μ-Dependent model reduction for uncertain discrete-time switched linear systems with average dwell time

μ-Dependent Model Reduction for Uncertain Discrete-Time Switched Linear Systems with Average Dwell Time

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Abstract

In this paper, the model reduction problem for a class of discrete-time polytopic uncertain switched linear systems with average dwell time switching is investigated. The stability criterion for general discrete-time switched systems is first explored, and a *μ-dependent* approach is then introduced for the considered systems to the model reduction solution. A reduced-order model is constructed and its corresponding existence conditions are derived via LMI formulation. The admissible switching signals and the desired reduced model matrices are accordingly obtained from such conditions such that the resulting model error system is robustly exponentially stable and has an exponential H_∞ performance. A numerical example is presented to demonstrate the potential and effectiveness of the developed theoretical results.

Keywords: average dwell time, linear matrix inequalities, model reduction, switched linear systems

1 Introduction

Switched systems have been widely studied in the past decades and many issues have been tackled, see for example, [4, 7, 8, 14, 17]. For a switched system, switching signals are crucial to determine system behavior, which might depend on either time or system state, or both, or other supervisory decision procedures [2, 10, 12, 16, 19]. Usually, the switching in systems or control are classified into autonomous and controlled ones, which result respectively from system itself and the designers' intervention [10, 12].

For the autonomous switched systems, one of the basic problems is to find out less conservative stability conditions, especially considering the switching signals are arbitrary. On this issue, the multiple Lyapunov functions (MLF) approach has been well deemed less conservative in contrast with the global Lyapunov function (GLF) [1]. Although the general MLF idea describes how the stability of switched system is ensured, it is still hard to derive the numerically checkable stability criteria. The switched Lyapunov function (SLF) approach [2], as a special kind of MLF, attracts the poly-quadratic stability idea such that some issues such as stability, control and filtering, etc., are solved for a class of discrete-time nominal or uncertain switched linear systems under arbitrary switching [21, 22, 23].

For controlled switched systems, on the other hand, stability analysis usually needs to specify dwell (or average dwell) time of the switching signal, which thereby can be also viewed as a design problem

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of switching laws and are frequently encountered in the switching control practice [9, 18]. Some special switching signals have been studied in this context such as dwell (average dwell) time switching, hysteresis switching, etc. [12], and nowadays, the average dwell time switching has been recognized to be more flexible and efficient in system stability analysis [6, 10]. Often, the switched systems with dwell (or average dwell) time are also viewed slowly switched system in the literature, and many useful results on stability analysis and control synthesis for the systems have been available within the continuous-time context including both linear case and nonlinear case, see for example, [6, 13, 15]. However, to the best of the authors' knowledge, the relevant problems on model reduction for high-order switched systems with dwell (or average dwell) time switching have not been investigated, with or without uncertainties. Note that with the switching feature, the model reduction results for switched systems can not be simply obtained by means of the existing model simplification theories of general dynamic systems. Note also that both the approaches GLF and SLF will be not quite suitable to analyze the slowly switched systems due to the stricter requirements on the Lyapunov function values at the switching instants.

In this paper, we are interested in the model reduction for a class of polytopic uncertain switched linear discrete-time systems under average dwell time switching. The discrete-time counterpart of the stability result of general switched systems is firstly presented. Then, a reduced-order model for the underlying system is constructed and the corresponding existence conditions are derived via LMI formulation. The obtained conditions are dependent on μ, which is the increasing degree of the Lyapunov-like functions associated to the different subsystem at switching instants. The admissible switching signals and the desired reduced model matrices can be obtained from such conditions for a given decay degree such that the resulting model error system is robustly exponentially stable and achieves an exponential H_∞ performance.

The remainder of the paper is organized as follows. The problem of model reduction for discrete-time uncertain switched linear system with average dwell time switching is formulated in Section 2. In Section 3, the stability result for general discrete-time switched systems, the exponential H_∞ performance analysis for the underlying systems and the corresponding H_∞ reduced-order model solution are developed as the main results of the paper. Section 4 provides an illustrative example and Section 5 concludes the paper.

Notation: The notation used in this paper is fairly standard. The superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{N}$ represents the set of nonnegative integers, the notation $\| \quad \|$ refers to the Euclidean vector norm. $l_2[0,\infty)$ is the space of square summable infinite sequence and for $u = \{u(k)\} \in l_2[0,\infty)$, its norm is given by $\|u\|_2 = \sqrt{\sum_{k=0}^{\infty} |u(k)|^2}$. $\mathcal{C}^1$ denotes the space of continuously differentiable functions, and a function $\beta : [0,\infty) \to [0,\infty)$ is said to be of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, unbounded, and $\beta(0) = 0$. In addition, in symmetric block matrices or long matrix expressions, we use * as an ellipsis for the terms that are introduced by symmetry and $diag\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $P > 0$ (≥ 0) means P is real symmetric and positive (semi-positive) definite. I and 0 represent respectively, identity matrix and zero matrix.

2 Problem Formulation and Preliminaries

Consider a class of uncertain discrete-time switched linear systems given by

$$
\begin{aligned}
x(k+1) &= A_{\sigma(k)}(\lambda)x(k) + B_{\sigma(k)}(\lambda)u(k) \\
y(k) &= C_{\sigma(k)}(\lambda)x(k) + D_{\sigma(k)}(\lambda)u(k)
\end{aligned} \tag{1}
$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^l$ is the input vector which belongs to $l_2[0, \infty)$, $y(k) \in \mathbb{R}^m$ is the measurement output vector. $\sigma(k)$ is a piecewise constant function of time, called a switching signal, which takes its values in the finite set $\mathcal{I} = \{1, \ldots, N\}$, $N > 1$ is the number of subsystems. At an arbitrary discrete time k, $\sigma(k)$, denoted by σ for simplicity, is dependent on k or $x(k)$, or both, or other switching rules. As in [2], we assume that the sequence of subsystems in switching signal σ is unknown *a priori*, but its instantaneous value is available in real time. Meanwhile, for the switching times sequence $k_0 < k_1 < k_2 < ...$ of switching signal σ, the holding time between $[k_l, k_{l+1}]$ is called the dwell time of the currently engaged subsystem, where $l \in \mathbb{N}$. In addition, when $\sigma(k) = i \in \mathcal{I}$, the matrices $(A_i(\lambda),\ B_i(\lambda),\ C_i(\lambda),\ D_i(\lambda))$ denote the ith subsystem and λ is a varying uncertain parameter. It is assumed that $(A_i(\lambda),\ B_i(\lambda),\ C_i(\lambda),$ $D_i(\lambda)) \in \Re_i$, where $\Re_i$ is a given convex bounded polyhedral domain described by s vertices in the ith subsystem.

$$\begin{aligned} \Re_i &\triangleq \{[A_i(\lambda),\ B_i(\lambda),\ C_i(\lambda),\ D_i(\lambda)] \\ &= \sum\nolimits_{m=1}^{s} \lambda_m [A_{i,m},\ B_{i,m},\ C_{i,m},\ D_{i,m}];\ \sum\nolimits_{m=1}^{s} \lambda_m = 1,\ \lambda_m \geq 0,\ i \in \mathcal{I}.\Big\} \end{aligned} \tag{2}$$

Remark 1 *The parameters and structure of the uncertainties in practice are usually the same throughout either the multi-models or switched control systems [3, 11], thus we assume both the number of vertices and uncertain parameter λ_m in each subsystem to be equal here (not $\lambda_{i,m}$) without loss of generality.*

For switching signal $\sigma(k)$, we revisit the average dwell time property in the following definition.

