Boris Novozhilov: Life and contribution to the physics of combustion

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Boris Novozhilov: Life and Contribution to the Physics of Combustion

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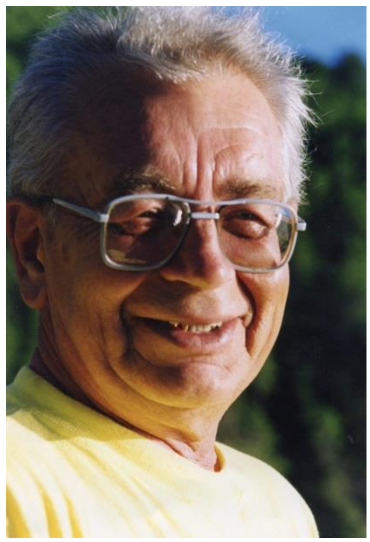

Abstract

Professor Boris Novozhilov (1930-2017) passed away on February 19th, 2017 in Moscow. The present paper provides brief account of his life and contributions to the physics of combustion. From extensive scientific legacy left by Boris, several major achievements are discussed here: Zeldovich-Novozhilov (ZN) theory of unsteady solid propellant combustion, contributions to thermal explosion theory, the theory of spin combustion, discovery of propellant combustion transition to chaotic regimes through Feigenbaum period bifurcation scenario.
Keywords: Propellants, ZN Theory, Thermal Explosion, Spin Combustion

Professor Boris Novozhilov (1930-2017) passed away on February 19th, 2017 in Moscow. He is mostly known for his outstanding fundamental contribution to the theory of solid propellant combustion.

He is survived by his wife Ludmila, his daughter Natalia and son Vasily.

Born on the 8th of July 1930 in Alma-Ata, Kazakhstan (that time part of the Soviet Union, and now independent Republic of Kazakhstan), Boris soon had to move with his family to Tomsk, and then to Novosibirsk, where Boris lived during the outbreak of WWII.

His interest in physics led him to enter the course at the Leningrad Polytechnic Institute (currently Peter the Great St. Petersburg Polytechnic University), which was at the time (1948) very highly ranked in physics education in the Soviet Union. Boris graduated with honors in Applied Physics in 1953.

From 1954 Boris started his scientific career in Moscow at the Institute of Chemical Physics (currently The Semenov Institute of Chemical Physics), USSR and later Russian Academy of Sciences. He worked there full time until his death. He joined the theoretical group of famous theoretician A.S. Kompaneyets and was assigned first to work on some projects related to nuclear energy. Boris' PhD thesis on the subject written nearly 6o years ago (1959) is still classified.

Communication with academician Nikolai Semenov, founding director of the Institute and Nobel Prize Laureate in Chemistry, was extremely important for Boris. Apart from Semenov, Boris benefited much from his interaction with Kirill Shchelkin, a famous Russian scientist who made outstanding contributions to the studies of detonation.

Following his PhD, Boris obtained the highest scientific degree in the Soviet Union, Doctor of Sciences (in physical and mathematical sciences) in 1968.

Fig. 1. Boris Novozhilov in the 60s

From 1976 to 1992 he occupied the post of the Head of the Laboratory of Mathematical Methods in Chemical Physics at the Institute. Many, if not most of his staff came from a formal mathematical background, and with some Boris used to have occasional heated arguments when he felt that excessive mathematical formalism overshadowed clear physical reasoning.

Soviet school of theoretical physics influenced greatly Boris' style of approaching problems (for example, he used to attend for some time the famous L.D. Landau theoretical seminar) and this fact helps to understand why he was so successful in developing fundamental theoretical concepts.

Physical insight into the problem has always remained a defining drive for Boris. He loved entire physics as a science, and despite working in a relatively narrow field, his understanding of and intuition in many areas of physics were amazing.

Although working initially on some projects related to the Soviet military nuclear program, Boris got quickly interested in the combustion of solid propellants. His major achievement was the development of what is now known as Zeldovich-Novozhilov (ZN) theory of unsteady propellant combustion. ZN theory was essentially developed by Boris in the 60s, although he has been refining it up to the end of his life.

Like most Soviet scientists, Boris was severely restricted in his ability to communicate with his colleagues in the West. In the late 60s he developed strong collaboration with Numa Manson's group at ENSMA (École Nationale Supérieure De Mécanique et D'Aérotechnique) and the University of Poitiers. However his invitation to work at Poitiers for three years was blocked by the Soviet authorities. Another tragic episode occurred in 1978, when his permission to leave the Soviet Union for International Symposium on Combustion in Leeds was revoked just days before the event. Boris was unable to travel abroad, apart from one minor conference in Warsaw in the seventies, until 1989.

