IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

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THE APPLICATION OF A RANDOM COEFFICIENT MODEL TO THE PROBLEM OF ESTIMATING AGGREGATE PRODUCTION PARAMETERS

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1. INTRODUCTION

The publication of the Arrow, Chenery, Minhas and Solow (ACMS) paper in 1961 precipitated a burst of activity in production analysis. Empirical workers attempted to exploit the simple mathematical form of the marginal productivity conditions implied by the CES function in order to estimate the important elasticity of substitution parameter (σ). These studies have, however, not been as informative as had been originally hoped. The substantial differences between the various estimates of σ tends to indicate that it is a highly unstable parameter which is sensitive to both the data base and the particular functional form used.[1] "A variety of hypotheses have been advanced to explain the diversity of results, including cyclical changes in the utilization of factors (Nerlove, 1967), random measurement errors (Leontief, 1964), systematic variation of input prices and product prices (Nerlove, 1967), embodied and disembodied technical change and problems in the measurement of inputs (Griliches, 1967a; Hildebrand and Liu, 1965), simultaneous equation bias (Maddala and Kadane, 1966; Nerlove, 1967),

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1. See Caddy (1976) for a survey of these results.

serial correlation (Griliches, 1967a), and lagged adjustment (Griliches; 1967a, Lucas; 1969, Jorgenson; 1972)"[1]. This paper adds another to this list, namely the problem associated with data aggregation and the choice of an appropriate specification for the aggregate relationship. A random coefficient model which overcomes these problems is specified and applied to Australian data.

2. THE PRODUCTION MODEL AND AGGREGATION

Despite Theil's (1954) comprehensive work on aggregation many of the econometric models which are used in empirical estimation of macro relationships are generated by expediency rather than rigorous economic theory. In particular it is common for macro analysts to postulate that relationships based on axiomatically founded micro theories can also be entertained as economically meaningful relationships at the macro level. This is the procedure that has been followed in the production literature.

Although the classical production model is based on essentially micro concepts the form of the relationship and the interpretation of the parameters implied by this theory are projected directly into the macro sphere and estimated using aggregate data. In doing this it is hoped that the estimated coefficients will yield some information about the "average" relationship which exists between the micro units. Unfortunately there is generally no attempt to establish the nature of the

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1. Berndt (1976) p.59

relationship between these analogically derived specifications and the micro theory on which they are ostensibly based.

In the particular case of the CES production function the most common method of estimating the elasticity of substitution between labour and capital is via the labour marginal productivity side condition. The micro theory tells us that each firm will adjust its labour input so as to ensure that the following relationship holds:

$$\log \frac{v_i}{\ell_i} = \alpha_i + \sigma_i \log \frac{w_i}{\ell_i} + u_i \quad (i = 1, \ldots, M), \tag{1}$$

where v_i is real value added, ℓ_i is labour input, w_i is the real wages bill for firm i, and u_i is a random disturbance term. The parameter α_i is a combination of the CES "efficiency" and "distribution" parameters while σ_i is the elasticity of substitution. In the conventional model these parameters are considered to be non-random.

In practice the next pragmatic step is to postulate that a similar form of relationship can be meaningfully employed to relate the arithmetic aggregates of the variables;

$$\text{i.e.,} \quad \log \left(\frac{\sum v_i}{\sum \ell_i} \right) = \approx{\alpha} + \approx{\sigma} \log \left(\frac{\sum w_i}{\sum \ell_i} \right) + \approx{u} \quad , \tag{2}$$

where the coefficients $\approx{\alpha}$ and $\approx{\sigma}$ are interpreted as providing some form of summary measure of the corresponding micro coefficients. It is important to note that the step from (1) to (2) follows from analogy only. The equation (2) is not based on any independent macro economic theory nor does it follow mathematically from (1).

The use of arithmetic sums in (2), although perhaps convenient, is inappropriate if consistency between the micro theory and the macro equation is to be maintained. The importance of this question of the appropriate form of aggregate variable to use was first considered in a pioneering paper by Klein (1946). He claimed that "the problem of bridging the gap between the traditional theories based on individual behaviour and the theories based on community or class behaviour is, to a large extent, a problem of proper measurement" (p.93). The use of arithmetic sums does not provide this bridge. Klein suggests[1] that if the macro and micro specifications are to be consistent then the maximization of profit by the individual firms so that the marginal productivity equations hold under perfect competition, should also lead to the aggregative marginal productivity condition being satisfied. If we accept the micro relationships as being of the form given by (1) then the implied aggregative equation is

$$\sum_i \log\left(\frac{v_{it}}{\ell_{it}}\right) = \sum_i \alpha_i + \left\{\frac{\sum_i \sigma_i \log\left(\frac{w_{it}}{\ell_{it}}\right)}{\sum_i \log\left(\frac{w_{it}}{\ell_{it}}\right)}\right\} \sum_i \log\left(\frac{w_{it}}{\ell_{it}}\right) + \sum_i u_{it} , \qquad (3)$$

which can be written as

$$V = \alpha^* + \sigma^* W + U . \qquad (4)$$

(1) Klein (1946) p.94.