Definition 1 *[6] For switching signal and any $k_v > k_s > k_0$, let $N_{\sigma(k)}(k_s, k_v)$ be the switching numbers of $\sigma(k)$ over the interval $[k_s, k_v]$. If for any given $N_0 > 0$, $\tau_a > 0$, we have $N_{\sigma(k)}(k_s, k_v) \leq N_0 + (k_v - k_s)/\tau_a$, then τ_a and N_0 are called average dwell time and the chatter bound, respectively.*

Remark 2 *By average dwell time switching, we mean a class of switching signals satisfying that the average time interval between consecutive switchings is at least τ_a. Then, in the analysis and synthesis for such systems, a basic problem is to specify the admissible τ_a and herewith the admissible switching signals.*

To present the main objective of this paper more clearly, we also introduce the following definitions of the uncertain switched linear systems (1), which will be essential for the later development.

Definition 2 *The equilibrium $x = 0$ of system (1) is robustly exponentially stable under switching signal $\sigma(k)$ if for all admissible λ, there exist constants $K > 0, 0 < \delta < 1$ such that the solution $x(k)$ of the system satisfies $||x(k)|| \leq K\delta^{(k-k_0)}||x(k_0)||, \forall k \geq k_0$.*

Remark 3 *For switched systems under the dwell (or average dwell) time switching, the Lyapunov function values at switching instants are often considered to increase μ times ($\mu > 1$) to reduce the conservatism in system analysis and synthesis, which will lead to the normal input attenuation performance is hard to compute or check, even in linear setting. Therefore, we adopt the following exponential H_∞ performance criterion, which can be referred to [5] and [19] for more details, to evaluate the underlying system while obtaining the expected exponential stability.*

Definition 3 *Given scalars $\gamma > 0$ and $0 < \alpha < 1$, system (1) is said to be robustly exponentially stable with an exponential H_∞ performance γ if it is robustly exponentially stable and under zero initial condition, $\sum_{k=k_0}^{\infty}(1-\alpha)^k y^T(k)y(k) \leq \sum_{k=k_0}^{\infty} \gamma^2 u^T(k)u(k)$ for all nonzero $u(k) \in l_2[0, \infty)$.*

Remark 4 *The concept exponential H_∞ performance used here means the noise attenuation performance is different when the decay degree of the system is different. Note that the scalar α symbolizes the decreasing rate of the Lyapunov-like functions within each subsystem. If $\alpha \to 0$, the evaluated performance index will approach the normal H_∞ performance for the whole time domain.*

Here, we are interested in constructing a reduced-order switched system with the following form

$$\begin{aligned} \hat{x}(k+1) &= \hat{A}_i(\lambda)\hat{x}(k) + \hat{B}_i(\lambda)u(k) \\ \hat{y}(k) &= \hat{C}_i(\lambda)\hat{x}(k) + \hat{D}_i(\lambda)u(k) \end{aligned} \tag{3}$$

where $\hat{x}(k) \in \mathbb{R}^v$ is the state vector of the reduced-order system with $v < n$, and $(\hat{A}_i(\lambda), \hat{B}_i(\lambda), \hat{C}_i(\lambda), \hat{D}_i(\lambda), i \in \mathcal{I})$ are matrices with compatible dimensions to be determined, and belong to a convex polytope with the same structure as described in (2), namely,

$$[\hat{A}_i(\lambda), \hat{B}_i(\lambda), \hat{C}_i(\lambda), \hat{D}_i(\lambda)] = \sum_{m=1}^{s} \lambda_m [\hat{A}_{i,m},\ \hat{B}_{i,m},\ \hat{C}_{i,m}, \hat{D}_{i,m}] \tag{4}$$

In addition, the reduced-order model with the above structure will be switched homogeneously by the switching signal σ in system (1). Then, augmenting the model of system (1) to include the states of system (3), we can obtain the following error system

$$\begin{aligned} \xi(k+1) &= \bar{A}_i(\lambda)\xi(k) + \bar{B}_i(\lambda)u(k) \\ e(k) &= \bar{C}_i(\lambda)\xi(k) + \bar{D}_i(\lambda)u(k) \end{aligned} \tag{5}$$

where $e(k) = y(k) - \hat{y}(k)$ and

$$\begin{aligned} \xi(k) &= \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix},\ \bar{A}_i(\lambda) = \begin{bmatrix} A_i(\lambda) & 0 \\ 0 & \hat{A}_i(\lambda) \end{bmatrix},\ \bar{B}_i(\lambda) = \begin{bmatrix} B_i(\lambda) \\ \hat{B}_i(\lambda) \end{bmatrix}, \\ \bar{C}_i(\lambda) &= \begin{bmatrix} C_i(\lambda) & -\hat{C}_i(\lambda) \end{bmatrix},\ \bar{D}_i(\lambda) = D_i(\lambda) - \hat{D}_i(\lambda) \end{aligned}$$

Our objective in this paper is to design a reduced-order system model of form (3) and find admissible switching signals such that the resulting model error system (5) is robustly exponentially stable and guarantees an exponential H_∞ performance index.

Remark 5 *It is worth mentioning that if we assume $[\hat{A}_{i,m}, \hat{B}_{i,m}, \hat{C}_{i,m}, \hat{D}_{i,m}] \triangleq [\hat{A}_{i,l}, \hat{B}_{i,l}, \hat{C}_{i,l}, \hat{D}_{i,l}] \triangleq \ldots \triangleq [\hat{A}_{i,n}, \hat{B}_{i,n}, \hat{C}_{i,n}, \hat{D}_{i,n}]$ (where $1 \le l, m, n \le s$, i.e. the number of vertices is decreased) or select $[\hat{A}_{i,m},\ \hat{B}_{i,m},\ \hat{C}_{i,m},\ \hat{D}_{i,m}] \triangleq [\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i]$, or further select $[\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i] \triangleq [\hat{A}, \hat{B}, \hat{C}, \hat{D}]$ in (4), then we will obtain the corresponding reduced-order models as different special cases of our desired result.*

Before ending this section, we present the following lemmas which will play an important role in our further derivation.