Fig. 2. Photo presumably taken while defending Dr. Sci. Thesis in 1968

Luckily, the situation changed later in his life, and he was able to work intensively with his colleagues in Italy (being invited there many times from 1989 to 2004). He gave a number of lectures at Politecnico di Milano and National Council of Research, actively participated in workshops and conferences, and co-authored several papers. Boris also have had very close relationships with the researchers at the Institute of Space and Astronautical Science, Japan Aerospace Exploration Agency, over a period of about 10 years (from 1994 to 2004). Boris also visited and lectured in the USA and Australia for short periods in the mid-90s.

Over the years Boris theory acquired world-wide recognition (largely due to interaction with several key scientists in the West, such as Martin Summerfield, Edward Price, Forman Williams, Fred Culick, and others, who visited Soviet Union occasionally while Boris was unable to travel abroad). Eventually that recognition led The Combustion Institute to award Boris the Zeldovich Gold Medal for outstanding contribution to the theory of combustion in 1996.

Apart from ZN theory, Boris made versatile contributions to various areas of combustion science, some of which are discussed in more detail below. He also made a number of practical inventions and contributed to development of a number of defence-related technologies. For one of such important contributions he was awarded the Russian Federation Government Prize in Science and Technology in 2012.

Boris is the author of 5 books published in the Soviet Union. For many years he taught part-time as a Professor at the Moscow Institute of Physics and Technology, a leading Soviet and Russian University in physics education.

Due to limitations of space, we provide very little exact mathematical details as it would be impossible to fit into the manuscript proper discussion of mathematical formulations and solutions of many technical problems. We have tried to provide general description of the ideas involved, as well as some illustrations of major results.

Boris theory of unsteady solid propellant combustion was built on the initial ideas presented by Y. Zeldovich in the two papers published in 1942 and 1964 [1,2], although personal interaction between the two was always rather limited. Zeldovich assumed the surface temperature of the burning propellant to remain constant, and provided, under such an assumption, his results on calculating the nonsteady burning rate of the propellant, as well as on the stability of combustion. It was rather obvious from the beginning that the Zeldovich assumption of constant surface temperature is too restrictive. It was found that predictions of such theory contradicted with the experiments as no real systems really fulfill the constant temperature condition.

Extension of the theory to the case of variable surface temperature proved to be difficult, despite efforts of many scientists working in the field. Boris succeeded in extending theory of nonsteady combustion to the real case of variable surface temperature by demonstrating that in this case both the surface temperature and burning rate are determined by the instantaneous values of the pressure and the temperature gradient at the surface of the solid phase. This idea essentially extends the earlier proposition by Zeldovich to feed information from steady-state propellant combustion data into the calculation of unsteady burning regimes. Thus, in the general case of varying surface temperature, like in the original Zeldovich theory, there is no need for complicated theoretical models predicting all the features of propellant combustion phenomena. The necessary dependencies, "the steady-state burning laws" valid in fact for any nonsteady regimes, can be obtained experimentally at steady-state conditions, and would contain all the relevant "aggregated" information which is required to predict unsteady combustion process. The great advantage of this approach, emphasized by Boris in his earliest fundamental works on the problem, is that all the complex physico-chemical phenomena of real propellants combustion are automatically taken into account by employing experimental dependencies related to specific propellant in question. Consequently, any results obtainable for a particular propellant using detailed combustion model, may be derived from the general theory developed by Boris. To achieve this task, one just needs to write down the burning laws for that particular combustion model.

Another great advantage of the theory is that it is readily extendible to other types of propellant combustion problems, for example to erosive combustion of the propellant in the tangential gas stream. Here the gas velocity takes the place of pressure as a known external parameter in the equations, and no other changes are required. Thus, a variety of practical propellant combustion problems may be considered from a unified and consistent viewpoint.

Fundamentals of the theory, now known as Zeldovich-Novozhilov (ZN) theory, were presented by Boris in [3-5].

The major object under consideration in the ZN theory is a ballistic propellant whose combustion essentially follows the mechanism discovered experimentally by Belyaev [6], that is it heats up to thermal degradation (essentially boiling) temperature, vaporises, and the actual combustion proceeds then in the gas phase.