From (3) it can be seen that the appropriate aggregates are geometric rather than the ratios of arithmetic sums and that the coefficient on the explanatory variable should be interpreted as being a weighted average of the σ_i, with the weights being proportional to the observations on $\log\left(\frac{w_{it}}{\ell_{it}}\right)$. Since these weights will vary from sample to sample the economic significance of such a parameter is limited.

3. ESTIMATION OF AGGREGATED RELATIONSHIPS

The problems associated with aggregation not only relate to the specification but also impinge upon the estimation procedures. It can be shown that when the data consists of a single cross-section in which the observations are aggregates of subsets of the micro units,[1] the "average" aggregate coefficients yielded by the use of O.L.S. techniques will, in general, depend on a complicated combination of corresponding and non-corresponding micro coefficients. This problem rises because the assumption of basic homogeneity between the units of observation which underlies all cross-sectional estimation is unlikely to be satisfied. The reason for this is obvious from equation (3). Unless there is a one to one correspondence between the technical characteristics of the firms in the various subsets over which aggregation takes place and unless the weights given to each firm having a particular characteristic are identical for each subset we must expect the aggregate coefficients to vary

1. For example we might consider using data which consist of aggregates taken over firms within various statistical areas or States.

over the sample.

Even when the aggregation takes place over groups of firms within a narrowly defined industry classification the homogeneity of firm parameters necessary to ensure that the coefficients of each of the aggregate relationships are the same is unlikely to be realised. While firms within a given industry could, by definition, be expected to face similar technical production possibilities, differences in entrepreneurial ability possessed by the various firms would cause differences in the efficiency parameters (and hence α_i) even although the distribution and substitution parameters are the same. When the aggregates cover a wider spectrum of industries the assumption of constancy of these latter parameters would also appear to be untenable.

If this diversity of parameters does exist, and we have observations on N relationships of the form (4) the application of O.L.S. techniques gives

$$\hat{\beta} = (\underline{\underline{W}}^T \underline{\underline{W}})^{-1} \underline{\underline{W}}^T \underline{V} \quad , \tag{5}$$

where $\hat{\beta} = \begin{bmatrix} \hat{\alpha} \\ \hat{\sigma} \end{bmatrix}$ is a vector of estimated "average" coefficients, $\underline{V}$ is an N x 1 matrix of the dependent aggregate variable (the typical element being $V_j = \sum_i \log \left(\frac{v_{it}}{\ell_{it}}\right)$, where i identifies all the micro units in subset j), and $\underline{\underline{W}}$ is an N x 2 matrix in which the first column is a unit vector and the second consists of observations on the independent aggregate variable $W_j = \sum_i \log \left(\frac{W_{it}}{\ell_{it}}\right)$.

Substituting from (4) into (5) gives

$$\hat{\beta} = (\underline{\underline{W}}^T \underline{\underline{W}})^{-1} \underline{\underline{W}}^T \begin{bmatrix} \alpha_1^* + \sigma_1 W_1^* + U_1 \\ \vdots \\ \alpha_N^* + \sigma_N W_N^* + U_N \end{bmatrix} ,$$

or
$$\hat{\hat{\beta}} = (\underline{\underline{W}}^T \underline{\underline{W}})^{-1} \underline{\underline{W}}^T \overline{\underline{\underline{W}}} \begin{bmatrix} \alpha_1^* \\ \sigma_1^* \\ \vdots \\ \alpha_N^* \\ \sigma_N^* \end{bmatrix} + (\underline{\underline{W}}^T \underline{\underline{W}})^{-1} \underline{\underline{W}}^T U ,$$

where
$$\overline{\underline{\underline{W}}} = \begin{bmatrix} 1 & W_1 & 0 & 0 \ldots\ldots & 0 & 0 \\ 0 & 0 & 1 & W_2 \ldots\ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \ldots\ldots & 1 & W_N \end{bmatrix} .$$

Taking expectations we get

$$E(\hat{\hat{\beta}}) = (\underline{\underline{W}}^T \underline{\underline{W}})^{-1} \underline{\underline{W}}^T \overline{\underline{\underline{W}}} \begin{bmatrix} \alpha_1^* \\ \vdots \\ \sigma_N^* \end{bmatrix}$$

$$= Z \begin{bmatrix} \alpha_1^* \\ \vdots \\ \sigma_N^* \end{bmatrix} , \quad (6)$$

where
$$Z = (\underline{\underline{W}}^T \underline{\underline{W}})^{-1} \underline{\underline{W}}^T \overline{\underline{\underline{W}}} .$$

