006 by Bakirov and his associates (Bakirov et al 2006, Szekely et al 2007). Bakirov proposed the null and alternative hypotheses as:

H_0: $p(x,y) = p(x)\ p(y)$, H_1: $p(x,y) \neq p(x)\ p(y)$

Bakirov proposed a coefficient of statistical independence *I = I(X,Y) with* $0 \leq I \leq 1$ *and I = 0 if and only if X and Y are independent*. It is a non-parametric test, independent of distribution model of test statistic. Szekely et al (2007) proposed the concept of distance covariance, *dCov* (X,Y) and distance correlation, *dCor* *R*(X,Y), where:

$$\nu^2(X, Y) = \|p(x,y) - p(x)p(y)\|^2 \tag{5}$$

$$R^2(X,Y) = \frac{\nu^2(X,Y)}{\sqrt{\nu^2(X)\nu^2(Y)}}, \tag{6}$$

It can be seen from Eq. 5 that *dcov* is directly related to the definition of statistical independence. The authors proposed two tests called *dcov.test* and *mvI.test*. Both tests are implemented in the module *indep.test* of the package *"energy'"* developed by Rizzo and Szekely in R language which can be downloaded from http://cran.r-project.org/web/packages/energy/index.html (Unnisa et al 2014). The *mvI.test* takes longer (as many times as the number of elements) computing time than the dcov.test. They would yield the *p-value* of the test and it is widely accepted that H_0 should be rejected if p-value < 0.05. It should be noted that in statistical hypothesis test, p-value is viewed as a measure of reliability of the hypothesis test.

For time series, especially when temporal order does matter and data are not exchangeable, Zhou used similar approach as Bakirov group and has developed a criterion for statistical independence of time series (Zhou 2011) but a practical test is not yet available.

Implementation of ICA

Hyvärinen and associates have contributed greatly to the implementation of ICA (Hyvärinen 2013) and offered an algorithm for real number signals called *fastICA* available in R, Matlab, C++ and Python programming. *fastICA* was developd by Marchini, Heaton and Ripley and can be downloaded from http://research.ics.aalto.fi/ica/fastica/. Basically it employs an approximation of negentropy as the objective function in the optimization searching of the unmixing matrix **W** where **S** = **WX** under the constraints that **W** is an orthonormal matrix after the data has been centered, normalized and whitened. As the name implies it is a very fast algorithm, using fixed point iteration scheme for maximizing negentropy. It should be noted that the output of ICA (source signals, matrices **A** and **W**) are ambiguous as far as sign, scale and order are concerned.

This paper investigates the application of ICA, using simulated vibration response by finite elelment method (FEM) as signal mixtures to yield **S** which are then compared to force excitation signals.

FINITE ELEMENT SIMULATION OF VIBRATION

A simple structure in the form of a steel cantilever beam of 1 m long, 25 mm wide and 12 mm deep, of density 8324 kg/m3, Young's modulus of 210 GPa was studied. It was modeled by Timoshenko beam element with twenty elements of equal length as shown in Figure 1. First, a modal analysis was carried out to obtain natural frequencies and corresponding mode shapes. The natural frequencies within 1000 Hz were found to be 9.7, 61.0, 170.8, 334.5, 552.8 and 825.4 Hz. Force signals were input at two nodes in very short time steps of 0.5-1 milliseconds for a time duration between 0.5-5 seconds.

Newmark transient analysis by ANSYS software was carried out to obtain nodal displacement responses using integration time step of 0.5 to 3 microseconds. A three-dimensional FEM model of finer meshing was also studied by explicit solver of LsDyna software. The results from the two models were similar. The response signals were obtained in time steps of 0.5 or 1 millisecond and input into *fastICA* to find source signals.

A tacit assumption of ICA is the source signals should be statistically independent. Only force signals that satisfy the SI *indep.test* test developed by Bakirov and associates were used. Details can be found in (Unnisa et al 2013). Harmonic, saw-tooth, impact and random signals in various combinations were studied but only the case of two harmonic force excitations of different frequencies is reported here.