Experimental data suggests that during propellant's combustion surface interface remains plane (for a sufficiently large sample diameter). Based on this fact, the problem is considered as one-dimensional. Fundamental approximation of the ZN theory, the so-called t_c approximation, is that only thermal inertia of the condensed phase needs be retained (hence the subscript "c" referring to "condensed"). Relaxation times of the other two relevant zones (the one in which the condensed phase is transformed into intermediate gaseous products, and the one where the latter products are transformed into the final combustion products) are negligible. t_c approximation can be fully justified upon accurate estimation of the orders of magnitude of relaxation times of different zones.

Under the t_c approximation, mass and species balances at the interface may be shown [7] to have exactly the same form for either steady-state or unsteady combustion process. In both cases, any of the relevant variables can be expressed as a function of the just two parameters: temperature gradient (at the interface) in the condensed phase, and pressure. In particular, this is true for the propellant mass burning rate and its surface temperature:

$$m = m\left(f, p \right) \quad T_s = T_s\left(f, p \right) \tag{1}$$

where m is propellant mass burning rate, p pressure, T_s surface temperature of the propellant, f temperature gradient in the condensed phase, taken at the interface.

Pressure variation with time is considered as a given function.

The relations (1) play fundamental role in the theory. They can be obtained from steady-state propellant combustion experimental data and fed into the unsteady burning process calculations. In other words, during its evolution under unsteady combustion the system proceeds through the states such that the instantaneous values of the variables m, p, T_s, f at the interface would satisfy restrictions (1) taken for the steady-state combustion regime.

To furnish mathematical formulation, one further step is taken making note that steady-state burning regimes follow the Michelson distribution profiles (the solution of the heat transfer equation in a semi-infinite domain with the boundary possessing prescribed boundary temperature and moving into the domain with prescribed speed). Under steady-state conditions (denoted by the "0" subscript), combustion process is fully determined by pressure and initial temperature T_a of the propellant

$$m_0 = U(T_a, p) \quad T_{s0} = V(T_a, p) \tag{2}$$

Michelson solution allows initial propellant temperature to be eliminated from (2) and write steady-state burning laws in the form

$$\begin{aligned} m_0 &= U(T_{s0} - \chi f_0 / u_{c0}, p) \\ T_{s0} &= V(T_{s0} - \chi f_0 / u_{c0}, p) \end{aligned} \tag{3}$$

where u_c is propellant linear regression rate, χ thermal diffusivity. By virtue of the above argument, the same will be true for any unsteady combustion process (removing the subscript "0")

$$\begin{aligned} m &= U(T_s - f\chi / u_c, p) \\ T_s &= V(T_s - f\chi / u_c, p) \end{aligned} \tag{4}$$

In this form unsteady burning laws can be found from steady-state experimental data. Clearly, the laws (4) essentially have the same form as (1).

From this point it is obvious that, upon suitable scaling e.g. [7], the ZN theory equations can be written in the following non-dimensional form

$$\begin{gathered} \frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial\xi^2} - R\frac{\partial\theta}{\partial\xi} \quad ; \quad -\infty < \xi \le 0 \\ \theta(\xi,0) = \theta_i(\xi) \;;\; \theta(-\infty,\tau) = \theta_a \;;\; \theta(0,\tau) = \theta_s \\ R = R(\varphi, P); \quad \theta_s = \theta_s(\varphi, P) \end{gathered} \tag{5}$$

Appearing in (5) are non-dimensional counterparts of respective dimensional variables. We refer reader to the review [7] for more details.

One needs to find from (5) the linear burning rate $R(\tau)$ given known pressure variation with time and the burning laws (last line, equation (5), non-dimensional equivalent of (1)).

In most cases, temperature distribution in the body of the propellant is a by-product of the solution, and not actually needed. Based on this consideration, Boris also derived an alternative formulation of the ZN theory which often becomes useful. This formulation (which in full detail can be found in [7]) replaces the differential heat transfer equation in (5) with the following integro-differential equation written at the propellant surface

$$\begin{aligned} \sqrt{\pi}\theta_s(\tau) = \int_0^\tau &\left[\varphi(y) - R(y)\theta_s(y) + \theta_s(y)\mathrm{I}(\tau,y)/2(\tau-y)\right] \\ &\times \exp\left[-\mathrm{I}^2(\tau,y)/4(\tau-y)\right]dy/(\tau-y)^{1/2} \\ &\times \tau^{-1/2}\int_{-\infty}^{\infty}\theta_i(y)\exp\left[-\left(\mathrm{I}(\tau,0)+y\right)^2/4\tau\right]dy \end{aligned} \tag{6}$$

while retaining of course burning laws restrictions. Such formulation contains only mostly needed linear regression rate as an unknown to be solved for.