From (6) it can be seen that the expectation of a particular element of $\hat{\hat{\beta}}$ is not related in any straight-forward way to the corresponding aggregate parameters. The complexity of the relationship is more readily appreciated if (6) is expressed using summation notion. This gives[1]

$$E(\hat{\bar{\alpha}}) = \frac{\sum_i^N W_i^2 \left(\sum_i^N \alpha_i^* + \sum_i^N \sigma_i^* W_i\right) - \sum_i^N W_i \left(\sum_i^N \alpha_i^* W_i + \sum_i^N \sigma_i^* W_i^2\right)}{N \sum_i^N W_i^2 - \left(\sum_i^N W_i\right)^2}, \qquad (7a)$$

and

$$E(\hat{\bar{\sigma}}) = \frac{N \left(\sum_i^N \alpha_i^* W_i + \sum_i^N \sigma_i^* W_i^2\right) - \sum_i^N W_i \left(\sum_i^N \alpha_i^* + \sigma_i^* W_i\right)}{N \sum_i^N W_i^2 - \left(\sum_i^N W_i\right)^2}. \qquad (7b)$$

Thus the expected values of these estimated "average" parameters do not have the simple property that $\hat{\bar{\alpha}}$ depends only on the α_i^* and $\hat{\sigma}$ on the σ_i^* but are general weighted averages of all the parameters. Furthermore, since the weights are determined by the observation matrix, one would expect that the use of different time periods or areas of aggregation - each of which call for different observation matrices - would cause substantial changes in the estimated aggregate coefficients even although the micro parameters may be perfectly stable.

In view of the earlier comments on the appropriate form for the aggregate specification and the above estimation problems one is forced to conclude that the plethora of econometric studies which use arithmetic aggregates and which fail to deal with the consequent mis-specification are unlikely to provide parameter estimates which can be related in any meaningful way to the average or typical firm. Rather they provide estimates which are complex weighted averages of all the parameters in the model whose interpretation is obscure.

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1. See Klein (1972) pp. 356 - 357.

4. A RANDOM COEFFICIENT MODEL

In view of the above it is obvious that careful consideration must be given to the steps involved in moving from the micro theory to a macro estimating equation. This section specifies a random coefficient model which facilitates the estimation of a substitution parameter which, under the assumptions adopted, can be positively identified as representing the mean value of that parameter for all industries in the economy.

We will start with the theoretically justified micro relationship (1). In the previous section it was suggested that while the elasticity of substitution could be expected to be constant over firms within a given industry it is highly probable that the efficiency parameter will vary from firm to firm. In more concrete terms we are saying that the technology confronting all firms is the same in so far as it is technically possible for them all to substitute labour for capital with the same ease but, because managerial ability is not evenly distributed across firms, some will always produce greater levels of output than the others for a given level of inputs. If we assume that these managerial skills are randomly distributed over firms then the micro relationship for the i^{th} firm in industry j can be written as

$$\log \frac{v_i}{\ell_i} = \alpha^j + \lambda_i^j + \sigma^j \log \frac{w_i}{\ell_i} + u_i \quad , \tag{8}$$

where λ_i^j is a random element which reflects the above or below average efficiency achieved by the management of that firm. This random component is characterized by

$$\left.\begin{aligned} E\ (\lambda_n) &= 0 \\ Var\ (\lambda_n) &= \emptyset_\lambda \end{aligned}\right\} \forall n \quad ;$$

$$Cov\ (\lambda_n, \lambda_m) = 0\ (n \neq m) \quad , \tag{9}$$

$$Cov\ (\lambda_n, u_m) = 0\ (\forall n, m) \quad .$$

We can write (8) as

$$\log \frac{v_i}{\ell_i} = \alpha^j + \sigma^j \log \frac{w_i}{\ell_i} + \xi_i \quad , \tag{10}$$

where $\xi_i = \lambda_i + u_i$. The random terms have been merged into ξ_i which, since it is the sum of two independent random variables with mean zero, will also be a random variable with mean zero and variance $\emptyset_\xi = \emptyset_u + \emptyset_\lambda$. As reliable firm data is not available (10) does not constitute an operational form.

If we now sum over all firms in industry j we get

$$\sum_i \log \frac{v_i}{\ell_i} = N\ \alpha^j + \sigma^j \sum_i \log \frac{w_i}{\ell_i} + \sum_i \xi_i \quad ;$$

i.e.

$$\frac{1}{N}\left\{\sum_i \log v_i - \sum_i \log \ell_i\right\} = \alpha^j + \sigma^j \frac{1}{N}\left\{\sum_i \log w_i - \sum_i \log \ell_i\right\} + \sum_i \xi_i \quad ;$$

or, to simplify the notation

$$V^j = \alpha^j + \sigma^j W^j + \xi^j \qquad (j = 1,\ldots N). \tag{11}$$

It is immediately obvious that any attempt at empirical estimation of (11) will encounter data problems. The most readily accessible industry data consists of arithmetic sums. While some approximation error is unavoidable it would appear that there is sufficiently detailed published data available to allow acceptable estimates of the geometric means to be calculated. This is discussed in the following section on data.

