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## Application of Linear Programming for looped water supply pipe network design

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# APPLICATION OF LINEAR PROGRAMMING FOR LOOPED WATER SUPPLY PIPE NETWORK DESIGN

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**Abstract**
It has been indicated in the literature that the looped water distribution systems designed with Linear Programming (LP) optimisation technique are converted into tree like structure resulting in the disappearance of the original geometry in the final design. Looped networks are provided for system reliability, thus such a design approach will defeat the basic purpose of looped systems provision. Such a limitation has hindered the application of LP for the design of looped water supply networks. A method for the design of a looped water distribution system has been developed such that the loop configuration of the network is maintained by bringing all the pipes of the network in the optimisation problem formulation using LP optimisation method.

**Keywords**
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## INTRODUCTION

Generally urban water distribution systems have looped configurations and receive water from single or multi-input points (sources). The area of water distribution system design has attracted the hydraulic professionals the most as evident from the extensive literature that exists. Karmeli et al. (1968) for the first time used LP for single input source branched water network design. Some of the early examples for LP application in water network design are Schaake and Lai (1969), Quindry et al. (1981) and Bhave (1983). In general the looped networks were decomposed to branched network for LP problem formulation and the deleted pipe links were specified with minimum diameter or flow conditions. Thus, the original geometry of the pipe network is decomposed to branch network, which reduces the advantages of looped network.

Savic and Walters (1997) highlighted that the loops in normal circumstances would disappear leaving underlying tree like structure, when designed for least cost considerations. The looped configuration is maintained in the final design by specifying minimum pipe diameters. It has been seen that the non-linear algorithms for network optimisation do not guarantee global optima as they are based on a continuous diameter approach (Cunha and Sousa 1999). The conversion of continuous diameter to commercially available pipe sizes further reduces the quality of the solution, as this conversion is not based on any cost consideration. Todini (2000) indicated that the looped network increases the reliability of water availability at all the nodes. The looped network allows for redundancy, which ensures that there is sufficient capacity in the system to overcome local failures. Rowell and Barnes (1982) also addressed the problem of reliability through the provision of sufficient capacity in looped network pipes.

Wu et al. (2001) highlighted that many powerful numerical optimization models had been developed for water distribution systems and applied with varying level of success (Simpson et al. 1994; Lippai et al.1999; Boulos et al. 2000; Wu et al., 2000 and Wu and Simpson 2001). They also indicated that some of the most successful applications have used some form of genetic algorithms (GAs). GAs are generally search procedures inspired by natural evaluation and biological reproduction. They emulate the principles of survival of fittest and genetic recombination. Different GAs may vary in the way they imitate the evaluation process.

Sarbu and Kalmar (2002) presented a method for the looped water distribution design and also indicated that the looped configuration of pipelines is preferred over branched configurations due to high reliability and low risk from the loss of services. The computed pipe sizes using the approach were continuous in nature and were approximated to closest commercial sizes. Adopting nearest commercial sizes would result into an approximate solution. Boulos et al. (2004) described the various optimisation models applied to water distribution systems. Ostfeld and Karpibka (2005) included a wide literature on the application of GA and LP techniques in water supply network design.

The aim of this paper is to present a design method for looped water distribution system using LP technique that maintains the looped configuration in the final design for reliability consideration. This was achieved by bringing all the pipes of the network in the optimization problem formulation to preserve looped configuration.