As has been mentioned earlier, pressure can also be replaced by other relevant combustion controlling parameters, e.g. velocity of the tangential gas flow.

Let us consider several applications of the theory considered by Boris. The basic natural problem in propellant combustion is a combustion stability under constant pressure. Zeldovich theory assuming constant surface temperature led to combustion stability criteria which was found to contradict with experimental observations.

Boris successfully considered, in the linear approximation, a problem of stability of propellant combustion under constant pressure, and derived the two fundamental parameters describing the boundary of the stability region. These are

$$k = (T_{s,0} - T_a)\left(\frac{\partial \ln m_0}{\partial T_a}\right)_p \;; \quad r = \left(\frac{\partial T_{s,0}}{\partial T_a}\right)_p \tag{7}$$

The first of these parameters was introduced to the theory already by Zeldovich [1,2]. The stability boundary is presented in Figure 3.

Dashed region in Fig 3 corresponds to instability, in particular above the dashed curve in that region small perturbations grow in oscillatory manner; below dashed curve they grow exponentially without oscillations. The curve "s" which is described by the condition

$$r \ge \frac{(k-1)^2}{(k+1)} \tag{8}$$

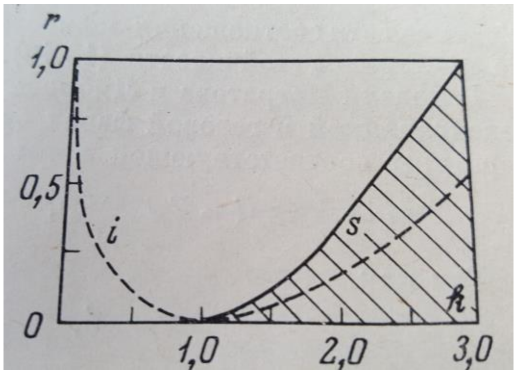

Fig. 3. Stability diagram for combustion under constant pressure

separates stable and unstable regions. Region to the left from the curve "s" corresponds to stable combustion conditions.

As follows from Fig. 3, in contrast to the earlier result by Zeldovich (stability condition $k<1$) in the proper model with variable surface temperature stable combustion is possible under restriction (8) for $k>1$ as well.

If any specific type of propellant combustion model is presented, its stability analysis would lead to the same stability criterion as above where parameters k, r must be calculated for that particular model. In this way, for example, the results of the general analysis are perfectly unified with the earlier flame model analysis by Denison and Baum (1961) [8]. Therefore, the stability analysis performed on the basis of the ZN theory is of completely universal nature.

Furthermore, the theory readily predicts that in a certain region of variation of the parameters (7), the propellant essentially behaves as an oscillator having its own natural frequency of oscillations ω and damping ratio λ:

$$\lambda = \left[r\left(k+1\right) - (k-1)^2 \right] / 2r^2$$
$$\omega = \left(k / r^2 - \lambda^2 \right)^{1/2} \qquad (9)$$

Further investigating implications of his theory, Boris considered a number of important problems concerning burning rate oscillations and associated nonlinear effects. Such are the problems of the propellant burning rate response to harmonically oscillating pressure, both in the linear and higher approximations, and investigations of associated linear and non-linear resonances.

It should be noted that in the general case, when pressure varies with time, in addition to k and r there are two additional parameters which get involved in the stability analysis results

$$\nu = \left(\frac{\partial \ln m_0}{\partial \ln P} \right)_{T_a} ; \quad \mu = \left(\frac{\partial T_{s,0}}{\partial \ln P} \right)_{T_a} \Big/ \left(T_{s0} - T_a \right) \qquad (10)$$

Considering most natural basic problem of harmonically oscillating pressure, Boris found burning velocity as a function of the pressure amplitude and frequency. Most interesting results are obtained when the frequency of oscillations is close to the natural propellant combustion frequency, i.e. in the case of linear resonance. It was shown that as usual the phase changes by the value of $\pi/2$ upon passing the resonance.

Next problem considered (in a linear approximation) was non-acoustic (low frequency) propellant combustion stability in a semi-enclosed volume, i.e. in a combustion chamber. Importance of such a problem arises from the fact that in chambers of relatively small size, and at relatively low pressures, combustion instability manifests itself in pressure and burning rate oscillations at frequencies much lower than the chamber acoustic frequencies. Here Boris obtained stability conditions and demonstrated that his theory if far more consistent than the earlier results of Zeldovich on the same problem obtained under unrealistic assumption of constant propellant surface temperature. ZN theory, taking proper account of the surface temperature variation, predicts substantial widening of the stability region compared to the earlier Zeldovich results. In particular, stable combustion is possible for $k>1$ which is often observed in a reality.