The random coefficient concept can also be gainfully employed to allow for the anticipated differences between the parameters of the various industry functions. In this case we write

$$\alpha^j = \bar{\alpha} + \pi^j \qquad (j = 1,\ldots.,N),$$

$$\sigma^j = \bar{\sigma} + \eta^j \qquad (j = 1,\ldots.,N),$$

where $\bar{\alpha}$ and $\bar{\sigma}$ are the mean values for the parameters over the whole economy and π^j and η^j are random variables which reflect the extent to which the parameters for industry j deviate from the economy wide average. The random variables have the following specification:

$$\left.\begin{array}{ll} E(\pi^j) = 0 \ , & E(\eta^j) = 0 \\ Var(\pi^j) = \emptyset_\pi \ , & Var(\eta^j) = \emptyset_\eta \end{array}\right\} \forall j \qquad ; (12)$$

$$Cov(\pi^j, \pi^k) = 0 \ , \quad Cov(\eta^j, \eta^k) = 0 \quad (j \neq k) \ ,$$

$$Cov(\pi^j, \xi^k) = 0 \ , \quad Cov(\eta^j, \xi^k) = 0 \quad (\forall j,k) \ . \quad (13)$$

Equation (11) can now be written as

$$V^j = (\bar{\alpha} + \pi^j) + (\bar{\sigma} + \eta^j)\, W^j + \xi^j \qquad (j = 1,\ldots,N). \quad (14)$$

Since the equation error ξ^j cannot be distinguished from the random component of the intercept term (i.e. π^j) the two are combined to form a composite random term.

$$\pi^j + \xi^j = \Gamma^j \ ;$$

$$Var(\Gamma) = \emptyset_\pi + \emptyset_\xi = \emptyset_\Gamma \ .$$

We can now write

$$V^j = \bar{\alpha} + \bar{\sigma} W^j + \ell^j , \qquad (15)$$

where

$$\ell^j = \Gamma^j + W^j \eta^j .$$

Assuming W to be non-stochastic and the various random components to be independent we can write :

$$E(\ell^j) = E(\Gamma^j) + W^j E(\eta^j) , \qquad (16a)$$

$$= 0 ;$$

$$Var(\ell^j) = \phi_\Gamma + (W^j)^2 \phi_\eta = K_j , \qquad (16b)$$

$$Cov(\ell^j, \ell^k) = 0 . \qquad (16c)$$

5. ESTIMATION

Considering the equation (15) and the error specification (16a-c) we see that we have arrived at a linear regression model with a hetroskedastic disturbance term. Using linear regression techniques it should be possible to obtain unbiassed estimates of the average value of the production parameters for the whole economy (i.e. the macro parameters $\bar{\alpha}$ and $\bar{\sigma}$). In view of the hetroskedasticity of the error structure the efficient estimating procedure is to use generalized least squares (G.L.S.):

$$\text{i.e.} \quad \hat{\beta}_{GLS} = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} V , \qquad (17)$$

where

$$\hat{\beta} \begin{bmatrix} \hat{\bar{\alpha}} \\ \hat{\bar{\sigma}} \end{bmatrix} , \quad X = \begin{bmatrix} 1 & W^1 \\ 1 & W^2 \\ \vdots & \vdots \\ 1 & W^N \end{bmatrix} , \quad V = \begin{bmatrix} V^1 \\ V^2 \\ \vdots \\ V^N \end{bmatrix} ,$$

$$\Omega = \begin{bmatrix} K_1 & 0 & \cdots & 0 \\ 0 & K_2 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_N \end{bmatrix} .$$

From (16b) we can see that Ω is made up of unknown parameters $\emptyset_\Gamma$ and $\emptyset_\eta$ which must be estimated before the G.L.S. estimates can be obtained. The procedure adopted below follows that suggested by Hildreth and Houck (1968).

Let $\hat{u}$ be the vector of ordinary least squares (O.L.S.) residuals,

$$\text{i.e.,} \quad \hat{u} = V - X\,\hat{\beta}_{OLS} = Au \quad , \tag{18}$$

where $\quad A = I_N - X(X'X)^{-1}X'$ and $u = V - X\beta \quad ,$ (19)

$$\hat{u}\;\hat{u}' = Au\,u'A' \quad ,$$

$$E(\hat{u}\;\hat{u}') = A\,E(u\,u')\,A' = A\;\Omega\,A' \quad .$$

Therefore $\quad E\,(\hat{u}_n^2) = n^{th}$ row of A multiplied by the n^{th} column of $\Omega A'$.

i.e. the nn^{th} element of $E(\hat{u}\,\hat{u}')$ $= \begin{bmatrix} a_{n1} & a_{n2} & \cdots & a_{nN} \end{bmatrix} \begin{bmatrix} K_1 & a_{n1} \\ K_2 & a_{n2} \\ \vdots & \\ K_N^a & a_{nN} \end{bmatrix}$

$$= a_{n1}^2 K_1 + a_{n2}^2 K_2 + \cdots + a_{nN}^2 K_N .$$

We will now define a matrix $\dot{A}$ in which all the elements are the square of the corresponding elements in A, i.e.,

$$\dot{A} = \begin{bmatrix} a_{11}^2 & a_{12}^2 & \cdots & a_{1N}^2 \\ \vdots & \vdots & & \vdots \\ a_{N1}^2 & a_{NN}^2 & \cdots & a_{NN}^2 \end{bmatrix} = \begin{bmatrix} \dot{A}_1 \\ \dot{A}_2 \\ \vdots \\ \dot{A}_N \end{bmatrix} , \qquad (20)$$

where we have defined $\dot{A}_n$ as the n^{th} row of $\dot{A}$. Define

$$\approx{K} = \begin{bmatrix} K_1 \\ K_2 \\ \vdots \\ K_N \end{bmatrix} . \qquad (21)$$