## DESIGN PROBLEM FORMULATION

### Maintaining Looped Configuration

The looped water distribution networks are indirectly decomposed to branched network even if the nodal headloss constraint equations were developed for all the nodes in the network in LP problem formulation for optimal design. The nodal headloss constraints equations are used to minimise cost function using optimisation techniques. The development of cost function and constraint are explained in the network design section. The common approach for the formulation of nodal headloss constraint equations in a looped network is demonstrated using a simple pumping water distribution network containing 5 pipes, 5 nodes and one loop as shown in Figure 1.
To formulate the headloss nodal constraint equations for all nodes, the following pipes will be used (No headloss constraint equation is required for node 1 being an input source node):

Headloss constraint equation for node 2: based on headloss in pipe 1 only
Headloss constraint equation for node 3: based on headloss in pipes 2 and 1
Headloss constraint equation for node 4: based on headloss in pipes 3 and 1
Headloss constraint equation for node 5: This can be based on have one of the following combinations of pipes in constraint equation:
(1) Headloss in pipes 4, 2, 1 or
(2) Headloss in pipes 5, 3, 1.

Thus, either pipe 4 or pipe 5 will not be included in the nodal headloss constraint equations in optimization problem formulation. As shown in Figure 1 (B), pipe 5 is not included here in optimization problem formulation. In order to reduce the cost, the optimization algorithms provide zero pipe diameters to these pipes and looped networks are forcefully converted to branch network for optimization purposes. A minimum pipe diameter is generally adopted for these pipe links in the final design. If a pipe network has $k_L$ loops, then the $k_L$ pipes will not be included in the cost optimization problem formulation and a minimum pipe size will be adopted for all these pipes. Thus, the looped water distribution systems designed with least-cost consideration only, are converted into tree like structure resulting in the disappearance of the original geometry in the final design.

This problem can be overcome if every pipe of the network can be included in the optimisation problem formulation. In this simple example if both the above mentioned headloss constraint equations for node 5 are included in the optimisation problem formulation, that will bring all the pipes in the design process and the loop network configuration can be maintained in the final design. Thus, the total number of headloss constraint equations will be equal to total number of pipes and not total number of nodes. For a looped water distribution network the headloss constraints equations will be equal to $i_L$ (total number of pipe links in the network) and not $j_L$ (total number of pipe nodes in the network). In this way, there will $k_L$ number of additional headloss constraint equations generating from various nodes. Thus, there will be some nodes with two or more headloss constraint equations. This will bring all the pipes in the optimisation problem formulation and the network will not convert into a branch system during the optimisation process. The formulations of such nodal heasdloss constraint equations for a large pipe network are described in network design section of the paper.

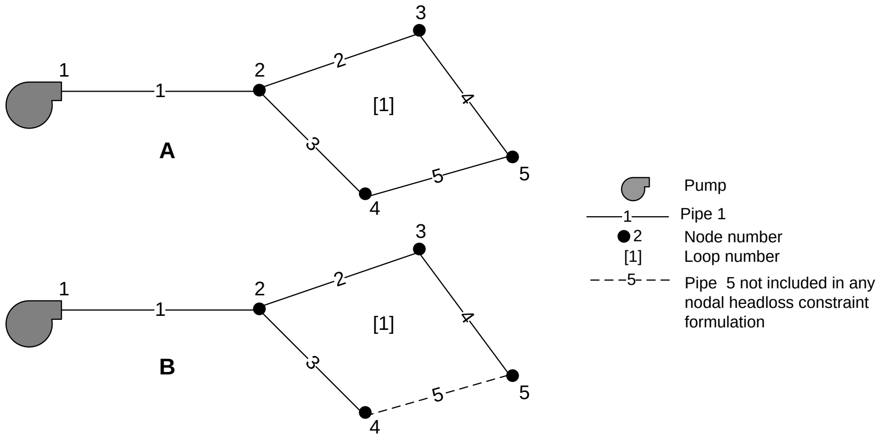


**Figure 1**: Simple single looped water distribution network

## Network Description

In order to describe the algorithm properly, a water distribution network as shown in Figure 2 is considered. The geometry of the network is described by pipe link, input (source) point, nodal elevations and looped forming pipes data.