Fundamental results were obtained by Boris upon investigating non-linear effects associated with oscillating regimes of propellant combustion. In particular, he obtained propellant burning velocity and its temperature distribution in the third approximation, and investigated associated nonlinear resonances. Four different types of resonance curves (Fig. 4) were identified. It turned out that the properties of non-linear resonance in a distributed (i.e. having infinite number of degrees of freedom) systems, such as burning propellant, are similar to those observed for mechanical and electrical systems with the finite number of degrees of freedom. In particular, at the resonance the amplitude of the first harmonics is of the order of the cubic root of the pressure (analogue of the external force for mechanical systems); zeroth (constant) and the second harmonics are quadratic functions of the amplitude of the first harmonics. Moreover, combustion regimes analogous to auto-oscillations have been found. It turns out that it is possible to describe propellant, by analogy with mechanical and electrical systems, quantitatively as an auto-oscillator in terms of oscillating system, energy source, and the regulator that controls energy supply to the system.

Closely related to the above problematic is a problem of quantification of acoustic admittance of the propellant burning surface.

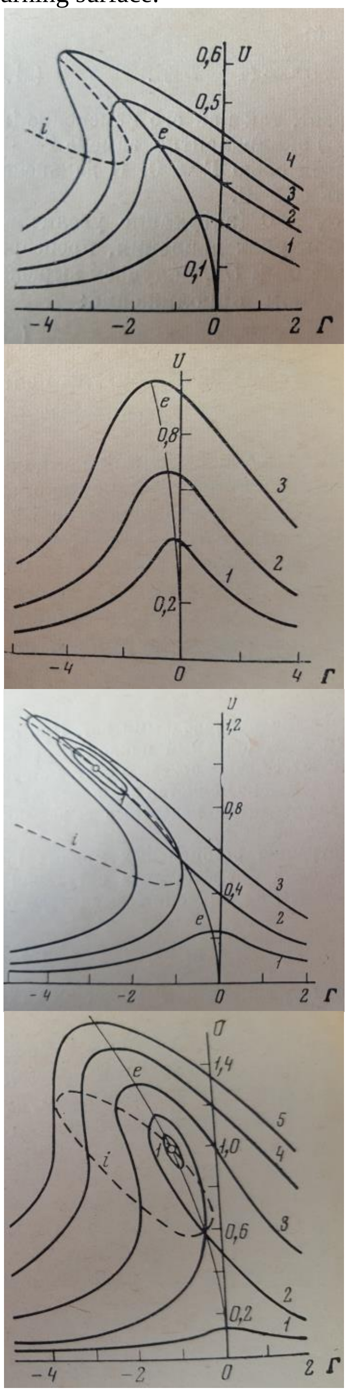

Fig. 4. Four types of non-linear resonances. Propellant combustion under harmonically oscillating pressure. U is proportional to the burning rate perturbation

This problem constituted large body of work in the Boris' investigations. In particular, in the linear approximation, he expressed acoustic admittance in terms of pressure- and velocity-coupled burning rate response functions. Response functions relate pressure and burning rate, or tangential gas velocity and burning rate oscillations at the surface. Note that outside of the ZN approach, in the Flame Model paradigm, similar exact analytical relations are still not known. Further, Boris introduced non-linear, i.e. quadratic response functions and obtained corresponding expressions [9,10]. Not having an opportunity to discuss this problematic in more detail here, we refer reader to the comprehensive review [11] and the papers [9,10].

A big advantage of the ZN theory is that it offers a unified view point for various phenomena associated with propellant combustion. As an example, the problem of transient propellant extinction under pressure drop was considered. An integro-differential formulation of the theory (6) was used, and numerical analysis was applied to solve the equation (6) for the burning velocity. Computed characteristics of the extinction process were found to be in close agreement with experimental results.

