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Aggregating the Human Development Index: A Non-compensatory Approach

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Aggregating the Human Development Index: A Non-compensatory Approach

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Abstract

The United Nations' Human Development Index remains a widely used and accepted measure of human development. Although it has been revised over the years to address various critiques, a remaining concern is the way the three dimensions are aggregated into the single index. A deterioration in one dimension can be compensated for by an improvement in another. Since compensability is inextricably linked with trade-offs and intensity of preferences, a non-compensatory (i.e., Condorcet) approach to aggregation is employed in this paper. Although non-compensatory approaches have been employed previously, this paper adds to the literature by undertaking an application of the Condorcet approach to the entire HDI. This approach, which does not use intensities of preferences, ensures that the degree of compensability connected with the aggregation model is at the minimum possible level. To achieve this, country level rankings are then compared to those for the 2020 Human Development Index which aggregates dimensions using a geometric mean. The findings demonstrated substantial changes in rank-order between the HDI and Condorcet approach. This outcome provides empirical evidence which demonstrates that the non-compensatory Condorcet approach can mitigate issues of compensation present within the geometric aggregation technique currently employed by the HDI. These findings have potential implications in aiding the identification and employment of potential policy priorities—specifically, the notion that policy should emphasise the development of a country as opposed to economic growth alone.

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1 Introduction

Since its inception in 1990, the Human Development Index (HDI) of the United Nations Development Program (UNDP) has become one of the best known and widely used measures of human development. Enthusiasm for the HDI derives from the fact it provides a broader measure of a country's development than the traditional measure of income per capita. It is also relatively straightforward to calculate and provides a transparent tool for comparing progress across a large number of countries. At the same time the HDI has been criticised on a number of grounds. These criticisms include (but are not limited to) the neglect of other important dimensions of human development (Dasgupta & Weale, 1992; Liang et al., 2019), the quality of data from which it is calculated (Herrero et al., 2012; Srinivasan, 1994), the subjective choices made during the construction process (Mariano et al., 2021), the index adding little to the value of the individual variables comprising it (McGillivray, 1991; Morse, 2014), the use of national averages and the masking of inter-country inequalities (Hicks, 1997; Klugman, Rodriguez & Choi, 2011; Sagar & Najam, 1998) and the use of arbitrary weightings (Desai, 1991; Liang et al., 2019). Yet the HDI has stood the test of time and is still widely used by policymakers, academic researchers and the media.

Over time, the methods used to calculate the HDI have been revised, with the most notable being undertaken for the 20th anniversary edition (i.e., the 2010 edition of the HDI). While retaining the same three-dimensional structure with equal weights, the 2010 version of the HDI adopted *inter alia* different variables to proxy its dimensions as well as a different aggregation procedure (Klugman et al., 2011). More specifically, the sub-indices of the HDI were aggregated using a geometric mean instead of an arithmetic mean. The aggregation approach was changed due to the issue of perfect substitutability between its dimensions (Herrero et al., 2012). This was a problematic assumption of the old HDI formula, because it implied that falls in the attainment of one of the HDI components could be perfectly offset by an equal improvement in the attainment of another. When adopting a geometric mean, perfect substitutability no longer applies although some degree of substitutability still exists.

This paper posits that the aggregation issue continues to be a weakness of the HDI. It does not criticise the components of the HDI such as equal weighting and the arbitrary parameters of the component indices which also impact the final rank order of nations. Instead, this paper focuses on the method in which the HDI is aggregated and how this contributes to a flawed ranking result of countries given that falls in one component can still be compensated for by improvements in another. Ravallion (2010) argued that these trade-offs, measured by the marginal rates of substitution across HDI dimensions, are troubling on ethical and other grounds. He demonstrates that the implicit monetary values attached to an extra year of life vary from as little as $0.51 per year for Zimbabwe to almost $9,000 per year for the richest countries. Such issues can be problematic when optimal resource allocation decision making occurs in the context of trade-offs.

Although non-compensatory approaches have been employed previously (Alaimo & Seri, 2023; Goerlich & Reig, 2021), this paper adds to the literature by undertaking an

application of the Condorcet approach to the entire HDI. Under this approach, the weights of the HDI dimensions are interpreted as importance weights rather than marginal rates of substitution which is more in line with the original spirit of the HDI. Moreover, there is no compensation allowed for changes in the dimension values using this approach. Country rankings are then compared to those for the latest HDI. The results showed substantial changes in rank-order between the HDI and Condorcet approach.

The remainder of this paper is structured as follows. Section 2 describes the main changes to the HDI methodology. Section 3 discusses the weakness of the aggregation approach used in the HDI and outlines the Condorcet aggregation approach adopted by this paper. Section 4 provides the results of the changes to the rank-order of countries from the HDI geometric mean aggregation compared to the Condorcet aggregation technique. Finally, Sect. 5 concludes with some policy implications arising from the research.