Using this notation we have

$$E\,(\hat{u}_n^2) = \dot{A}_n\,\tilde{\tilde{K}} \quad , \qquad (22)$$

and

$$E\begin{bmatrix}\hat{u}_1^2\\ \hat{u}_2^2\\ \vdots\\ \hat{u}_N^2\end{bmatrix} = \begin{bmatrix}\dot{A}_1\\ \dot{A}_2\\ \vdots\\ A_N\end{bmatrix}\tilde{\tilde{K}} = \dot{A}\,\tilde{\tilde{K}} \quad ;$$

or

$$E\,(\dot{\hat{u}}) = \dot{A}\,\tilde{\tilde{K}} \quad . \qquad (23)$$

We now write

$$\dot{\hat{u}} = \dot{A}\,\tilde{\tilde{K}} + r \quad , \qquad (24)$$

where

$$r = \dot{\hat{u}} - E(\dot{\hat{u}}) \quad , \quad E(r) = 0 \quad \& \quad E(rr') = \Psi \quad .$$

Using a similar notation to that employed above we have (from 16b)

$$K_n = \begin{bmatrix}1^2 & W_n^2\end{bmatrix}\begin{bmatrix}\emptyset_r\\ \emptyset_\eta\end{bmatrix} = \dot{X}_n\,\emptyset$$

and

$$\tilde{\tilde{K}} = \dot{X}\,\emptyset \quad . \qquad (25)$$

Substituting (25) into (24) we get

$$\dot{\hat{u}} = \dot{A}\,\dot{X}\,\emptyset + r \quad ,$$

$$= G\,\emptyset + r \quad , \qquad (26)$$

where

$$G = \dot{A}\,\dot{X} \quad .$$

Since $\dot{\hat{u}}$, $\dot{A}$ and $\dot{X}$ are known (or at least can be calculated) (26) is in a form suitable for O.L.S. estimation;

$$\emptyset_{HH} = (G'G)^{-1} G' \dot{\hat{u}} \quad . \qquad (27)$$

Hildreth and Houck have proved $\hat{\emptyset}_{HH}$ to be an unbiassed and consistent estimator of $\emptyset$. However as $\emptyset$ is a variance the appearance of negative estimates, which is likely to occur with the above estimator, is undesirable. In order to overcome the problem Hildreth-Houck suggest the use of a truncated estimator

$$\hat{\emptyset}_T = \max (0, \hat{\emptyset}_{HH}) \quad , \qquad (28)$$

Although this estimator is biassed it is shown to have a smaller mean squared error than $\hat{\emptyset}_{HH}$.

A third estimator ($\hat{\emptyset}_{QP}$) which minimizes the sum of squares subject to $\emptyset > 0$ has also been proposed:

$$\min_{\emptyset} (\dot{\hat{u}} - G\,\emptyset)' (\dot{\hat{u}} - G\emptyset) \quad . \qquad (29)$$

The results of a Monte Carlo experiment carried out by Froehlich (1973) indicates that $\hat{\emptyset}_{QP}$ is preferable in terms of mean squared error to $\hat{\emptyset}_T$. The rapid rate at which the relative efficiency of $\hat{\emptyset}_{QP}$ over $\hat{\emptyset}_T$ increases as the sample size falls "suggests that in a practical situation where the sample size is likely to be small, the calculation of $\hat{\emptyset}_{QP}$ is well worth the effort."

These estimates can then be substituted into the covariance matrix Ω and the G.L.S. procedure applied to obtain an estimate of β. The resulting estimator will be consistent.

6. DATA

The variables on which observations are necessary for the application of the above estimating technique are those appearing in the V and X matrices. The composition of these matrices are given in (17) and (11). From these we can see we require observations on the following variables for each industry:

$\exp\left[\frac{1}{N}\sum \log v_i\right]$ = geometric mean value added per firm in industry j.

$\exp\left[\frac{1}{N}\sum \log \ell_i\right]$ = geometric mean labour input per firm in industry j,

and

$\exp\left[\frac{1}{N}\sum \log w_i\right]$ = geometric mean wages bill per firm in industry j.

If the deviations of the variables from their respective arithmetic means are relatively small then it can be shown that the geometric mean is approximately related to the arithmetic mean $(\bar{x})$ by the following formula:

$$\frac{1}{N}\sum_i \log x_i \approx \log \{\bar{x}(1 - \frac{1}{2}\frac{\sigma^2}{\bar{x}^2})\} \quad . \tag{30}$$

The Australian Bureau of Statistics publication <u>Manufacturing Establishments: Selected Items of Data Classified by Industry and Employment Size, Australian Economic Censuses 1968-9</u>, provides data at a more disaggregated level than is normally used in estimation. This publication identifies the number of firms in each industry falling into nine different size categories (where size is determined by the number of persons employed) and also records the total wages bill, value added and people employed for each size category.