Upon discovery of ZN theory in the 60s [3,4] many, including its author Boris believed that it would be quickly superseded by a consistent macroscopic model of chemical physics that could be applied (numerically) to any desired propellant. Forty years later in his review [11] Boris stated (translated from Russian by the author of the present paper) that "…progress beyond the ZN theory occurs very slowly due to sheer complexity of propellant combustion (even for homogeneous systems) and in particular of the phase change from condensed phase to gaseous, complicated further by chemical transformation". From late 80s through to 00s Boris proposed an extension of the ZN theory that takes into account influence of the gas-phase inertia on burning stability. First, he noted [7] that in certain situations, even if formal conditions (inequalities comparing characteristic relaxation time scales of different zones) required by the ZN approach are fulfilled, one may arrive at inherently contradictory results. In such situations it is incorrect to neglect inertia of the gas phase and/or propellant reaction zone. Before the studies [12,13] no analytical approaches to the problem of combustion stability at constant pressure, allowing for time relaxation of small-inertia zones existed. Based on propellant combustion model due to Belyaev, Boris obtained such a solution [12] as well as stability conditions. Further, he proposed what he called t_r approximation [7] as a general way for expansion of the ZN theory. In essence t_r approximation introduces time lags due to small-inertia zones into the burning laws (4). The elaborate results cannot be communicated here due to limitation of space but may be found in [7,12,13]. It is sufficient to present here Fig. 5 illustrating major results. The curve "1" in this Figure corresponds to the curve "s" Fig. 3, that is to stability boundary in the t_c approximation. Curves "2" and "3" correspond to increasing value of the parameter that measures relative inertia of the gas phase compared to the condensed phase. Thus even for respective relaxation times obeying $t_g << t_c$ allowance for the gas phase relaxation extends the stability region substantially. Also the natural frequency of oscillations changes compared to the t_c approximation.

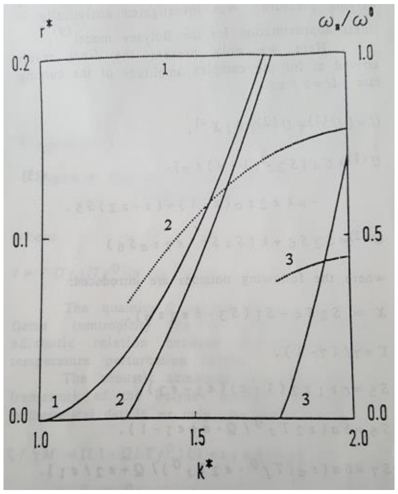

Fig. 5. Widening of combustion stability region under t_r approximation. Constant pressure

Boris contributions to the physics of combustion extends far beyond the ZN theory. Several examples of such contributions are discussed below.

Boris contributed substantially to the development of classical theoretical problem of thermal explosion (known also as thermal runaway). Here he considered several problems extending classical studies of Semenov and Frank-Kamenetskii. The range of problems considered in this area covers thermal explosion in a mixture with non-uniform initial reactants concentrations, thermal explosion of non-premixed reactants [14,15], thermal explosion under concurrent homogeneous and heterogeneous chemical reactions [16], thermal explosion under forced convection conditions [17-19]. The latter studies on thermal explosion in dynamic conditions are especially interesting. Despite apparent practical importance of this problem, studies on the topic, before the papers [17-19] were limited to just one paper. In [17-19] the authors obtained analytical solutions that allow to predict influence of the forced mixing intensity on critical conditions for thermal explosion in chemical reactors. Examples of the flow configurations they considered are presented in Fig. 6.

Critical conditions for the problem of thermal explosion under forced convection are presented in Fig 7.

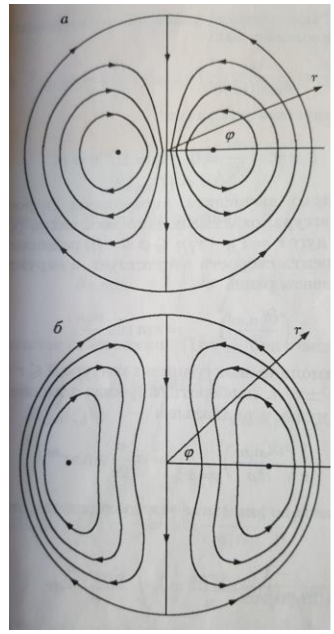

Fig. 6. Flow configurations in the problem of thermal explosion under forced convection conditions [17-19]

This problem arises, for example, in chemical reactors where mixture needs be stirred in order to increase heat dissipation rate and avoid unlimited temperature rise due to chemical reaction. In such a device, flow similar to that in Fig. 6 will be enforced by a number of mechanical blades which rotate and generate a number of vortices in the flow. Different curves in Fig. 7 correspond to different number of blades (forced vertices). The horizontal axis shows distance from the centre of the reactor to centres of vortices. Conditions leading to thermal explosion are above the corresponding curve.