2 Reviewing the Main Methodological Changes to the HDI

From 1990 until 2009, a long and healthy life, which comprises the first HDI dimension, was measured using life expectancy at birth. The second dimension: knowledge, was measured using the adult literacy rate (with a weighting of two-thirds); and the combined gross enrolment ratio for primary, secondary and tertiary schooling (with a weighting of one-third). The third, and final, dimension was a decent standard of living, which was measured using GDP per capita in US dollars adjusted for Purchasing Power Parity (PPP). The variable was logged to reflect the diminishing importance of income for human development at higher levels of GDP per capita. The variables for the three dimensions were normalised using a fixed minima and maxima to achieve the property of scale neutrality using the following formula:

$$\text{Dimension index} = (\textit{Actual value} - \textit{Minimum value})/$$
$$(\textit{Maximum value} - \textit{Minimum value})$$

To arrive at the final HDI score, an arithmetic mean using equal weights of one-third for the three-dimensional indices was applied.

In 2010, the HDR methodology was revised with the following changes: The aggregation of the HDI involved the geometric mean of the normalised indices (Deb, 2015); the proxy for knowledge combined the mean years of schooling for adults 25 years and older with the expected years of schooling of children commencing schooling; and the proxy for standard of living was replaced with the log of per capita Gross National Income (GNI) adjusted for PPP. Another significant revision was the use of a constant maxima for the normalization of the indices, instead of the observed maxima. The specified upper limits were: 85 years for life expectancy, 15 years for mean years of schooling, 18 years for expected years of schooling, and \$75,000 for GNI per capita.[1] The scores for the three dimensional indices (I) were aggregated using the geometric mean rather than the arithmetic mean to provide the overall HDI score:

$$\text{HDI} = \left(I_{\text{Health}} * I_{\text{Education}} * I_{\text{Income}} \right)^{1/3} \tag{1}$$

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[1] For more detail on the methodological changes in the HDI see Deb (2015).

3 Index Aggregation: Additive Method, Geometric Mean and the Condorcet Approach[2]

As Goerlich and Reig (2021) point out, the key to distinguishing aggregation methods centres on whether they implement a compensatory or a non-compensatory logic. In this light, this section reviews the three main aggregation approaches: (i) additive method; (ii) geometric mean aggregation approach utilised by the HDI; and (iii) the Condorcet aggregation technique.

3.1 Additive Method

Up until 2009, the HDI employed an additive aggregation approach, via an arithmetic mean, to determine its value. The approach is based on ordinal information, where a country's rank is summed for each of the indicators. As Munda and Nardo (2005a) state, this approach is simple to use, does not consider potential interactions across the dimensions and is insensitive to outliers. This has led to arguably its main criticism which is that shortfalls in one dimension can be compensated for by a strong outcome in another dimension (Sagar & Najam, 1998). Studies by McGillivray and White (1993), Dijkstra and Hanmer, (2000) and Cahill (2005) highlight how strong GDP outcomes can overshadow poor health or education outcomes potentially leading to policies that result in a sub-optimal allocation of resources. As Desai (1991) points out, the additive approach implied perfect substitutability between the three HDI dimensions reflects a reductionist viewpoint making it an inappropriate approach. Given these shortcomings, the HDI moved to a geometric aggregation approach from 2010. This approach is reviewed below.

3.2 HDI Geometric Mean Aggregation

Sagar and Najam (1998) put forward an aggregate calculation based on a multiplicative scheme where the HDI is calculated by the geometric mean of the component values. With the application of a geometric mean the UNDP, 2010, (p. 15) notes:

Poor performance in any dimension is now directly reflected in the HDI, and there is no longer perfect substitutability across dimensions. This method captures how well rounded a country's performance is across the three dimensions. As a basis for comparisons of achievement, this method is also more respectful of the intrinsic differences in the dimensions than a simple average is. It recognizes that health, education and income are all important, but also that it is hard to compare these different dimensions of well-being and that we should not let changes in any of them go unnoticed.

As asserted by Sagar and Najam (1998) this property of the HDI computed using a geometric mean is consistent with the original purpose of HDI which was to control for the trade-offs between each dimension. By doing so, each dimension was treated as being vital and non-substitutable. However, under this multiplicative scheme, a compensatory effect still occurs where one dimension that fares poorly can be offset by a good result in another dimension. As Greco et al. (2019) assert, the use of a geometric mean is an improved approach compared to the additive aggregation via an arithmetic mean (i.e.,

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[2] The Condorcet approach is also referred to as non-compensatory multi-criteria analysis aggregation technique.

linear approach). However, they add, that it is not ideal as it is akin to being between compensatory and non-compensatory techniques (Zhou et al., 2010). Since the objective is to assign weights based on their importance to the model, such a trade-off is theoretically inconsistent with this objective. This perspective is echoed by Goerlich and Rigg, 2021, (p. 3), who opined that, "… a geometric aggregation represents a middle-of-the-way approach, between full compensability and non-compensability (Blancas et al., 2013; Nardo et al., 2008; Van Puyenbroeck & Rogge, 2017; Zimmermann & Zysno, 1983)". They also state that the limitation of this method (i.e., inferior compensability) is most apparent when indices have lower values. In such cases, a non-compensatory multi-criteria approach is preferred.