Lemma 1 *[20] Consider the discrete-time switched system $x_{k+1} = f_{\sigma(k)}(x_k), \sigma(k) \in \mathcal{I}$ and let $0 < \alpha < 1, \mu > 1$ be given constants. Suppose that there exists $\mathcal{C}^1$ functions $V_{\sigma(k)} : \mathbb{R}^n \to \mathbb{R}$, $\sigma(k) \in \mathcal{I}$, and two class $\mathcal{K}_\infty$ functions β_1 and β_2 such that $\forall \sigma(k) = i \in \mathcal{I}$,*

$$\beta_1(|x|) \le V_i(x) \le \beta_2(|x|) \tag{6}$$

$$\Delta V_i(x) \le -\alpha V_i(x) \tag{7}$$

and $\forall(\sigma(k_l)=i,\sigma(k_l-1)=j)\in\mathcal{I}\times\mathcal{I}\ ,\ i\neq j,$

$$V_i(x_{k_l})\leq \mu V_j(x_{k_l}) \tag{8}$$

then the system is globally asymptotically stable for any switching signals with the average dwell time

$$\tau_a\geq\tau_a^*=-\frac{\ln\mu}{\ln(1-\alpha)}. \tag{9}$$

Remark 6 *The proof of Lemma 1 can be obtained following the similar lines in section 3.2 of [10]. Note that it can be seen from Lemma 1 that when we increase the value of* μ*, the existence likelihood of the multiple Lyapunov function for the system stability will be increased, which means the stability of system can be ensured at the expense of increasing* μ*. In other words, for a given* α*, the the system stability will be directly dependent on* μ*.*

Lemma 2 *[20] Consider the uncertain switched linear system (5) and let* $\alpha>0$*,* $\gamma>0$ *and* $\mu>1$ *be given constants. If there exist matrix functions* $P_i(\lambda)>0$*,* $\forall i\in\mathcal{I}$ *such that*

$$\begin{bmatrix} -P_i(\lambda) & 0 & P_i(\lambda)\bar{A}_i(\lambda) & P_i(\lambda)\bar{B}_i(\lambda)\\ * & -I & \bar{C}_i(\lambda) & \bar{D}_i(\lambda)\\ * & * & -(1-\alpha)P_i(\lambda) & 0\\ * & * & * & -\gamma^2 I\end{bmatrix} < 0 \tag{10}$$

$$P_i(\lambda)-\mu P_j(\lambda) \leq 0 \tag{11}$$

then the model error system (5) is robustly exponentially stable and has an exponential H_∞ *performance for all admissible uncertainties satisfying (2) and any switching signals with the average dwell time satisfying (9).*

Remark 7 *Lemmas 2 gives the exponential* H_∞ *performance criterion for the uncertain switched linear system in discrete-time context. Its proof can be readily obtained by Lemma 1 and constructing a parameter dependent multiple Lyapunov function. It is easily seen that the performance index* γ *will depend on* μ*, the increasing degree of the Lyapunov-like functions at switching instants. In what follows, we will give the* μ*-dependent reduced model solution for the underlying systems.*

3 Exponential H_∞ Model Reduction

The following Theorem presents sufficient conditions for the existence of an exponential H_∞ reduced-order model in the form of (3).

Theorem 1 *Consider the uncertain switched linear system (1) and let* $\alpha>0$*,* $\gamma>0$ *and* $\mu>1$ *be given constants. If there exist matrices* $\bar{P}_{1i,m}>0$*,* $\bar{P}_{3i,m}>0$ *and matrices* $\bar{P}_{2i,m}$*,* $R_{i,m}$*,* $S_{i,m}$*,* T_i*,* $\check{A}_{i,m}$*,* $\check{B}_{i,m}$*,* $\check{C}_{i,m}$*,* $\check{D}_{i,m}$*,* $\forall i\in\mathcal{I}$*,* $1\leq m\leq s$ *such that*

$$\Xi^i_{m,n}+\Xi^i_{n,m}<0,\ \ (1\leq m\leq n\leq s) \tag{12}$$

$$\begin{bmatrix} \bar{P}_{1i,m}-\mu R_{i,m}^T-\mu R_{i,m} & \bar{P}_{2i,m}-\mu S_{i,m}-\mu ET_i & R_{i,m}^T & ET_j\\ * & \bar{P}_{3i,m}-\mu T_i^T-\mu T_i & S_{i,m}^T & T_j\\ * & * & -\mu^{-1}\bar{P}_{1j,m} & -\mu^{-1}\bar{P}_{2j,m}\\ * & * & * & -\mu^{-1}\bar{P}_{3j,m}\end{bmatrix} \leq 0, i\neq j \tag{13}$$

where,

$$\Xi_{m,n}^{i} \triangleq \begin{bmatrix} \Lambda_{11,m}^{i} & \Lambda_{12,m}^{i} & 0 & R_{i,m}^{T}A_{i,n} & E\check{A}_{i,m} & R_{i,m}^{T}B_{i,n}+E\check{B}_{i,m} \\ * & \Lambda_{22,m}^{i} & 0 & S_{i,m}^{T}A_{i,n} & \check{A}_{i,m} & S_{i,m}^{T}B_{i,n}+\check{B}_{i,m} \\ * & * & I & C_{i,m} & -\check{C}_{i,m} & D_{i,m}-\check{D}_{i,m} \\ * & * & * & -(1-\alpha)\bar{P}_{1i,m} & -(1-\alpha)\bar{P}_{2i,m} & 0 \\ * & * & * & * & -(1-\alpha)\bar{P}_{3i,m} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Lambda_{11,m}^{i} \triangleq \bar{P}_{1i,m} - R_{i,m}^{T} - R_{i,m}, \quad \Lambda_{12,m}^{i} \triangleq \bar{P}_{2i,m} - S_{i,m} - ET_i,$$

$$\Lambda_{22,m}^{i} \triangleq \bar{P}_{3i,m} - T_i^{T} - T_i, \; E \triangleq \begin{bmatrix} I & 0 \end{bmatrix}^{T}, I \in \mathbb{R}^{v}$$

then, there exist a reduced-order model system such that the corresponding model error system (5) is robustly exponentially stable with an exponential H_∞ performance for all admissible uncertainties satisfying (2) and any switching signals with the average dwell time satisfying (9). Moreover, if the LMIs (12)-(13) have a feasible solution, then an admissible reduced-order model in the form (3) can be given by

$$\begin{bmatrix} \hat{A}_{i,m} & \hat{B}_{i,m} \\ \hat{C}_{i,m} & \hat{D}_{i,m} \end{bmatrix} \triangleq \begin{bmatrix} T_i^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \check{A}_{i,m} & \check{B}_{i,m} \\ \check{C}_{i,m} & \check{D}_{i,m} \end{bmatrix} \tag{14}$$

Proof. By Lemma 2, system (5) is robustly exponentially stable with a prescribed exponential H_∞ noise-attenuation level bound γ if the following inequalities hold

$$\begin{bmatrix} -P_i(\lambda) & 0 & P_i(\lambda)\bar{A}_i(\lambda) & P_i(\lambda)\bar{B}_i(\lambda) \\ * & -I & \bar{C}_i(\lambda) & \bar{D}_i(\lambda) \\ * & * & -(1-\alpha)P_i(\lambda) & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{15}$$

$$P_i(\lambda) - \mu P_j(\lambda) \le 0 \tag{16}$$

where, $\bar{A}_i(\lambda)$, $\bar{B}_i(\lambda)$, $\bar{C}_i(\lambda)$, $\bar{D}_i(\lambda)$ are described in (5).