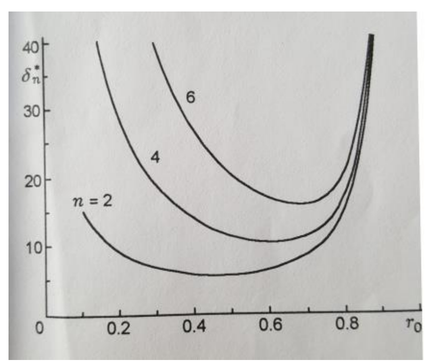

Fig. 7. Critical conditions for thermal explosion. Dynamic mixture with forced convection [19]

Another area of combustion science where Boris made a fascinating contribution is a theory of spin combustion. Spin is a peculiar combustion wave spiralling (while also moving laterally) around the surface of cylindrical sample. It is observed for specific solid fuels in gaseous oxidizers. First attempts of theoretical explanation went on using perturbation analysis to consider loss of stability by stationary planar combustion front. This approach reveals so-called "weak" spin, which has actually been never observed experimentally.

In contrast, Boris was the first to provide rigorous theory of the real observable "strong" spin. His extremely elegant and simple text book – style analysis [20,21] was to consider basic heat balance in the direction of the spin propagation (Fig. 8), i.e. transversal to the axis of the sample.

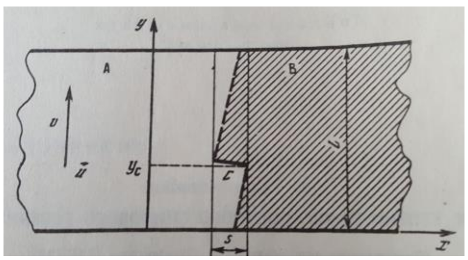

Fig. 8. Sketch illustrating theory development for spin combustion. A – unburnt sample, B – products of combustion, C – reaction zone. Temperature is averaged over narrow strip of the width s (spin width). Heat transfer equation is being written in the y - direction.

This balance with suitable boundary conditions leads upon detailed analysis [20,21] to analytical solution that predicts all the important features of spin propagation. Fig. 9 presents non-dimensional transversal spin speed against the non-dimensional parameter which is proportional to the sample diameter.

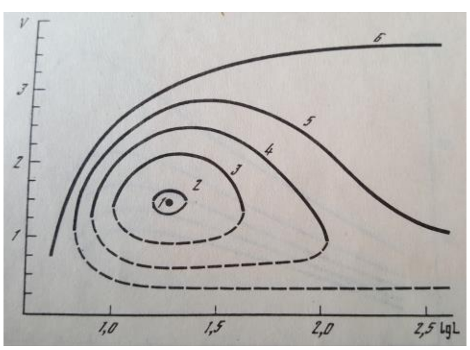

Fig. 9. Transversal spin speed as a function of the sample diameter for different intensities of heat losses. Dashed curves represent unstable solutions

Fig. 10 presents transversal to longitudinal velocity ratio for different values of the thermal diffusivity of the sample.

One of fascinating results of the theory is that the ratio of squared spin velocity along the axis of the cylinder and the spin frequency must be constant and of the same order of magnitude as the sample's thermal diffusivity. This fact agrees very well with experimental data. Therefore, a clock is all one needs to measure thermal diffusivity of spin-propagating samples! The theory also readily predicts conditions of existence and characteristics of multi-head spins.

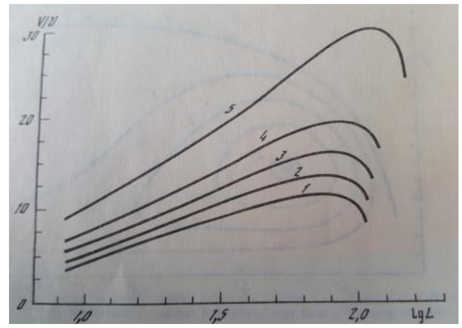

Fig. 10. Transversal to longitudinal velocity ratio for spin propagation. Different values of the thermal diffusivity of the sample.

Finally, in the later period of his activity Boris made another very important discovery. Making systematic analysis of propellant combustion behavior beyond stability region (here we return back to the ZN theory!) he discovered chaotic regimes of propellant combustion. Both combustion at constant pressure, beyond stability boundary, and under large amplitudes of harmonically oscillating pressure lead upon parametric bifurcations to the famous Feigenbaum period bifurcation scenario (in this case in a distributed combustion systems). These investigations are reported in publications [11,22].