The pipe link $i$ has two end points with the nodes $J_{1i}$ and $J_{2i}$; and a length $L_i$ for $i$ = 1, 2, 3......$i_L$. The elevation of nodal point $j$ is $z_j$ for $j$ = 1, 2, 3...... $j_L$. The pipe link population load is $P_i$ and diameter of pipe $i$ is $D_{i.}$ The pipe link $i$ is the part of two loops $k_1(i)$ and $k_2(i)$. As there are no loops in a branched pipe configuration $k_1(i)$ and $k_2(i)$ are zero. However, the description of loops is not independent information and can be generated from pipe-node connectivity data (Swamee and Sharma 1991; Sharma and Swamee 2005). The total number of loops in the network is $k_{L.}$ There are $N_p(j)$ pipe links meeting at the node $j$. These pipe links are numbered as $I_p(j,l)$ with $l$ varying from 1 to $N_p(j)$. Scanning the network data, the node pipe connectivity data can be formed. The water distribution shown in Figure 2 has 59 pipes, 37 nodes and 23 loops. The input source nodes are 35, 36 and 37.

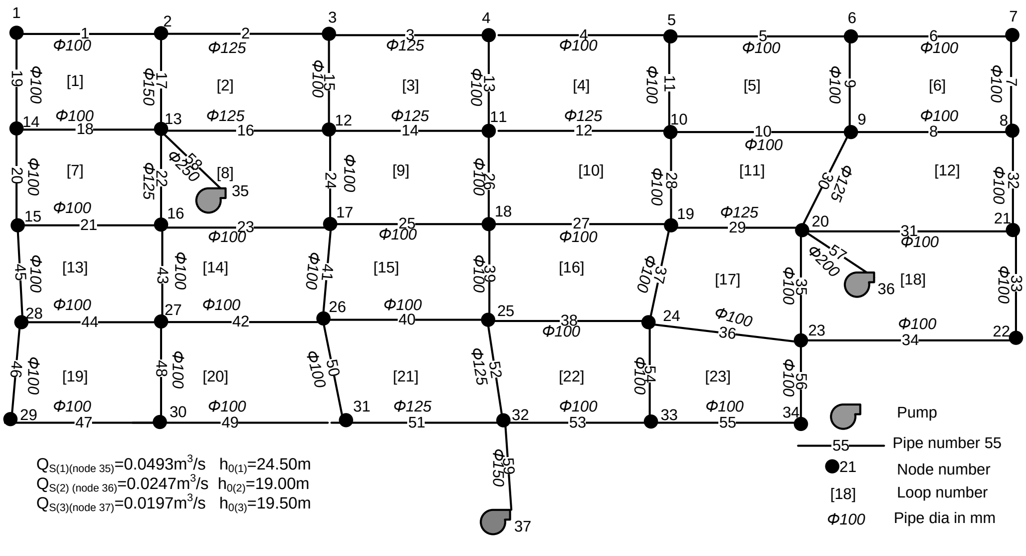


**Figure 2:** Water supply distribution network

**Table 1**: Network pipe link data

| Pipe, *i* /Node, *j* | Node 1 $J_{1i}$ | Node 2 $J_{2i}$ | Loop 1 $k_{1i}$ | Loop 2 $k_{2i}$ | Length $L_i$ (m) | Population $P_i$ (no.) | Pipe size $D_i$ (mm) | Elevation $Z_j$ (m) |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 | 700 | 350 | 100 | 101.0 |
| 2 | 2 | 3 | 2 | 0 | 700 | 350 | 100 | 101.5 |
| 3 | 3 | 4 | 3 | 0 | 700 | 350 | 100 | 100.5 |
| 4 | 4 | 5 | 4 | 0 | 700 | 350 | 100 | 100.5 |
| 5 | 5 | 6 | 5 | 0 | 700 | 350 | 100 | 101.5 |
| 6 | 6 | 7 | 6 | 0 | 700 | 350 | 100 | 100.0 |
| 7 | 7 | 8 | 6 | 0 | 500 | 250 | 100 | 100.5 |
| 8 | 8 | 9 | 6 | 12 | 700 | 350 | 100 | 99.5 |
| 9 | 6 | 9 | 5 | 6 | 500 | 250 | 100 | 100.5 |
| 10 | 9 | 10 | 5 | 11 | 700 | 350 | 100 | 100.0 |
| 11 | 5 | 10 | 4 | 5 | 500 | 250 | 100 | 100.5 |
| 12 | 10 | 11 | 4 | 10 | 700 | 350 | 100 | 100.5 |
| 13 | 4 | 11 | 3 | 4 | 500 | 250 | 100 | 100.5 |
| 14 | 11 | 12 | 3 | 9 | 700 | 350 | 100 | 100.0 |
| 15 | 3 | 12 | 2 | 3 | 500 | 250 | 100 | 100.5 |
| 16 | 12 | 13 | 2 | 8 | 700 | 350 | 150 | 100.0 |
| 17 | 2 | 13 | 1 | 2 | 500 | 250 | 100 | 100.5 |