Then, consider an arbitrary matrix function $G_i(\lambda)$, $\forall i \in \mathcal{I}$ with compatible dimension, we have the fact

$$\begin{aligned} (P_i(\lambda) - G_i(\lambda))^T P_i^{-1}(\lambda)(P_i(\lambda) - G_i(\lambda) &\ge 0 \\ (P_j(\lambda) - G_i(\lambda))^T P_j^{-1}(\lambda)(P_j(\lambda) - G_i(\lambda) &\ge 0 \end{aligned}$$

thus we have

$$\begin{aligned} P_i(\lambda) - G_i(\lambda) - G_i^T(\lambda) &\ge -G_i^T(\lambda)P_i^{-1}(\lambda)G_i(\lambda) \\ P_j(\lambda) - G_i(\lambda) - G_i^T(\lambda) &\ge -G_i^T(\lambda)P_j^{-1}(\lambda)G_i(\lambda) \end{aligned}$$

therefore, if the following inequalities hold

$$\begin{bmatrix} P_i(\lambda) - G_i(\lambda) - G_i^T(\lambda) & 0 & G_i^T(\lambda)\bar{A}_i(\lambda) & G_i^T(\lambda)\bar{B}_i(\lambda) \\ * & -I & \bar{C}_i(\lambda) & \bar{D}_i(\lambda) \\ * & * & -(1-\alpha)P_i(\lambda) & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{17}$$

$$P_i(\lambda) - \mu\left[G_i(\lambda) + G_i^T(\lambda) - G_i^T(\lambda)P_j^{-1}(\lambda)G_i(\lambda)\right] \le 0 \tag{18}$$

then (18) implies (16). Also, from (17), we can obtain

$$\begin{bmatrix} -G_i^T(\lambda)P_i^{-1}(\lambda)G_i(\lambda) & 0 & G_i^T(\lambda)\bar{A}_i(\lambda) & G_i^T(\lambda)\bar{B}_i(\lambda) \\ * & -I & \bar{C}_i(\lambda) & \bar{D}_i(\lambda) \\ * & * & -(1-\alpha)P_i(\lambda) & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0$$

Performing a congruence transformation to above formula via $diag\{G_i^{-1}(\lambda)P_i(\lambda), I, I, I\}$ yields (15) (note that $G_i(\lambda)$ will be invertible if it satisfies (17)). In addition, by Schur complement, (18) is equivalent to

$$\begin{bmatrix} P_j(\lambda) - \mu G_i(\lambda) - \mu G_i^T(\lambda) & G_i^T(\lambda) \\ * & -\mu^{-1}P_j(\lambda) \end{bmatrix} \leq 0 \tag{19}$$

Now, let us show that conditions (12) and (13) ensure respectively that (17) and (19) are satisfied. Firstly, if (13) holds, we have

$$\begin{bmatrix} \bar{P}_{1i}(\lambda) - \mu R_i^T(\lambda) - \mu R_i(\lambda) & \bar{P}_{2i}(\lambda) - \mu S_i(\lambda) - \mu E T_i & R_i^T(\lambda) & ET_j \\ * & \bar{P}_{3i}(\lambda) - \mu T_i^T - \mu T_i & S_i^T(\lambda) & T_j \\ * & * & -\mu^{-1}\bar{P}_{1j}(\lambda) & -\mu^{-1}\bar{P}_{2j}(\lambda) \\ * & * & * & -\mu^{-1}\bar{P}_{3j}(\lambda) \end{bmatrix} \leq 0 \tag{20}$$

Also, if (12) hold, we have

$$\Xi^i(\lambda) = \sum\nolimits_{m=1}^{s}\sum\nolimits_{n=1}^{s}\lambda_m\lambda_n\Xi_{m,n}^i = \sum\nolimits_{m=1}^{s}\lambda_m^2\Xi_{m,m}^i + \sum\nolimits_{m=1}^{s-1}\sum\nolimits_{n=m+1}^{s}\lambda_m\lambda_n(\Xi_{m,n}^i + \Xi_{n,m}^i) < 0$$

i.e.

$$\begin{bmatrix} \Lambda_{11}^i(\lambda) & \Lambda_{12}^i(\lambda) & 0 & R_i^T(\lambda)A_i(\lambda) & E\check{A}_i(\lambda) & R_i^T(\lambda)B_i(\lambda) + E\check{B}_i(\lambda) \\ * & \Lambda_{22}^i(\lambda) & 0 & S_i^T(\lambda)A_i(\lambda) & \check{A}_i(\lambda) & S_i^T(\lambda)B_i(\lambda) + \check{B}_i(\lambda) \\ * & * & I & C_i(\lambda) & -\check{C}_i(\lambda) & D_i(\lambda) - \check{D}_i(\lambda) \\ * & * & * & -(1-\alpha)\bar{P}_{1i}(\lambda) & -(1-\alpha)\bar{P}_{2i}(\lambda) & 0 \\ * & * & * & * & -(1-\alpha)\bar{P}_{3i}(\lambda) & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{21}$$

where,

$$\Lambda_{11}^i(\lambda) \triangleq \bar{P}_i(\lambda) - R_i^T(\lambda) - R_i(\lambda),\ \Lambda_{12}^i(\lambda) \triangleq \bar{P}_{2i}(\lambda) - S_i(\lambda) - ET_i,\ \Lambda_{22}^i(\lambda) \triangleq \bar{P}_{3i}(\lambda) - T_i^T - T_i$$

Note that from (21), we also know that

$$\bar{P}_{3i}(\lambda) - T_i^T - T_i < 0$$

thus we can infer that $T_i^T + T_i > 0$, which implies T_i is nonsingular. Then, one can always find nonsingular matrices G_{3i} and G_4 satisfying $T_i = G_4^T G_{3i}^{-1} G_4, \forall i \in \mathcal{I}$. Now, introduce the following matrix variables related to G_{3i} and G_4:

$$V_i \triangleq \begin{bmatrix} I & 0 \\ 0 & G_{3i}^{-1}G_4 \end{bmatrix},\ G_i(\lambda) \triangleq \begin{bmatrix} R_i(\lambda) & S_i(\lambda)G_4^{-1}G_{3i} \\ G_4E^T & G_{3i} \end{bmatrix}$$