Although less compensatory than additive aggregation, the compensability nature of the multiplicative scheme does not, this paper insists, go far enough to align with the original goals of the UNDP, which was to mirror the real needs of the nations under analysis rather than to provide a distorted image through an uneven, or overly compensatory, dimension contribution. This is why an absence of conflict amongst the variables is preferred.

For aggregation purposes, the nature of the data is an important consideration. Since the HDI data is not on a ratio scale, the aggregation options are quite limited. According to Ebert and Welsch (2004), a weighted geometric mean of the un-normalised data can produce a purposeful index so long as all the variables are strictly positive with a natural origin.[3] The 2010 revised HDI employed the geometric mean as one part of its revisions outlined in Sect. 2 above. According to Klugman et al. (2011), this approach seems to mirror the alternative put forward by Herrero, Martinez and Villar (2005, 2010).

Although the notion of a purposeful index through the use of a geometric mean is welcome, its purposefulness could be further improved by addressing remaining aggregation issues. Specifically, empirical studies have showed that the geometric mean poses other issues such as not knowing the degree of compensability (Alaimo & Seri, 2023), penalising unbalanced or skewed attainment across the HDI dimensions (Mangaraj & Aparajits, 2020), imperfect substitutability (Mazziotta & Pareto, 2022). In addition, Pinar's (2022) analysis of HDI demonstrated that similar HDI scores were received for most nations with both aggregation techniques–geometric and arithmetic mean. Furthermore, these similarities in scores also were displayed when there was great variations across the three dimensions. Consequently, from a conceptual and methodological perspective, the aggregative-compensative approach is seen as inappropriate (Alaimo & Seri, 2021; Alaimo et al., 2022).

Furthermore, prior studies such as Herrero et al. (2012), Klugman et al. (2011), Zambrano (2011), Ravallion (2010) and Morse (2014) also examined the impact of the HDI modifications to its methodology on country rankings. While all agree that the refinements to the HDI has led to improved measurement, there is also mention of inconsistencies and improvements that are still required. For example, Morse (2014) argued that the changes in the HDI methodology led to increased turbulence in country rankings. This increased turbulence can result in enhanced press reporting of the rank order changes which can be construed as a positive. However, it could also result in a country either rising rapidly without necessarily invoking anything tangible or declining greatly despite initiating a set of good policies designed to help with human development.

Ravallion (2010) claims that the relaxation of perfect substitutability between the three dimensions has led to 'troubling trade-offs' which an alternative aggregation technique that

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[3] Interestingly, Bohringer and Jochem (2007) assessed eleven sustainability indices that possessed ratio scale noncomparability scaled variables – of which the HDI was one. Of the eleven, only the Living Planet Index failed to use the geometric mean aggregation procedure set out in Ebert and Welsch's (2004) article.

still allows for imperfect substitution could resolve. In addition, Herrero et al. (2012) argues that although the geometric mean is the right choice, improvements are required regarding how raw variables are numbered and the manner in which the income log variable is explained. Similar arguments have been made by Deb (2015) and Anand (2018). Unfortunately, change in the HDI aggregation approach reduces, but does not solve the compensability issue between the different dimensions as it does not penalise unbalanced achievements across dimensions strongly enough.[4] This supports the earlier assertion by Goerlich and Reig (2021) that the geometric aggregation scheme lies between full compensability and non-compensability.[5]

Given that the principle of the symmetrical importance of variables needs to be retained, this paper argues the HDI needs to shift away from the use of a geometrical mean, since it does not solve the 'trade-off' issue (Greco et al., 2019; Zhou et al., 2010). Given this, a move towards adopting a non-compensatory multi-criteria approach is preferred. This approach avoids complete compensability and suggests a theoretical assurance that the HDI weights are interpreted as measures of importance rather than marginal rates of substitution (Bouyssou, 1986; Munda, 2005). From a compensatory logic perspective, the marginal rates of substitution is problematic when the importance of the weights for individual indicators are actually measuring substitutability (Paruolo et al., 2013).

3.3 Non-compensatory Multi-criteria analysis (Condorcet Approach)

Our approach tries to resolve the compensability issue by using a discrete multi-criteria approach that includes the lack of preference independence (Munda, 1995, 2012; Roy, 1996). A pair-wise comparison of selected countries across the HDI variables is performed and ranked from best to worst. This Condorcet-type ranking procedure occurs via a complete pre-order by a mathematical formulation (Natoli & Zuhair, 2011).

Under the Condorcet approach, the intensity of preferences are not used, thus minimising compensability with the HDI aggregation, with the pair-wise comparisons interpreted as a voting matrix (Munda & Nardo, 2005a; Nardo et al., 2005). As Munda (2005, 2012) points out, unlike linear or geometric aggregation, the Condorcet approach finds compromises between two or more legitimate goals assuring non-compensability which is also supported via the social choice literature (Arrow & Raynaud, 1986; Moulin, 1988; Munda & Nardo, 2005b).[6]

The main drawback to this method, according to Munda (2005), is that the number of permutation calculations increases exponentially, when many countries are analysed. Despite this drawback, employing a non-compensability aggregation approach to the HDI is desirable due to its theoretical consistency and its ability to have no limitation on the

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[4] Although as Gasparatos, El-Haram and Horner (2008) opine, it would appear on a lower level than the arithmetic mean aggregation employed by the HDI.