A perusal of the data indicates that there are significant variations in the wage rates and value added per man between firms within the one industry. Consequently the use of arithmetic means in the estimation will yield erroneous results due to measurement errors in the variables. In an attempt to reduce such errors approximations to the geometric means are calculated under the assumption that each size category identified in the above publication is sufficiently narrowly defined to allow the variance within that class to be considered to be zero. For example if in industry j there are y firms employing between five and nine people and these firms have a total wages bill of \$z, then in the calculation of the geometric mean wages bill this is treated as if we had y individual observations of firms having a wages bill of \$ z/y.

Data is provided for approximately 160 industry classifications (i.e. all manufacturing industries at the four digit ASIC level). While it would be preferable to utilise the data at this level of disaggregation[4] it has been decided to aggregate up three digit level (35 industries) for this study.

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4. This study defines an industry as consisting of those activities for which the substitution and distribution parameters are the same. The higher the degree of disaggregation used the more appropriate this definition is likely to be. Also aggregating data involves a loss of information, information which could be used to improve the precision of the estimates.

7. RESULTS

Before proceeding with the estimation it is necessary to consider the complications arising due to the fact that the above data measures the variables in money terms, while the model outlined in Section 4 defines relationships between real values.[1]
Multiplying (8) by output price we get the following relationship in which the variables are expressed in money terms:

$$\log \frac{v_i p^j}{\ell_i} = \alpha^j + \lambda_i + \sigma^j \log \frac{w_i p^j}{\ell_i} + (1 - \sigma^j) \log p^j + u_i \quad , \qquad (31)$$

where p^j is the price of output from industry j.

Summing over firms and using the random coefficient specification to allow for parameter differences between industries (ref. p.11) we get the "money" equivalent to (14):

$$V_*^j = (\bar{\alpha} + \pi^j) + (\bar{\sigma} + \eta^j) W_*^j + (1 - \bar{\sigma} - \eta^j) P^j + \xi^j \quad , \qquad (32)$$

where

$$V_*^j = \frac{1}{N} \left\{ \sum_i \log \frac{v_i p^j}{\ell_i} \right\} \quad ,$$

$$W_*^j = \frac{1}{N} \left\{ \sum_i \log \frac{w_i p^j}{\ell_i} \right\} \quad ,$$

and

$$P^j = \log p^j \quad .$$

Rearranging the various random components we get

$$V_*^j = \bar{\alpha} + \bar{\sigma} W_*^j + (1 - \bar{\sigma}) P^j + \ell_*^j \quad , \qquad (33)$$

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1. This problem has been mentioned by McKinnon (1963).

where

$$\ell_*^j = \Gamma^j + W^j\eta^j - P^j\eta^j \quad .$$

(Equation (33) can be compared to (15)).

It is obvious that a specification error will occur if a relationship of the form (15) is used with data which measures the variables in money terms. Unless output prices either vary randomly across industries (in which case the price term could be absorbed into the error structure) or are the same for all industries (which would allow $(1 - \bar{\sigma})P^j$ to be treated as part of the constant) the exclusion of prices from the regression will result in an unaccounted for non-randomness in the quasi-disturbance term.

Although not normally considered to be of particular relevance to cross-sectional studies the Durbin-Watson statistic can, on some occasions, be helpful in identifying specification errors. The relatively low value that resulted when prices were omitted from the estimating equation (ref. Table 1) can be rationalized as resulting from a systematic influence being exerted by the price variable.

TABLE 1[(1)] : ESTIMATES OF CES SIDE RELATION USING RANDOM COEFFICIENTS MODEL WITHOUT PRICE CORRECTION - 35 MANUFACTURING INDUSTRIES

Estimating Equation: $V_*^j = \bar{\alpha} + \bar{\sigma} W_*^j + \ell^j$

$\hat{\bar{\alpha}}$	= 0.6496 (0.112)	$\hat{\phi}_\Gamma$	= 0.1534
$\hat{\bar{\sigma}}$	= 1.187 (0.138)	$\hat{\phi}_\eta$	= 0.0283
d	= 1.409	$\bar{R}^2$	= 0.64

If observations on the price of output for each industry were available the mis-specification could be corrected in the obvious

(1) Standard errors in brackets.

manner. Unfortunately the implementation of such a procedure is precluded because of the conceptual and practical problems associated with the measurement of the "price of a unit of real value added" for each industry.

In order to mitigate the problem to some extent it was decided to split the industries into two groups and to allow for a different "average" price level in each. It was then postulated that the output prices for industries within each group are randomly distributed about the group mean (i.e. $P_i^j = \overline{P}_i + \psi_i^j$ where P_i^j is the price for industry j in group i, $\overline{P}_i$ is the average price for group i and ψ_i^j is the deviation of the industry j price from the group average).