Then, by further performing a congruence transformation to (21) and (20) via $diag\{V_i^{-1}, I, V_i^{-1}, I\}$ and $diag\{V_i^{-1}, V_j^{-1}\}$, respectively, and setting matrix functions

$$P_i(\lambda) \triangleq V_i^{-T}\bar{P}_i(\lambda)V_i^{-1} = V_i^{-T}\begin{bmatrix} \bar{P}_{1i}(\lambda) & \bar{P}_{2i}(\lambda) \\ * & \bar{P}_{3i}(\lambda) \end{bmatrix} V_i^{-1}$$

$$\begin{bmatrix} \hat{A}_i(\lambda) & \hat{B}_i(\lambda) \\ \hat{C}_i(\lambda) & \hat{D}_i(\lambda) \end{bmatrix} \triangleq \begin{bmatrix} G_4^{-T} & 0 \\ * & I \end{bmatrix}\begin{bmatrix} \check{A}_i(\lambda) & \check{B}_i(\lambda) \\ \check{C}_i(\lambda) & \check{D}_i(\lambda) \end{bmatrix}\begin{bmatrix} G_4^{-1}G_{3i} & 0 \\ * & I \end{bmatrix} \tag{22}$$

we can obtain that (17) and (19).

Meanwhile, from (22) we know an admissible reduced-order model for the underlying system can be given by

$$\hat{A}_{i,m} = G_4^{-T}\check{A}_{i,m}G_4^{-1}G_{3i},\ \hat{B}_{i,m} = G_4^{-T}\check{B}_{i,m},\ \hat{C}_{i,m} = \check{C}_{i,m}G_4^{-1}G_{3i},\ \hat{D}_{i,m} = \check{D}_{i,m} \tag{23}$$

Now, denote the reduced-order model transfer function from $u(k)$ to $e(k)$ by

$$T(\mathbf{z}) = C_{Fi}(\mathbf{z}I - A_{Fi})^{-1}B_{Fi} + D_{Fi}$$

By substituting the matrices ($\hat{A}_{i,m}$, $\hat{B}_{i,m}$, $\hat{C}_{i,m}$, $\hat{D}_{i,m}$) in (23) and considering $T_i = G_4^T G_{3i}^{-1} G_4$, we have

$$\begin{aligned} T(\mathbf{z}) &= \hat{C}_{i,m}G_4^{-1}G_{3i}(\mathbf{z}I - G_4^{-T}\hat{A}_{i,m}G_4^{-1}G_{3i})^{-1}G_4^{-T}\hat{B}_{i,m} + \hat{D}_{i,m} \\ &= \hat{C}_{i,m}(\mathbf{z}I - T_i^{-1}\hat{A}_{i,m})^{-1}T_i^{-1}\hat{B}_{i,m} + \hat{D}_{i,m} \end{aligned}$$

which implies that an admissible reduced-order model can be given by (14), this completes the proof. □

Remark 8 *Note that the matrices of the desired reduced-order model can be solved from the LMIs in (12)-(14), as well as the switching signals can be found from (9), (12)-(13). Then, the original system and the obtained reduced- order model will be switched by the switching signals satisfying (9).*

Remark 9 *From (12)-(14), it can be obviously seen that the reduced model matrices will be indirectly dependent on μ, which resembles, to some extent, the **delay-dependent** issues in time-delay system to determine delay-dependent controller or filter, etc. Therefore, a new concept **μ-dependent** approach for the underlying system is introduced here, and the results developed with this concept will present less conservatism compared to the existing results that one can refer to as "**μ-independent**", such as those based on the GLF or the SLF approaches (the switching signal is arbitrary therein).*

Remark 10 *In addition, conditions (12)-(13) are formulated in terms of a set of LMIs, which are not only over the matrix variables but also the scalar γ^2. Therefore, the scalar γ can be optimized by a **μ-dependent** convex optimization problem for a fixed system decay degree as follows.*

Problem 1:

$$\begin{gathered} \textit{Min } \delta \text{ subject to (12)-(13)}, \forall i \in \mathcal{I},\ 1 \leq m \leq s \text{ with } \delta = \gamma^2 \\ \text{over } R_{i,m},\ S_{i,m},\ T_i,\ \check{A}_{i,m},\ \check{B}_{i,m},\ \check{C}_{i,m},\ \check{D}_{i,m},\ \bar{P}_{1i,m},\ \bar{P}_{2i,m},\ \bar{P}_{3i,m}. \end{gathered}$$

The minimum exponential noise attenuation level bound is then obtained by setting $\gamma = \sqrt{\delta^*}$,where δ^* is the optimal value of δ, and the corresponding reduced system matrices are given by (14).

4 Numerical Example

Consider the following uncertain discrete-time switched linear systems consisting of two uncertain subsystems, where there are two vertices in subsystem 1:

$$A_{11} = \begin{bmatrix} 0.195 & 0.33 & -0.195 & 0.120 \\ 0.075 & -0.045 & 0.285 & -0.090 \\ -0.105 & -0.075 & -0.06 & -0.180 \\ -0.255 & 0.315 & 0.045 & 0.420 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 0.285 \\ -0.270 \\ 0.240 \\ -0.120 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} 1.800 & 0.750 & 0.195 & 0.091 \end{bmatrix}, \quad D_{11} = 0.120$$

$$A_{12} = \begin{bmatrix} -0.195 & 0.33 & -0.195 & 0.120 \\ 0.075 & -0.045 & 0.285 & -0.090 \\ -0.105 & -0.075 & -0.06 & -0.180 \\ -0.255 & 0.315 & 0.045 & -0.420 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0.285 \\ 0.270 \\ -0.240 \\ 0.120 \end{bmatrix},$$

$$C_{12} = \begin{bmatrix} 1.800 & -0.750 & 0.195 & -0.091 \end{bmatrix}, \quad D_{12} = 0.135$$

and the two vertices in the subsystem 2:

$$A_{21} = \begin{bmatrix} 0.165 & 0.330 & -0.195 & 0.120 \\ 0.075 & -0.045 & 0.225 & -0.090 \\ -0.105 & -0.045 & -0.06 & -0.180 \\ -0.255 & 0.315 & 0.045 & 0.300 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 0.345 \\ -0.195 \\ 0.240 \\ -0.060 \end{bmatrix},$$

$$C_{21} = \begin{bmatrix} 1.800 & 0.750 & 0.195 & -0.061 \end{bmatrix}, \quad D_{21} = -0.105$$

$$A_{22} = \begin{bmatrix} -0.165 & 0.330 & -0.195 & 0.120 \\ 0.075 & -0.045 & 0.225 & -0.090 \\ -0.105 & -0.045 & -0.06 & -0.180 \\ -0.255 & 0.315 & 0.045 & -0.300 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0.345 \\ 0.195 \\ -0.240 \\ 0.060 \end{bmatrix},$$

$$C_{22} = \begin{bmatrix} 1.800 & -0.750 & 0.195 & 0.061 \end{bmatrix}, \quad D_{22} = -0.135$$

Our purpose is to find a reduced-order model in the form of (5) and the admissible switching signals for the above uncertain switched system such that the resulted model error system is robustly exponentially stable with a *μ-dependent* exponential H_∞ performance, for a given decay degree α. By solving *Problem 1*, we can obtain the different optimal γ^* for different μ as shown in Table 1.