[5] Arguably, the geometric-mean aggregation approach is less of a concern than the functional form of the HDI's components. For instance, regarding the functional form of the income variable. The UNDP's choice of using a logarithmic transformation is based on marginal gains in income being more important at lower levels. However, Sagar and Najam (1998) question this approach since even at high levels, income is able to expand peoples' choice sets. They further state that, "In fact, it could be argued that income has to reach a certain threshold before it becomes tradable for human development (Sagar and Hajam, 1998, pp. 253-54)". Hence, if other functional forms (such as exponential or power functions) were applied to income this would result in changes to the HDI country rankings.

[6] In fact, Arrow and Raynaud (1986, p. 77) state that, "… for aggregating an algorithm, the highest feasible multi-criterion ranking must be Condorcet.".

Table 1 Cycles

Voter	1st Preference	2nd Preference	3rd Preference
V_1	C_1	C_2	C_3
V_2	C_2	C_3	C_1
V_3	C_3	C_1	C_2

measurement scale of variables (Munda & Nardo, 2005a). These benefits should reduce the uncertainty and imprecision in the HDI index. The Condorcet technique is detailed below. Note, that while the HDI weights should be viewed as trade-offs in compensatory aggregation methods, they are viewed as 'importance coefficients' in non-compensatory aggregation methods.

An axiomatic setting for the non-compensatory approach is outlined below. When comparing development across countries, it is common to find that one country outperforms another in one area but not in another. Overcoming these conflicting results in a non-compensatory manner enables this paper to appropriately rank the selected countries favouring a Condorcet approach.[7] However, one drawback involves the issue of cycles.[8]

Regarding cycles, we can consider a voting situation with three voters, V_1, V_2 and V_3 and three candidates C_1, C_2 and C_3. The preferences are given in table 1 below where candidates are given in decreasing order of preference.

For instance, if C_3 is the winner, It is possible to argue that C_2 should have been the winner because V_1 and V_2 prefer C_2 to C_3 and only V_3 prefer C_3 to C_2. It then follows that C_1 is preferred to C_2 and C_3 is preferred to C_1 with a two to one margin in each case. This is a case of cycling where C_1 is preferred to C_2 which is preferred over C_3 which is preferred over C_1.

Cycles can occur quite regularly with macroeconomic data and is dealt with via the Condorcet-Kemeny-Young-Levenglick (CKYL) ranking procedure.[9] However, as Munda (2005) and Munda and Nardo (2005a) point out, one still must accept that rank reversals may occur. A recent study by Su et al. (2023), examined the rank-reversal problem through employing a subgroup dominance-based benefit-of-doubt (SD-BoD) model. The model compared and analysed the HDI of 28 European nations and found that the severity of rank reversals was largely dictated by the employed common-weight BoD method. Although rank reversals are at variance with Arrow's axiom of independence of irrelevant alternatives, Young (1988) states that the locally stable nature of the CKYL ranking procedure maintains the ranking of alternatives. As Munda (2005) states, the CKYL ranking procedure for composite indices is obtained by a simple, yet formal ranking algorithm that supports the maximum likelihood ranking of countries. To achieve this, the maximum number of individual indicators for each pair-wise comparison is summed over all pairs of countries involved as per the below ranking algorithm.

Given a set of individual indicators $G = \{g_m\}, m = 1, 2, ..., M$, and a defined set $A = \{a_n\}, n = 1, 2, ..., N$ of countries; each country a_n is evaluated with respect to an

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[7] The foundations for the Condorcet approach can be found in: *Essai sur l'application de l'analyse a la probabilite des decisions rendues a la pluralite des voix* (Condorcet, 1785), which is cited in Munda (2005) and Munda and Nardo (2005a).

[8] Munda (2005, pp. 962–964) and Munda and Nardo (2005a, 2005b, pp. 6–7) display the formulas and summary.

[9] In addition to Condorcet the CKYL ranking procedure utilises the works of Kemeny (1959) as well as Young and Levenglick (1978).

individual indicator g_m from an assumed *ordinal, interval* or *ratio* measurement scale. Here, individual indicators with higher values are preferred to a lower one as outlined below:

$$\begin{cases} a_j P a_k \Leftrightarrow g_m(a_j) > g_m(a_k) \\ a_j I a_k \Leftrightarrow g_m(a_j) = g_m(a_k) \end{cases} \tag{2}$$

where P and I specify a preference and an indifference relation accordingly, both fulfilling the transitive property.

From there, a set of individual indicator weights $W = \{w_m\}, m = 1, 2..., M$, with $\sum_{m=1}^{M} w_m = 1$, are derived as importance coefficients. To rank in a complete pre-order, from best to worst with the available information, the following mathematical aggregation procedure is required: (i) pair-wise comparison of countries according to the whole set of individual indicators used; and (ii) ranking of countries in a complete pre-order. To perform a pair-wise comparison of countries for the whole set of HDI indicators, an axiomatic system is adapted from Arrow and Raynaud (1986, pp. 81–82) is required.