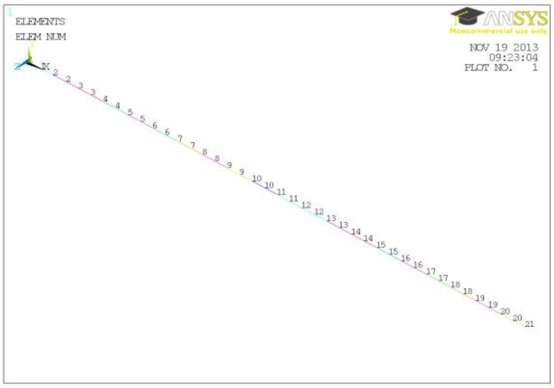

Figure 1. FEM model showing node numbers

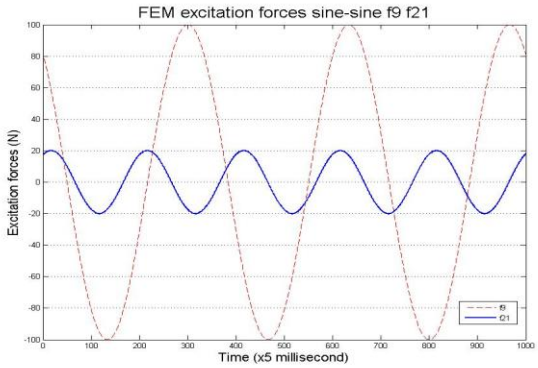

Figure 2. Excitation force **F**

A sinusoidal excitation force, termed F1, of 100 N amplitude, frequency of 0.6 Hz was acting at node 9, another sinusoidal excitation force, termed F2, of 20 N amplitude, frequency of 1 Hz, lagging by 66 degrees was acting at node 21 as shown in Figure 2. Let **F** be the original excitation force vector formed from F1 and F2. The displacement response mixture signals at node 10 and 21, termed v10 and v21 were given by transient dynamic solution of ANSYS software and shown in Fig. 3.

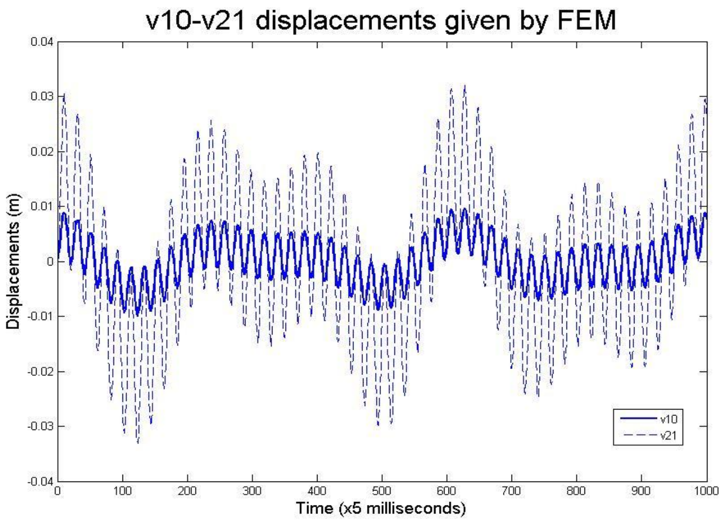

Figure 3. Displacement mixture signals v10 and v21 given by FEM solution.

It can be seen that the FEM solution for v10 and v21, as a linear combination of F1 and F2, are expected to be also sinusoidal, but they are affected by beat phenomenon caused by their close excitation frequencies of 0.6 and 1 Hz.

SOURCE SIGNAL EXTRACTION BY *fastICA*

The mixture signals v10 and v21 were then input into the *fastICA* algorithm to extract the source signals which are shown separately in Figure 4, and together in Figure 5. Note that source signal vector **S** extracted by *fastICA* are inherently arbitrary in scale, sign and order. Taking into account beat phenomenon by FEM simulation, it can be seen that *fastICA* was successful in extracting **S**.

A Proposal to Compare S with Original Excitation Forces F

The question is how would one compare **S** with the original excitation signal **F** when source signals extracted by ICA are arbitrary in scale, sign and order. On first thought using probability distribution function (pdf) may be a convenient way as pdf is normalised, however as it does not reflect the fact that the temporal order of a force excitation signal does influence the resulting displacement response.

This property of vibration signals is termed convolutionary. In other words, a signal obtained by permuting (or shuffling) the values of a force variable would have the same pdf but their resulting displacement response would be different. In statistics term, pdf is more suitable for exchangeable data than for time series. A better approach for random variables is to compare their power distribution:

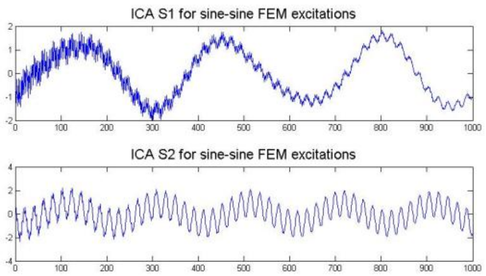

Figure 4. S1 and S2 separately shown

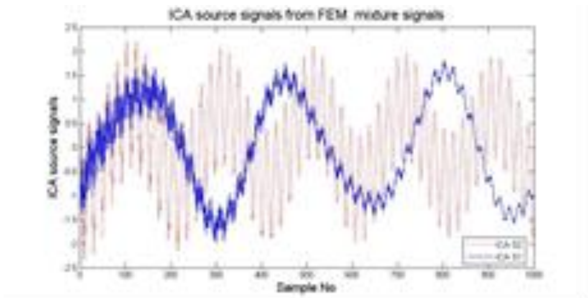

Figure 5. S1 and S2 shown together

Power Spectrum Density

First the cross-spectral density function (CSDF), G_{xy}, between two variables **X** and **Y** is defined as : $S_{xy(f)} = \int_{-\infty}^{\infty} R_{xy}(\tau) e^{-j2\pi f\tau} d\tau$, where R_{xy} is the cross-correlation function between x and y. A special case when x = y leads to auto spectral density function (ASDF) G_{xx} or G_{yy} which reflects the distribution of power in frequency domain. ASDF is also known as power spectrum density function (psdf). The CSDF should be tested for combination between **S** and **F,** four such combinations in this study.

Coherence or coherency squared function defined as: $\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_{xx}(f)G_{yy}(f)}$ also gives an idea on how related the two variables x and y are. These entities are evaluated for the case of excitation forces F1 and F2, and their (permuted) counterparts output by *fastICA*, S1 and S2. A typical plot of ASDF (in semi-log scale) is shown in Figure 6.

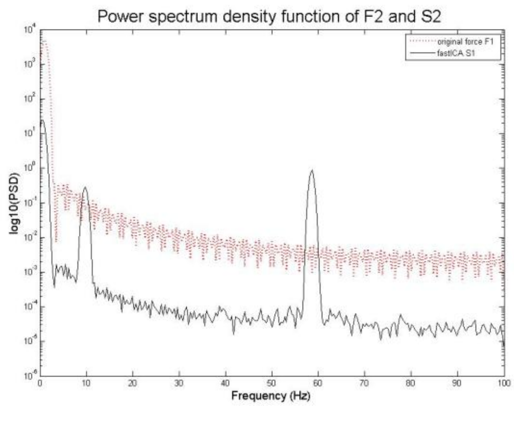

Figure 6. ASDF of F2, S2

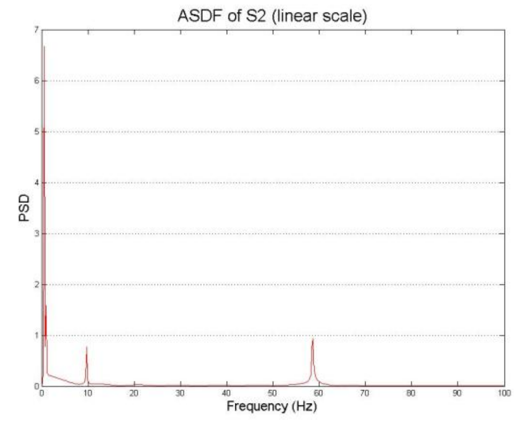

Figure 7. ASDF of S2 in linear scale

It should be noted that in the FEM simulation, the sampling frequency of response signals used was 200, hence the Nyquist frequency of 100 Hz used in Figure 6. Within this range there are only two natural frequencies of 9.7 and 61.0 Hz found. As expected, most of the power is concentrated around 1 Hz for F2. For S2, while a large part is concentrated around 1 Hz, some small portion is found near the above natural frequencies of the beam. The relative small portion is better shown in linear scale as in Figure 7. This is to be expected as response signals reflect not only frequency content of excitation forces but also the dynamic characteristics of the structure. As these are used as input for *fastICA*, the extracted source signals would reflect the influence of the structures. By evaluating cross-correlation functions, it can be shown that while F1 and F2 are completely uncorrelated, S1 and S2 are coarsely related around those peaks of natural frequencies. Further, on computing kurtosis of these signals, it was found that values for F1 and F2 (1.4985 and 1.5000 respectively) are smaller than those of S1 and S2 (1.6904 and 2.2271 respectively). This shows that *fastICA* algorithm gives source signals that are more non-Gaussian than the original forces even though they are less statistically independent. This suggests that *fastICA* based on non-Gaussianity objective function does not ensure more degree of SI, which is the originating platform of ICA.

CONCLUSION

This study shows that *fastICA* using real signals can be used in vibration studies to obtain characteristics of excitation force source signals when only response mixture signals are available. There are still a number of issues that require further studies before ICA can be fully employed to vibration studies:

1. Developing a suitable criterion of SI for convolutionary time signals of vibration, such as proposed by (Zhou 2011), and a rigorous test from statistics point of view, similar to *indep.test* by Rizzo and Szekely.
2. Developing an ICA algorithm that can cater for complex values in frequency domain by combining proposals of (Junhongand Zhuobin 2011) and (Hyvärinen 2013), preferably with no normalization involved so that the matrix **A** and **W** are uniquely determined. As these matrices reflect the dynamic characteristics of the system, they would be of great interest in system identification and structural damage detection.

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