μ	1.01	1.05	1.10	1.15	1.20
τ_a^*	9.94	48.76	95.26	139.69	182.23
γ^*	0.1961	0.1604	0.1416	0.1305	0.1228

a) $\alpha = 0.001$

μ	1.01	1.05	1.10	1.15	1.20
τ_a^*	1.98	9.73	19.01	27.88	36.37
γ^*	0.1974	0.1614	0.1425	0.1313	0.1235

b) $\alpha = 0.005$

μ	1.01	1.05	1.10	1.15	1.20
τ_a^*	0.1940	0.9512	1.8581	2.7248	3.5545
γ^*	0.2126	0.1737	0.1533	0.1413	0.1330

c) $\alpha = 0.05$

μ	1.01	1.05	1.10	1.15	1.20
τ_a^*	0.0144	0.0704	0.1375	0.2016	0.2630
γ^*	0.5786	0.4836	0.4288	0.3950	0.3727

d) $\alpha = 0.5$

Table 1 *μ-dependent* optimal γ^* for given different α

Obviously, it can be seen from Table 1 that the obtained exponential H_∞ performance γ^* of the error system is dependent on μ (it is straightforward from (9) that the average dwell time also depends on μ). Note that although the larger μ corresponds to the smaller obtained γ^*, it will be at the expense of longer average dwell time in the system yet.

In addition, by giving $\alpha = 0.05$, the corresponding *μ-dependent* reduced-order system can also be solved by *Problem 1*, e.g. for $\mu = 1.2$, the $2nd$-order reduced system model for the underlying system are obtained with the following matrices:

$$\begin{bmatrix} \hat{A}_{11} & \hat{B}_{11} \\ \hat{C}_{11} & \hat{D}_{11} \end{bmatrix} = \left[\begin{array}{cc|c} 0.1148 & 0.5127 & -0.2877 \\ 0.0998 & -0.2377 & 0.2622 \\ \hline -1.8747 & -0.6980 & 0.1091 \end{array}\right], \quad \begin{bmatrix} \hat{A}_{12} & \hat{B}_{12} \\ \hat{C}_{12} & \hat{D}_{12} \end{bmatrix} = \left[\begin{array}{cc|c} -0.0919 & 0.4599 & -0.2816 \\ -0.0174 & -0.2403 & -0.2659 \\ \hline -1.7697 & 0.8644 & 0.1427 \end{array}\right]$$

$$\begin{bmatrix} \hat{A}_{21} & \hat{B}_{21} \\ \hat{C}_{21} & \hat{D}_{21} \end{bmatrix} = \left[\begin{array}{cc|c} 0.1227 & 0.4567 & -0.3450 \\ 0.1225 & -0.1842 & 0.1998 \\ \hline -1.7770 & -0.5105 & -0.1038 \end{array}\right], \quad \begin{bmatrix} \hat{A}_{22} & \hat{B}_{22} \\ \hat{C}_{22} & \hat{D}_{22} \end{bmatrix} = \left[\begin{array}{cc|c} -0.0715 & 0.4174 & -0.3495 \\ -0.0138 & -0.1743 & -0.1996 \\ \hline -1.8012 & 0.8707 & -0.1262 \end{array}\right]$$

Then, consider the input signal $u(k) = 0.8\exp(-0.4k)$, Figure 1 and 3 show the output trajectories of the original system and $2nd$-order reduced model by randomly giving different uncertain parameters λ in (2), and Figure 2 and 4 present the output errors between original system and the reduced-order system. It can be observed from simulation curves that the obtained reduced model approximate the original system very well against varying parameter uncertainties under the corresponding average dwell time switching signal ($\tau_a = 4$ for both σ_1 and σ_2).

5 Conclusions

The problem of exponential H_∞ model reduction for a class of discrete-time uncertain switched linear system with average dwell time switching is investigated in this paper. Firstly, the stability result for general discrete-time switched systems is presented and a *μ-dependent* approach is introduced for the considered systems to the model reduction solution. Then, a reduced-order model is constructed and the corresponding LMI-based existence conditions are derived. The admissible switching signals and reduced-order model are consequently obtained from such conditions for a given decay degree such that the resulting model error system is robustly exponentially stable and has an exponential H_∞ performance. A numerical example is presented to demonstrate the applicability and effectiveness of the developed theoretical results.

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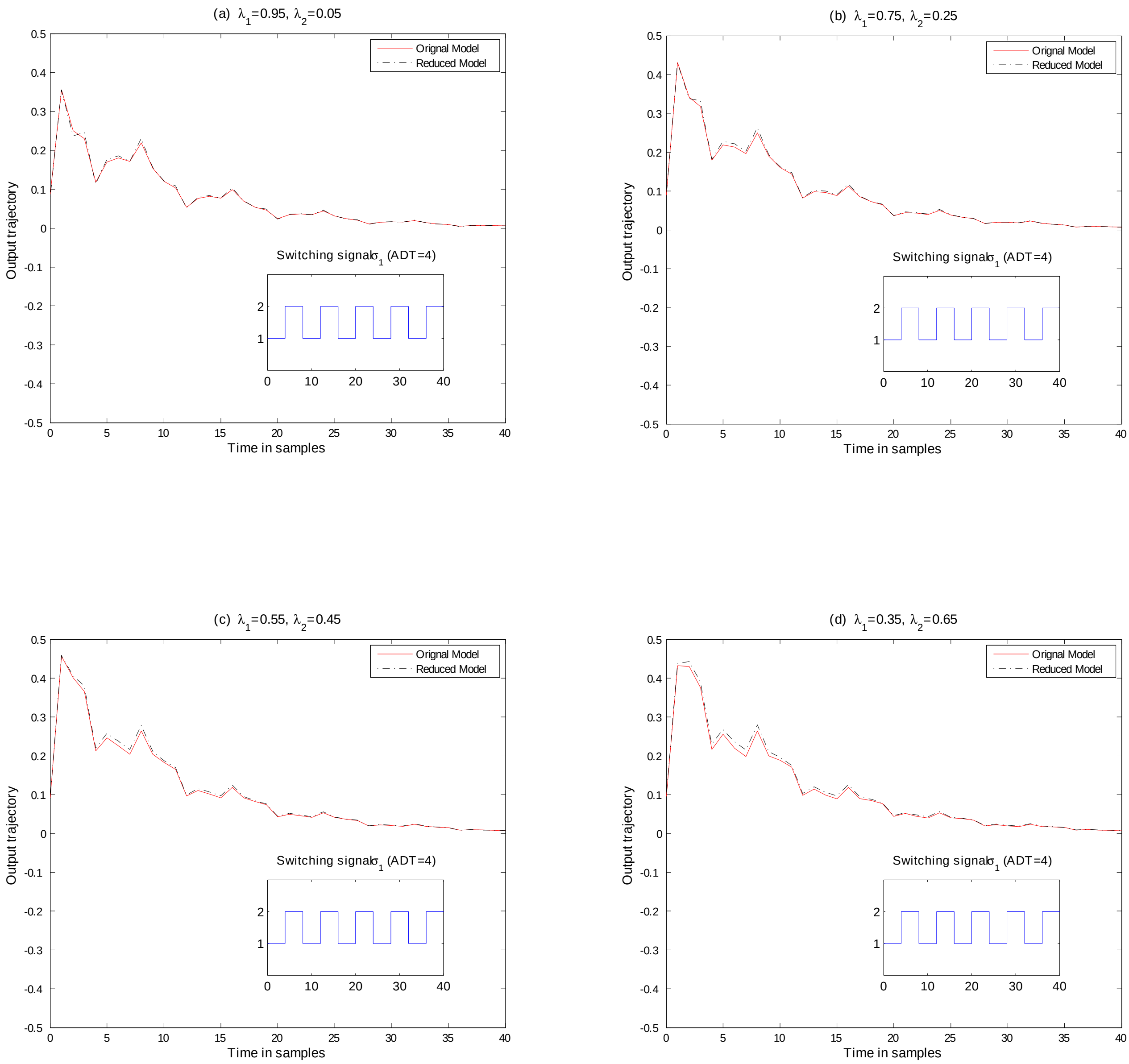