A partition, which differentiates between industries which produce "heavy" durable goods (group 1) and those that produce other (essentially non-durable) goods (group 2), does not appear unreasonable[1] and is generally consistent with the observed pattern of residuals from the initial regression. Using this grouping and the above price specification equation (33) can be written as:[2]

$$V_*^j = \{ \overline{\alpha} + (1 - \overline{\sigma}) \overline{P}_1 \} + \{ (1 - \overline{\sigma}) \overline{P}_\nabla \} \delta_2^j + \overline{\sigma} W_*^j + \ell_{**}^j , \qquad (34)$$

where $\overline{P}_\nabla = \overline{P}_2 - \overline{P}_1$, δ_i^j (i = 1,2) is a dummy variable which takes the value of one if industry j is in group i and zero otherwise, and ℓ_{**}^j is a hetroskedastic error term with

$$\mathrm{Var}\ (\ell_{**}^j) = \emptyset_{\Gamma_*} + \emptyset_\eta (W_*^j)^2 + \emptyset_{\psi_1} (\delta_1^j)^2 + \emptyset_{\psi_2} (\delta_2^j)^2 ,$$

and $\mathrm{Cov}\ (\ell_{**}^j, \ell_{**}^k) = 0$.

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1. The two groups are listed in Appendix 1.
2. For the derivation and full interpretation of this equation see Appendix 2.

Following the general estimating procedure detailed in Section 5 the vector of O.L.S. residuals $\hat{u}$ was obtained for equation (34). This was used in conjunction with the G_* matrix in the optimization problem (29) to obtain estimates of the variances of the random components.[1] These were then utilized to obtain the following G.L.S. estimates of the production parameters.

TABLE 2[(2)] : ESTIMATES OF CES SIDE RELATION USING RANDOM COEFFICIENTS MODEL WITH PRICE CORRECTION - 35 MANUFACTURING INDUSTRIES

Estimating Equation: $V_*^j = \bar{\alpha}_* + (\Delta\bar{\alpha}_*)\,\delta_2^j + \bar{\sigma}W_*^j + \ell_{**}^j$

$\hat{\bar{\alpha}}_*$	= 0.6255 (0.1020)	$\hat{\phi}_{\Gamma_*}$	= 0
$\Delta\hat{\bar{\alpha}}_*$	= -0.1267 (0.0566)	$\hat{\phi}_\eta$	= 0.0273
$\hat{\bar{\sigma}}$	= 1.276 (0.1408)	$\hat{\phi}_{\psi_1}$	= 0.118×10^{-5}
$\bar{R}^2$	= 0.70	$\hat{\phi}_{\psi_2}$	= 0.00283
d	= 1.748		

The overall goodness of fit of the relationship as measured by the $\bar{R}^2$ value is relatively good for cross-sectional data. Some degree of confirmation of the hypothesis that there is a systematic price variation between the two industry groupings is provided by the fact that the coefficient on the dummy variable introduced to allow for such variations (i.e. $\Delta\bar{\alpha}_*$) differs significantly from zero. The average value of the elasticity of substitution

1. The matrix manipulations involved in calculating the G_* matrix were carried out using the SUPERPASSION Program developed at the Harvard Economic Research Project (Timson 1968). The solution to the Q.P. Problem was obtained using the FLEXIPLEX Program (See: Himmelblau 1972) which is based on a flexible tolerance solution algorithm suggested by Paviani and Himmelblau (1968).

2. Standard errors in brackets.

for Australian manufacturing industries is estimated to be 1.276, with the 95% confidence interval extending from .988 to 1.564.[1] This result is considerably higher than those of Tran Van Hoa (1968) and Sampson (1968) whose estimates of the "aggregate" substitution elasticity for Australian Manufacturing were 0.785 and 0.770 respectively. However such discrepancies are common when time series estimates (such as those of Tran Van Hoa and Sampson) are compared with estimates made from cross-sectional data.

8. SUMMARY AND CONCLUSION

This paper suggests that the observed instability in the coefficients estimated for various "aggregate" production models may result from biases introduced by the aggregation procedure used rather than any inherent instability in the underlying production parameters. This could account for results such as those obtained by Zarembka and Chernicoff (1975). These results give estimates of σ using data at two different levels of aggregation. In several cases the estimated value for an industry group as a whole falls outside the values posited for each of the activities that go to make up that industry group. If the aggregate parameter is interpreted as representing the "typical" value for that industry such a result is unacceptable.

The model estimated above uses a random coefficient

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1. On the basis of these results the hypothesis that the production function is of the Cobb-Douglas form (i.e. $\sigma = 1$) cannot be rejected.

specification and geometric aggregates to avoid the aggregation problems and to provide a coherent link between the micro theory and the equation used to estimate the macro parameters. The model allows the estimation of an aggregate substitution elasticity (whose relationship to the various industry elasticities is clearly defined) from cross-sectional data. This data base is more extensive than the time series data that is usually used for aggregate studies. In addition the use of observations at a single point in time avoids the necessity of making arbitrary assumptions about the nature of technological change over time. Variations in efficiency are adequately handled by the random coefficient specification.