Axiom 1: Diversity

Each individual indicator is a total order on the defined set *A* of countries to be ranked, and there is no restriction on these variables; they can be any total order on *A*.[10]

Axiom 2: Symmetry

The individual indicators are non-comparable scales that contain ordinal pair-wise preferences. This primarily means that intensity of preferences and compensability are side-stepped, while the weights are symmetrical importance coefficients. It also means that a normalisation step is not needed since this axiom (symmetry) reduces uncertainty and imprecision.

Axiom 3: Positive Responsiveness

For two countries *a* and *b* the level of performance between them depends on how many indicators and their associated weights that rank *a* before *b*.[11] As a result, the equal treatment of all individual indicators is broken (Munda, 2005; Munda & Nardo, 2005a) and a trade-off occurs between anonymity and decisiveness (i.e., a ranking must be selected). Of the two, decisiveness is favoured.[12]

The three axioms (diversity, symmetry and positive responsiveness) allow a *N x N* matrix, *E*, called an *outranking matrix* to be established, which assumes to contain all available information (Arrow & Raynaud, 1986; Roy, 1996). Any generic element of $E : e_{jk}, j \neq k$ arises from the pair-wise comparison of all the *M* individual indicators between countries *j* and *k*. To arrive at a global pair-wise comparison, the equation below is employed:

$$e_{jk} = \sum_{m=1}^{M} \left(w_m(P_{jk}) + \frac{1}{2} w_m(I_{jk}) \right) \tag{3}$$

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[10] In the original, Arrow and Raynaud (1986) discuss a finite set X of alternatives, with no restriction condition on the criteria that can be any total order on X.

[11] In this instance, Arrow and Raynaud (1986) originally discussed the intensity of preferences between two alternatives x_i and x_j.

[12] Furthermore, as Munda (2005) posits, to avoid the aggregation becoming lexicographic, no indicator should weigh more than 50 per cent.

where, $w_m(P_{jk})$ and $w_m(I_{jk})$ refer to the relation between preference P and indifference I based on the weights of individual variables. Hence, we state that:

$$e_{jk} + e_{kj} = 1 \tag{4}$$

The outranking matrix E comprise all the $N(N-1)$ pair-wise comparisons. The notation represents R the set of all $N!$ possible complete rankings of alternatives, $R = \{r_s\}, s = 1, 2, ..., N!$. For each r_s the corresponding score ϕ_s is calculated as the sum of e_{jk} over all the $\binom{N}{2}$ pairs j, k of alternatives, i.e. $\phi_s = \sum e_{jk}$, where $j \neq k, s = 1, 2, ...N!$ and $e_{jk} \in r_s$.

The final ranking $(r*)$ maximises the equation below:

$$r* \Leftrightarrow \phi_* = \max \sum e_{jk} \text{ where } e_{jk} \in R \tag{5}$$

The CKYL approach also contains other formal properties which include (Munda, 2005; Young, 1988; Young & Levenglick, 1978):

- *Neutrality:* no country is favoured over another, thus all countries are treated equally.
- *Unanimity:* if all the individual indicators favour country a to country b, then country a should be chosen.
- *Monotonicity:* there are no defiers, hence if country a is preferred in any pair-wise comparison and *only their* variables are improved, then country a should continue to be the higher rank ordered country.
- *Reinforcement:* if the set A of countries is ranked by two subsets G_1 and G_2 of the individual indicator set G, such that the ranking is the same for both G_1 and G_2, then $G_1 \cup G_2 = G$ should still supply the same ranking. This notion of reinforcement is crucial when initially applying the variables to each single dimension and then aggregating them to the general model.

As Natoli and Zuhair (2011) point out, given the importance of reinforcement, the only Condorcet consistent rule that can uphold this property is the maximum likelihood ranking procedure. A property, which Arrow and Raynaud (1986) assert, whose definite ethical construct is critical to welfare economics.

It was found that if equal values are used for the weightings in determining the ranking using the Condorcet approach, the ranking may not be unique. With equal weightings, it is found that the non-uniqueness of the ranking occurs within a few ranking positions. Thus, as pointed out by Natoli and Zuhair (2011), when undertaking any aggregation technique, practical compromises need to take place. Hence, a small value, such as $\pi/100$, was used to adjust different weightings while still maintaining $\sum_{m=1}^{M} w_m = 1$. For example, if $M = 3$, traditional weightings would be $w_1 = w_2 = w_3 = 1/3$. In the present work, we use $w_1 = \frac{1}{3} + \frac{\pi}{100}, w_2 = \frac{1}{3}, w_3 = \frac{1}{3} - \frac{\pi}{100}$. With an interchanging of weightings, the ranking positions will change within the few non-unique positions of those using the equal weightings. This does not impact on the Condorcet method since, in reality, equal weighting for all the indicators is a subjective choice and the small difference in weighting means that some indicators are slightly more favoured than the others.