Figure 1 Output trajectories of original system and reduced model under switching signal σ_1

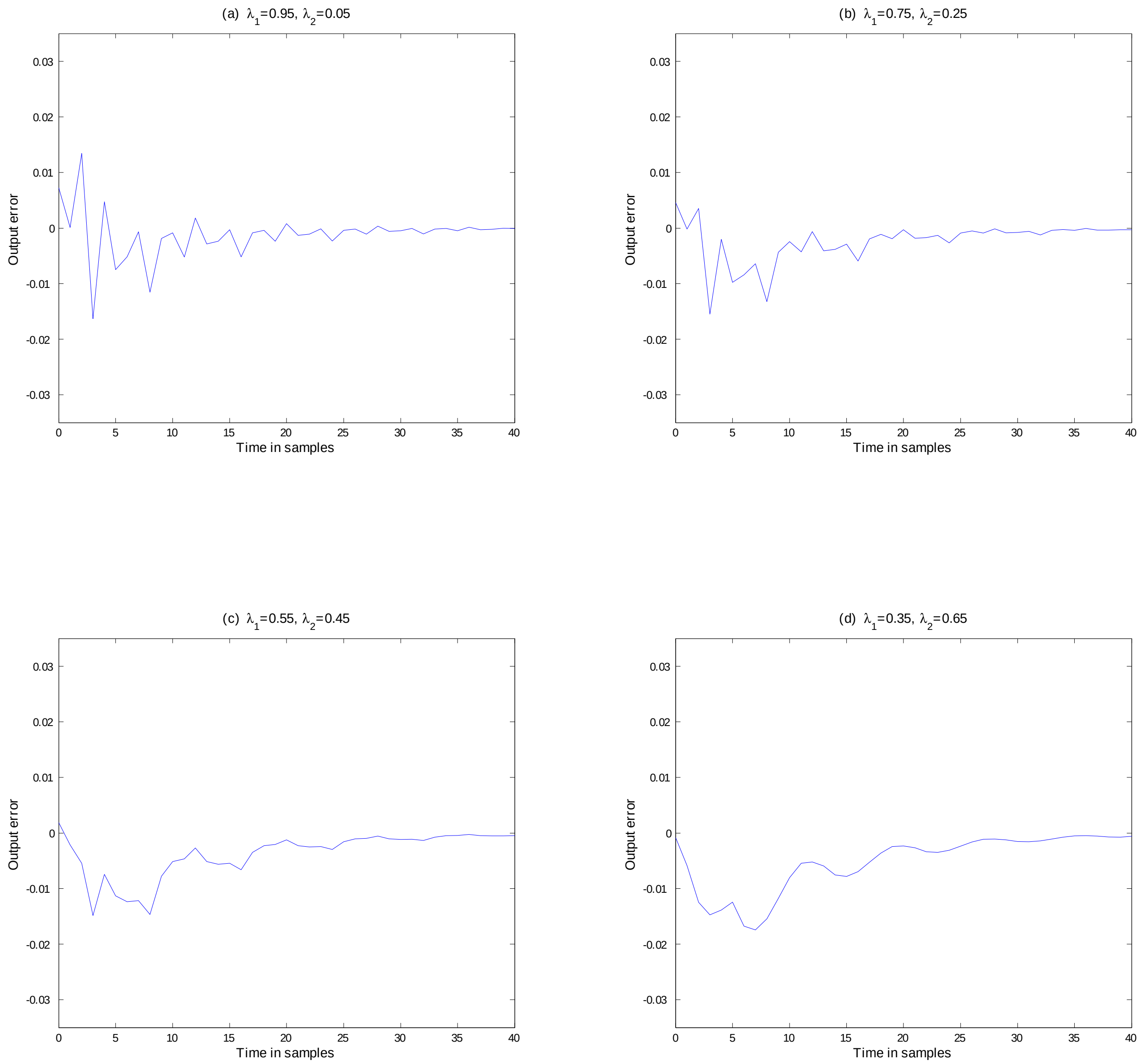

Figure 2 Output errors between original system and reduced model under switching signal σ_1