| 18 | 13 | 14 | 1 | 7 | 700 | 350 | 150 | 99.5 |
| 19 | 1 | 14 | 1 | 0 | 500 | 250 | 100 | 100.0 |
| 20 | 14 | 15 | 7 | 0 | 450 | 225 | 100 | 100.0 |
| 21 | 15 | 16 | 7 | 13 | 700 | 350 | 150 | 100.0 |
| 22 | 13 | 16 | 7 | 8 | 450 | 100 | 100 | 100.5 |
| 23 | 16 | 17 | 8 | 14 | 700 | 350 | 100 | 99.0 |
| 24 | 12 | 17 | 8 | 9 | 450 | 100 | 100 | 100.5 |
| 25 | 17 | 18 | 9 | 15 | 700 | 350 | 100 | 99.5 |
| 26 | 11 | 18 | 9 | 10 | 400 | 100 | 100 | 99.5 |
| 27 | 18 | 19 | 10 | 16 | 700 | 350 | 100 | 100.5 |
| 28 | 10 | 19 | 10 | 11 | 450 | 100 | 100 | 99.5 |
| 29 | 19 | 20 | 11 | 17 | 400 | 200 | 150 | 100.0 |
| 30 | 9 | 20 | 11 | 12 | 500 | 150 | 150 | 100.5 |
| 31 | 20 | 21 | 12 | 18 | 900 | 300 | 150 | 99.5 |
| 32 | 8 | 21 | 12 | 0 | 400 | 100 | 100 | 100.5 |
| 33 | 21 | 22 | 18 | 0 | 600 | 250 | 100 | 100.5 |
| 34 | 22 | 23 | 18 | 0 | 900 | 200 | 100 | 99.5 |
| 35 | 20 | 23 | 17 | 18 | 500 | 100 | 150 | 100.5 |
| 36 | 23 | 24 | 17 | 23 | 700 | 200 | 100 | 99.5 |
| 37 | 19 | 24 | 16 | 17 | 450 | 200 | 100 | 100.0 |
| 38 | 24 | 25 | 16 | 22 | 700 | 200 | 100 | - |
| 39 | 18 | 25 | 15 | 16 | 450 | 150 | 100 | - |
| 40 | 25 | 26 | 15 | 21 | 700 | 200 | 100 | - |
| 41 | 17 | 26 | 14 | 15 | 450 | 150 | 100 | - |
| 42 | 26 | 27 | 14 | 20 | 700 | 200 | 100 | - |
| 43 | 16 | 27 | 13 | 14 | 450 | 150 | 100 | - |
| 44 | 27 | 28 | 13 | 19 | 700 | 200 | 100 | - |
| 45 | 15 | 28 | 13 | 0 | 450 | 200 | 100 | - |
| 46 | 28 | 29 | 19 | 0 | 500 | 250 | 100 | - |
| 47 | 29 | 30 | 19 | 0 | 700 | 200 | 100 | - |
| 48 | 27 | 30 | 19 | 20 | 500 | 200 | 100 | - |
| 49 | 30 | 31 | 20 | 0 | 700 | 200 | 100 | - |
| 50 | 26 | 31 | 20 | 21 | 500 | 200 | 100 | - |
| 51 | 31 | 32 | 21 | 0 | 700 | 200 | 150 | - |
| 52 | 25 | 32 | 21 | 22 | 500 | 200 | 150 | - |
| 53 | 32 | 33 | 22 | 0 | 500 | 100 | 150 | - |
| 54 | 24 | 33 | 22 | 23 | 400 | 100 | 100 | - |
| 55 | 33 | 34 | 23 | 0 | 500 | 100 | 100 | - |
| 56 | 23 | 34 | 23 | 0 | 400 | 100 | 100 | - |
| 57 | 20 | 36 | 0 | 0 | 50 | 0 | 250 | - |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 58 | 13 | 35 | 0 | 0 | 50 | 0 | 250 | - |
| 59 | 32 | 37 | 0 | 0 | 50 | 0 | 250 | - |