Although both anonymity and information on the preference intensity of variables is lost, the non-compensatory approach preserves the theoretical importance of the coefficients and reduces the main source of imprecision and uncertainty (Munda & Nardo,

2005a). As Natoli and Zuhair (2011) posit, since progress measures can serve as a foundation to improve decisions on resource allocations, the Condorcet approach is preferred since such policy decisions should occur without trade-offs between dimensions.

4 Results and Discussion

4.1 Empirical Results

To explore whether there are rank-order issues prevalent within the 2020 HDI report derived from the geometric mean aggregation approach relative to the Condorcet approach, an analysis of rank-orders is required.

Table 2 contains the 189 countries in the 2020 HDI report and displays both their HDI and Condorcet rank-order. The results are also presented via Fig. 1. Although rank-order change occurs throughout, as shown in Fig. 1, the most pronounced variation between the HDI and Condorcet rank-orders seems to be concentrated among the mid-range countries with variations at the high-range and low-range being less apparent. This finding is similar to those highlighted by Greco et al. (2019) in a study by Munda (2012) who applied a CKYL non-compensatory approach to the Environmental Sustainability Index (ESI). Munda (2012) identified noticeable difference in rankings between his CKYL approach and the linear approach used by the ESI, which was mostly evident in the middle positions and less evident among those ranked first and last.

A study by Mangaraj and Aparajita (2020) undertook a generalised HDI (GHDI) as well as a measure of relative GHDI (GHDIR) with a multi-index ranking model. They compared the top 30 countries as ranked via the HDI with the GHDI and the GHDIR. Their findings showed that the rank-order changes of the GHDI did not deviate much from the ranks of the HDI. This reinforces the notion of little deviation when the focus is on either the high-end or low-end of nations in the HDI table.

Table 3 provides a snapshot of the number of countries which experience a sizable change in rank order. As Table 3 demonstrates, of the 189 countries, just over half (104, or 55% of all countries in the HDI) experience a rank order change of at least six positions. In addition, just over one-quarter (27.5%, or 52 countries in the HDI) experienced a movement of at least 10 rank-order positions. At the highest level of rank-order changes, 11 countries (or 5.8% of all HDI countries) experience a rank-order change of at least 19 positions.

In keeping with the highest level of rank-order changes, Table 4 identifies the top ten countries to experience the highest level of variability in rank-order changes in terms of both improvement and deterioration. As identified earlier, most of the countries that experienced the highest level of rank-order changes are those grouped in the middle of the HDR report (HDI original rank from 58 to 95).

As Table 4 shows, Morocco [MAR], Maldives [MDV], Costa Rica [CRI] and Cuba [CUB] benefited the most regarding a positive rank-order change under the Condorcet approach. For rank deteriorations, the most notable occurred to Bahamas [BHS], Trinidad & Tobago [TTO], Equatorial Guinea [GNQ] and Turkmenistan [TKM].

The magnitude of the changes in rank-order suggests that, as Anand (2018, p. 6) points out, that the geometric mean is not suitable since it allows "… changes in any of them [variables] to go unnoticed". Instead, due to the non-compensatory multi-criteria approach, the

Condorcet aggregation technique can identify compromises between more than one legitimate goal to assure non-compensability.

Table 5 shows the overall number of rank changes from the HDI rank-order to the Condorcet rank-order for all countries. Here, tied ranks refers to no change in a country's rank-order from the HDI and Condorcet approach, a positive rank refers to an improvement in a country's rank-order from the HDI to the Condorcet (e.g. 5 in HDI to 4 under a Condorcet approach), and a negative rank refers to a deterioration in a country's rank-order from HDI to Condorcet (e.g. 6 in HDI to 5 under a Condorcet approach). Of the 189 countries, there were only five occasions (2.6%) which resulted in a tied rank. Thus, the Condorcet aggregation approach results in almost all country rankings undergoing a change in rank-order.

4.2 Discussion

A case for the Condorcet aggregation method providing a better measure of country level human development can be made by examining the index scores across the three individual HDI components. For example, among countries with a 'very high' level of human development, Singapore is one of the countries that ranks much higher (3rd) under the Condorcet aggregation method than under the geometric mean approach (11th). This is because Singapore outranks many countries ranked higher in the HDI rankings with respect to both life expectancy and income. Singapore ranks less well for education. Under a geometric mean aggregation approach its overall HDI score and ranking are lower since the life expectancy and income index scores compensate for the lower education index score. Its higher rank under the Condorcet approach stems from the fact that Singapore ranks better on two of the three HDI components than nearly all other countries. We argue that unless there is a reason to believe that education is more important than income and health, Singapore should receive a higher overall ranking given its better performance in two of the three HDI components. In comparison, Switzerland which is ranked 3rd by the UNDP's HDI, ranks first under the Condorcet aggregation method. This is because it outranks Norway, Ireland and all other countries on at least two of the three HDI component indices.
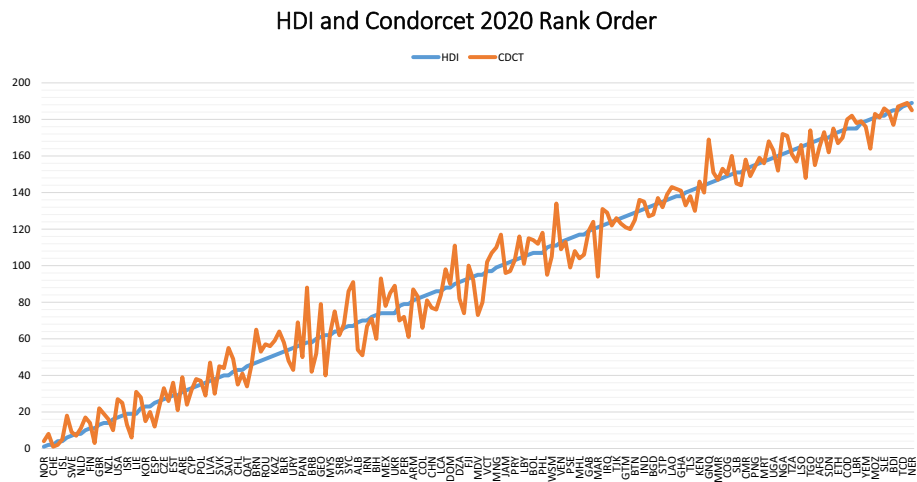