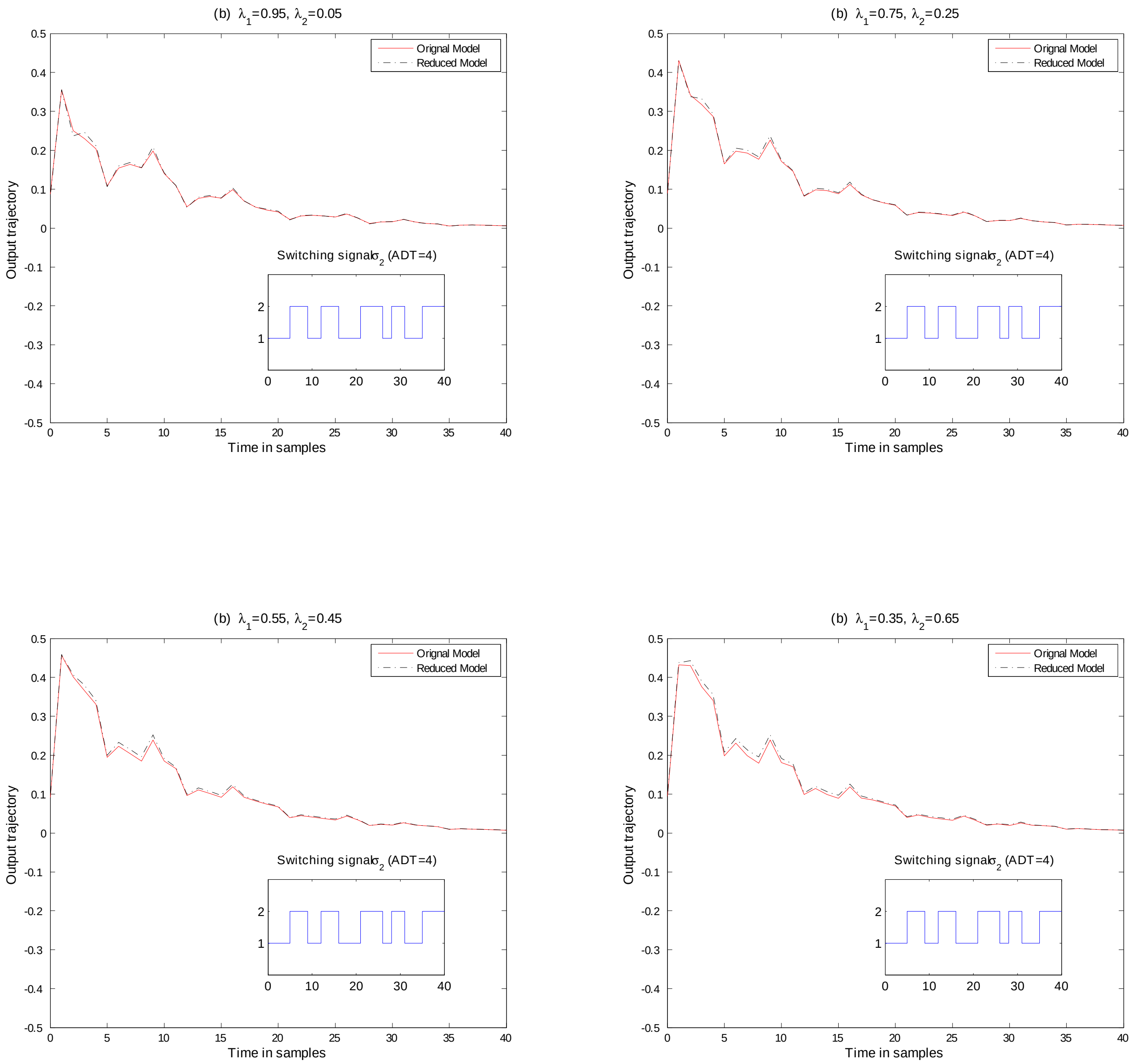

Figure 3 Output trajectories of original system and reduced model under switching signal σ_2

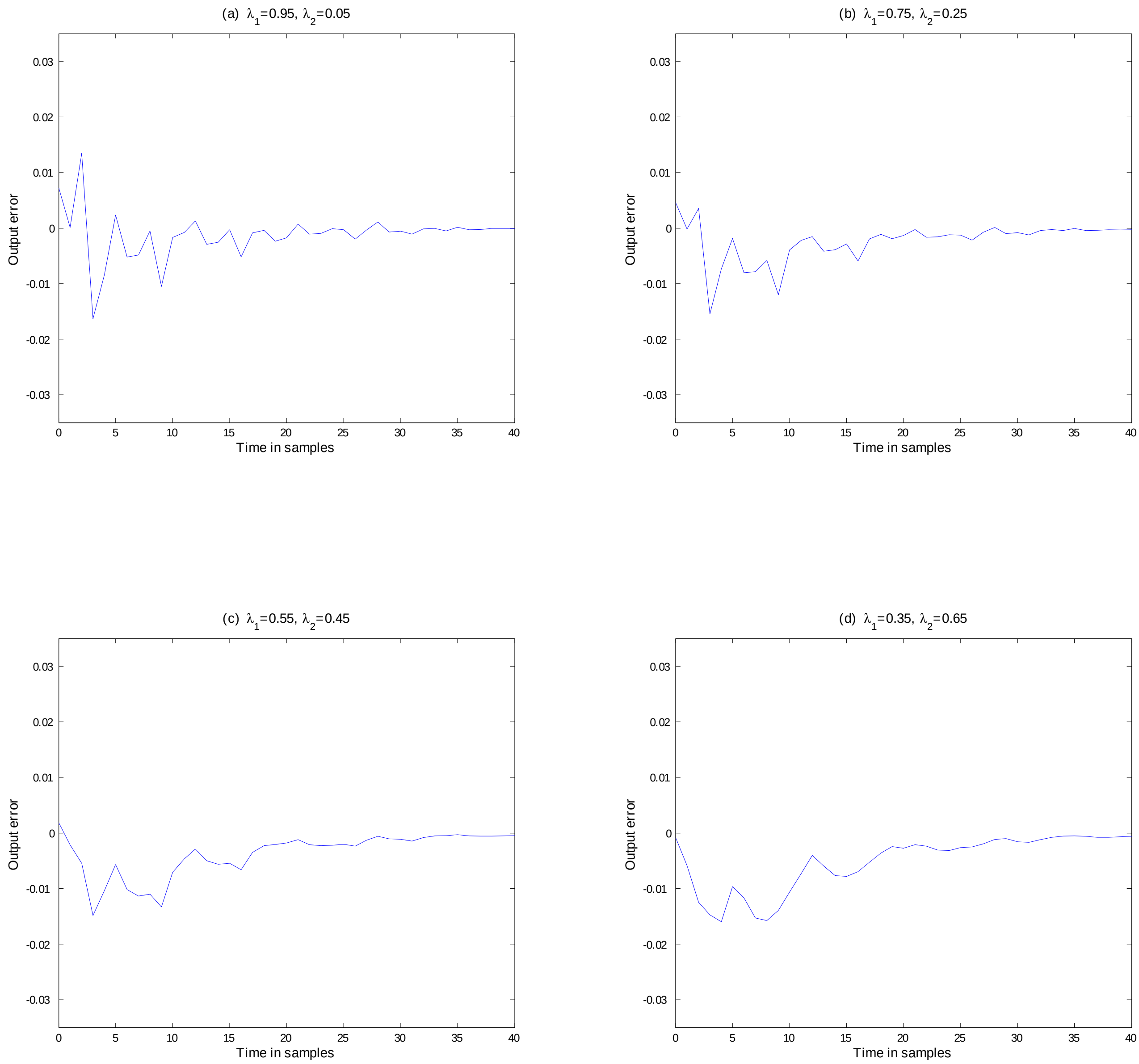

Figure 4 Output errors between original system and reduced model under switching signal σ_2