Transition to chaotic regimes are illustrated by a series of plots in the Figs. 11-13 for the constant pressure case. The transition is caused by the change in the parameter k (bifurcation parameter), keeping parameter r as constant (see Fig. 3). Top of the figures shows burning rate as a function of time. The bottom is corresponding trajectory in the phase space defined by variables (burning velocity, heat content in the condensed phase)

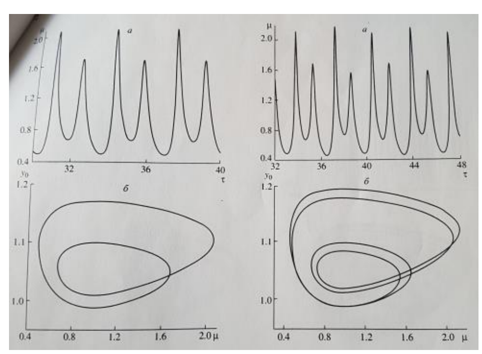

Fig. 11. Transition to combustion regimes of the period 2T (left) and 4T (right) from the original stable regime with the period T. Constant pressure.

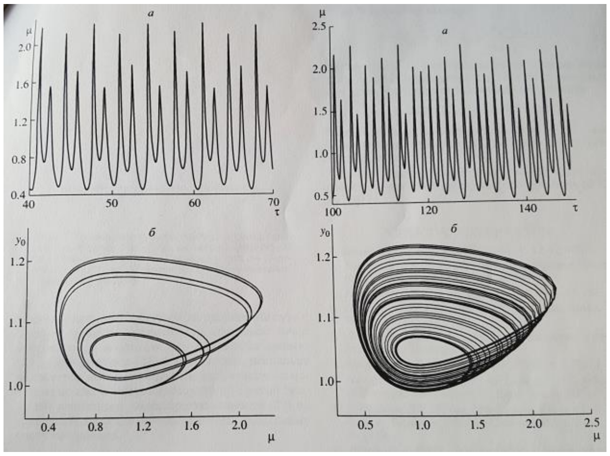

Fig. 12. Transition to combustion regimes of the period 8T (left) and chaotic regime on the right.

Another example of chaotic regime is presented in Fig. 13 which corresponds to the case of harmonically oscillating pressure leading to somewhat more complex and topologically different attractor.

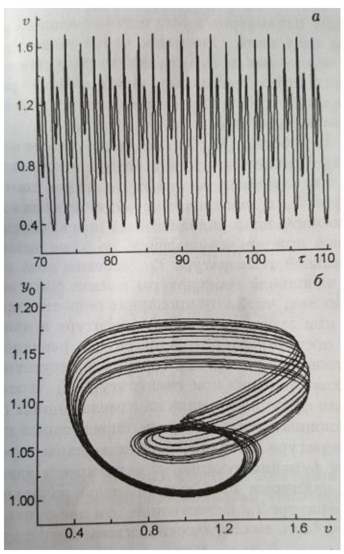

Fig. 13. Chaotic combustion behaviour in the case of harmonically oscillating pressure. Bifurcation parameter is an increasing pressure amplitude

Relevant calculations [11] involving successive values of parameters at which bifurcations occur show (within numerical errors) that the transition follows the classical universal scenario of Feigenbaum.

Finally, it should be noted that Lorenz predicted an existence, in the chaotic regime, of the approximate dependence

$$M_{s+1} = P\left(M_s\right) \qquad (11)$$

where M_s are the maxima of some time-dependent function. This fact can be observed in the chaotic propellant combustion as well.

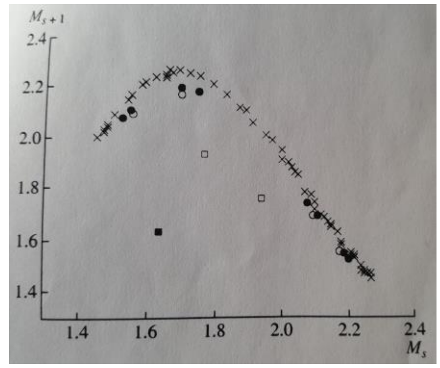

Fig. 14. Lorenz-type dependence (11) for chaotic propellant combustion regime under constant pressure.

Fig. 14 plots the maxima of the burning rate as a function of the preceding maximum. The stable regime (filled square marker) has a single maximum, and therefore just one point in Fig. 14, 2T regime (void square) has two points, etc. In the chaos limit (crosses) there is a continuum (in fact a Cantor set) of maxima values, and they collapse on a uniform functional dependence.

We hope that the present brief review gives an idea of Prof. Boris Novozhilov scientific endeavors and achievements over the span of his career. Boris will be remembered as an outstanding scientist, a caring family man, and a wonderful colleague. He will be deeply missed by numerous friends around the world who were very important for him.

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