Fig. 1 Rank order comparisons: HDI and condorcet

Table 2 HDI and condorcet 2020 rank order

Nation	CDCT	HDI	Nation	CDCT	HDI	Nation	CDCT	HDI	Nation	CDCT	HDI
CHE	1	2	BHR	49	42	JOR	97	102	SLB	145	151
HKG	2	4	PAN	50	57	AZE	98	88	KEN	146	143
SGP	3	11	CUB	51	70	PSE	99	115	MMR	147	147
NOR	4	1	OMN	52	60	FJI	100	93	DJI	148	166
ISL	5	4	MNE	53	48	LBY	101	105	PAK	149	154
JPN	6	19	ALB	54	69	VCT	102	97	COG	150	149
AUS	7	8	SAU	55	40	PRY	103	103	ZMB	151	146
IRL	8	2	PLW	56	50	MHL	104	117	RWA	152	160
SWE	9	7	ROU	57	49	WSM	105	111	AGO	153	148
CAN	10	16	BLR	58	53	VNM	106	117	PNG	154	155
NLD	11	8	KAZ	59	51	SUR	107	97	SEN	155	168
ESP	12	25	BIH	60	73	EGY	108	116	MRT	156	157
ISR	13	19	THA	61	79	VEN	109	113	MDG	157	164
FIN	14	11	SRB	62	64	MNG	110	99	CMR	158	153
KOR	15	23	MYS	63	62	MDA	111	90	COM	159	156
NZL	16	14	RUS	64	52	IDN	112	107	ZWE	160	150
DNK	17	10	BRN	65	47	ZAF	113	114	TZA	161	163
DEU	18	6	COL	66	83	BOL	114	107	SDN	162	170
BEL	19	14	IRN	67	70	UZB	115	106	UGA	163	159
LUX	20	23	MUS	68	66	TON	116	104	ERI	164	180
ITA	21	29	BGR	69	56	BWA	117	110	AFG	165	169
GBR	22	13	ATG	70	78	PHL	118	107	LSO	166	165
FRA	23	26	LKA	71	72	GAB	119	119	ETH	167	173
GRC	24	32	PER	72	79	NIC	120	128	BEN	168	158
AUT	25	18	MDV	73	95	GTM	121	127	GNQ	169	145
MLT	26	28	LBN	74	92	SLV	122	124	MWI	170	174
USA	27	17	KWT	75	64	CPV	123	126	CIV	171	162
SVN	28	22	ECU	76	86	KGZ	124	120	NGA	172	161
AND	29	36	CHN	77	85	BTN	125	129	HTI	173	170
PRT	30	38	MEX	78	74	TJK	126	125	TGO	174	167
LIE	31	19	GEO	79	61	HND	127	132	GMB	175	172
CYP	32	33	TUN	80	95	BGD	128	133	YEM	176	179
CZE	33	27	BRA	81	84	IRQ	129	123	BDI	177	185
QAT	34	45	DZA	82	91	NPL	130	142	LBR	178	175
CHL	35	43	MKD	83	82	GUY	131	122	GIN	179	178
EST	36	29	LCA	84	86	STP	132	135	COD	180	175
POL	37	35	KNA	85	74	VUT	133	140	BFA	181	182
LTU	38	34	SYC	86	67	TKM	134	111	GNB	182	175
ARE	39	31	ARM	87	81	IND	135	131	MOZ	183	181
CRI	40	62	BHS	88	58	NAM	136	130	MLI	184	184
HRV	41	43	UKR	89	74	KIR	137	134	NER	185	189
BRB	42	58	DOM	90	88	TLS	138	141	SLE	186	182
URY	43	55	TTO	91	67	FSM	139	136	SSD	187	185
HUN	44	40	DMA	92	94	KHM	140	144	TCD	188	187

Table 2 (continued)