The empirical results indicate that the average elasticity of substitution between labour and capital for Australian manufacturing industries (as measured by cross-sectional evidence from 1968-69) is 1.28.

APPENDIX 1

Classification of Industries

Group 1	Group 2
Basic Iron and Steel	Meat Products
Non-Ferrous Metal Basic Products	Milk Products
Fabricated Structural Metal Products	Fruit and Vegetable Products
Wood and Wood Products	Flour Mill and Cereal Food Products
Clay Products	Bread, Cakes and Biscuits
Cement and Concrete Products	Other Food Products
Other Non-Metallic Mineral Products	Textiles, Yarns and Woven Fabrics
Sheet Metal Products	Other Textile Products
Other Fabricated Metal Products	Knitting Mills
Motor Vehicles and Parts	Clothing
Other Transport Equipment	Footwear
Appliances and Electrical Equipment	Furniture and Mattresses
Industrial Machinery and Equipment	Paper and Paper Products
Rubber Products	Printing and Publishing
	Basic Chemicals
	Other Chemical and Related Products
	Glass and Glass Products
	Photographic, Professional and Scientific Equipment
	Leather and Leather Products
	Plastic and Related Products
	Other Manufacturing

APPENDIX 2

Start with the correctly specified "money" version of the random coefficient model as given in equation (32).

i.e. $$V_*^j = (\overline{\alpha} + \pi^j) + (\overline{\sigma} + \eta^j) \; W_*^j + (1 - \overline{\sigma} - \eta^j) \; P^j + \xi^j \qquad (1.1)$$

It is now assumed that the various industries can be dichotomized in such a way that one group has a consistently higher output price than the other. It is further assumed that the output prices for the individual industries in each group vary randomly about the group mean. Thus

$$P^j = \overline{P}_1 + \psi_1^j \text{ if industry j is in group 1,}$$

and $$P^j = \overline{P}_2 + \psi_2^j \text{ if industry j is in group 2,}$$

where ψ_i^j (i = 1, 2) are clasically well-behaved random terms which are uncorrelated with each other and with the other random terms specified in the model.

We can now write (1.1) as

$$V_*^j = (\overline{\alpha} + \pi^j) + (\overline{\sigma} + \eta^j) \; W_*^j + (1 - \overline{\sigma} - \eta^j) \; (\overline{P}_1 + \delta_1^j \; \psi_1^j) + \delta_2^j \; (\overline{P}_\nabla + \psi_2^j) + \xi^j; \qquad (1.2)$$

where δ_i^j (i = 1, 2) is a dummy variable which takes the value one if industry j is in group i and zero otherwise, and $\overline{P}_\nabla = \overline{P}_2 - \overline{P}_1$.

Rearranging (1.2) so as to collect the random terms we get

$$V_*^j = (\bar{\alpha} + (1 - \bar{\sigma})\ \bar{P}_1) + \{(1 - \bar{\sigma})\bar{P}_\nabla\}\ \delta_2^j + \bar{\sigma}W_*^j + [\{\pi^j - \eta^j\ \bar{P}_1 + \xi^j\}$$

$$+ \{\delta_2^j\ (\psi_2^j - \bar{\sigma}\psi_2^j - \eta^j\ \bar{P}_\nabla - \eta^j\ \psi_2^j)\} + \eta^j W_*^j$$

$$+ \{\delta_1^j\ (\psi_1^j - \bar{\sigma}\psi_1^j - \eta^j\psi_1^j\}] \qquad (1.3)$$

Since the covariances between the various random components are assumed to be zero the diagonal elements (K_*^j) in the variance/covariance matrix of the equation error (i.e. the portion of (1.3) in square brackets) can be written as

$$\begin{bmatrix} K_*^1 \\ K_*^2 \\ \\ K_*^N \end{bmatrix} = \begin{bmatrix} 1 & (\delta_2^1)^2 & (W_*^1)^2 & (\delta_1^1)^2 \\ 1 & (\delta_2^2)^2 & (W_*^2)^2 & (\delta_1^2)^2 \\ & & & \\ 1 & (\delta_2^N)^2 & (W_*^N)^2 & (\delta_1^N)^2 \end{bmatrix} \begin{bmatrix} \emptyset_{\Gamma_*} \\ \emptyset_{\psi_2} \\ \emptyset_\eta \\ \emptyset_{\psi_1} \end{bmatrix}, \qquad (1.4)$$

where $\emptyset_{\Gamma_*} = \text{Var}\ (\pi) - (\bar{P}_1)^2\ \text{Var}\ (\eta) + \text{Var}\ (\xi)$,

$\emptyset_{\psi_2} = \text{Var}\ (\psi_2)\ \{1 - (\bar{\sigma})^2 - \text{Var}\ (\eta)\} - (P_\nabla)^2\ \text{Var}\ (\eta)$,

$\emptyset\eta = \text{Var}\ (\eta)$,

and $\emptyset_{\psi_1} = \text{Var}\ (\psi_1)\ \{1 - (\bar{\sigma})^2 - \text{Var}\ (\eta)\}$

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