Nation	CDCT	HDI	Nation	CDCT	HDI	Nation	CDCT	HDI	Nation	CDCT	HDI
SVK	45	39	GRD	93	74	GHA	141	138	CAF	189	188
ARG	46	46	MAR	94	121	SWZ	142	138			
LVA	47	37	BLZ	95	110	LAO	143	137			
TUR	48	54	JAM	96	101	SYR	144	151			

NB: Nations are denoted by their current officially assigned ISO 3166–1 alpha-3 codes. *HDI* Human development index; *CDCT* Condorcet

Table 3 HDI, 2020 sizable rank-order changes

2020	Magnitude of change
HDI (n = 189)	104 countries ± 6 positions
	88 countries ± 7 positions
	74 countries ± 8 positions
	52 countries ± 10 positions
	26 countries ± 15 positions
	11 countries ± 19 positions

Table 4 HDI 2020 Top 10 rank-order changes

HDI (n = 189)			
Rank improvement		Rank deterioration	
Country	Rank-order change	Country	Rank-order change
Morocco [MAR]	121 to 94 [27]	Bahamas [BHS]	58–88 [30]
Maldives [MDV]	95 to 73 [22]	Trinidad & Tobago [TTO]	67–91 [24]
Costa Rica [CRI]	62 to 40 [22]	Equatorial Guinea [GNQ]	145–169 [24]
Cuba [CUB]	70 to 51 [19]	Turkmenistan [TKM]	111–134 [23]
Djibouti [DJI]	166 to 148 [18]	Moldova [MDA]	90–111 [21]
Lebanon [LBN]	92 to 74 [18]	Grenada [GRD]	74–93 [19]
Thailand [THA]	79 to 61 [18]	Seychelles [SYC]	67–86 [19]
Colombia [COL]	83 to 66 [17]	Georgia [GEO]	61–79 [18]
Barbados [BRB]	58 to 42 [16]	Brunei [BRN]	47–65 [18]
Palestine [PSE]	115 to 99 [16]	Botswana [BWA]	100–117 [17]

Displays the10 greatest rank-order improvements and deteriorations (between HDI and Condorcet outcomes) for the UNDP, 2020 HDR

Table 5 Overall rank changes

2020	Freq. (%)
HDI (n = 189)	
Tied ranks	5 (2.6%)
Positive ranks	88 (46.6%)
Negative ranks	96 (50.8%)

The Bahamas, another country with a 'very high' level of human development, drops 30 places in the rankings under the Condorcet method, mainly because its relatively high score for the income index can no longer compensate the relatively lower scores for life expectancy and education. A similar explanation applies to Equatorial Guinea, a country with a 'medium' level of human development which falls 24 places in the rankings (145th to 169th) when its relatively high income index score can no longer compensate its poorer life expectancy and education index scores. Conversely, Morocco, another country with a 'medium' level of human development ranks much better under the Condorcet approach (94th vs. 121st) because it has relatively higher life expectancy and income index scores than countries above it in the HDI index rankings.

There is relatively less change in rankings for countries with a 'low' level of human development. An exception is that of Eritrea which performs better when the Condorcet approach is applied since it performs relatively well according to life expectancy and income for countries at this level of development. Its rank improves from 180 to 164th when life expectancy and income are no longer compensate for lower education index scores bringing down its overall average HDI score. In sum, if we accept that the HDI components are equally important for human development and that shortfalls/achievements in one of the components cannot be compensated for (or penalised) by levels in another, our ranking according to the Condorcet method offers a more accurate depiction of the human development level of each country.

5 Conclusion and Policy Implications

This paper compares the country rankings of the UNDP's HDI (2020) with those resulting from a modified Condorcet ranking, a non-compensatory approach. Country HDI rankings using the Condorcet aggregation approach were compared to those reported in the 2020 Human Development Report which allow for shortfalls in one index component to be compensated for by high values in another. The paper compared the rank-order of the human development for the year 2020 with the Condorcet rank-order.

Findings suggest that the approach to aggregating the HDI is very important with respect to country rankings. Although a monotonic association was identified, the changes in rank-order between the HDI and Condorcet approach were substantial. This outcome provides empirical evidence regarding the problematic, and inconsistent, nature of using a geometric mean to aggregate the HDI where 'troubling trade-offs' occur due to the relaxation of perfect substitutability between the three dimensions. The substantial changes derived from the Condorcet approach can be partly explained by the fact that it is able to address this issue via its non-compensatory multi-criteria analysis.

An issue with geometric mean, as discussed above and in the literature, is that weakness in one criterion can be compensated by the strength in another criteria which could confuse the interpretation of the results and unable to provide clear direction to the policy makers. By removing this compensatory effect, Condorcet ranking is able to provide clearer direction on policy formulation for development.

The HDI, with its focus on people and their capabilities, can be used to query the policy choices of nations. For instance, the change in HDI rankings based on the Condorcet approach can capture the attention of policy makers, media, non-governmental organisations, etc. and further stimulate the debate surrounding the purpose of government policy.

Specifically, that policy should emphasise the development of a country as opposed to economic growth alone. It may also be used as one indicator amongst a suite-of-indicators to assess foreign aid or Official Development Assistance to low-income countries. This could result in alterations to the volume of aid received by some countries.

Declarations

Conflict of interest None.

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