**Network Design**

In the direct pumping systems, the system cost includes the cost of pipes, pumps, pumping (energy), and operation and maintenance. In order to use commercial pipe sizes directly in the design problem formulation, the Linear Programming (LP) technique has been selected as an optimization tool. To make application of LP possible, it is considered that each pipe link $L_i$ consists of two commercially available discrete sizes of diameters $D_{i1}$ and $D_{i2}$ having lengths $x_{i1}$ and $x_{i2}$ respectively. Thus the cost function $F$ is written as

$$F = \sum_{i=1}^{i_L}\left(C_{i1}x_{i1} + C_{i2}x_{i2}\right) + \rho g k_T \sum_{n}^{n_L} Q_{Tn} h_{on} \quad (1)$$

where $C_{i1}$ and $C_{i2}$ = the capitalised cost of one meter length of pipe of diameters $D_{i1}$ and $D_{i2}$ respectively, $Q_{Tn}$ = flow at input supply node $n$, $h_{on}$ = pumping head at input supply node $n$, $n_L$ = number of imput supply nodes, $\rho$ = mass density of water, $g$ = gravitational acceleration; and $k_T$ = pump and pumping cost coefficients (Swamee and Sharma 2008).
The network is subject to the following constraints:

- The sum of lengths $x_{i1}$ and $x_{i2}$ is equal to the pipe link length $L_i$. That is
  $x_{i1} + x_{i2} = Li$. for $i = 1, 2, 3, \ldots i_L$ (2)
- The pressure head at each node should be equal to or greater than the prescribed minimum head $H$. That is

$$\sum_{i\varepsilon T_j} \frac{8Q_i^2}{\pi^2 g}\left[\frac{1}{D_{i1}^4}\left(\frac{f_{i1}\,x_{i1}}{D_{i1}}\right) + \frac{1}{D_{i2}^4}\left(\frac{f_{i2}\,x_{i2}}{D_{i2}}\right)\right] - h_{on} \le z_{on} - z_j - H \quad (3)$$

where $f_{i1}$ and $f_{i2}$ are the friction factors for the two pipe sections of the link $i$; $z_{on}$ = elevation at $n^{th}$ input source; $z_j$ = ground level of node $j$; $H$ = prescribed terminal head and $T_j$ = a set of pipes (pipe track) connecting the node $j$ to an input source point $S(n)$. The algorithm for the identification of pipes connecting a node to a supply node is also described here.

As described in the subsection on Maintaining Looped Configuration, all the pipes of the network has to be brought in the headloss constraints inequation. This can be achieved by formulating the inequation (3) for all the originating nodes $J_t$ ($i$) of all the pipe tracks in the network, brings all the pipes of the network constraint equations for headloss (Sharma and Swamee 2005; Swamee and Sharma 2008).

As depicted in Figure 2, pipe 13 supplies water to node 4. Thus, the originating node of pipe 13 track is node 4 ($J_t$.(13) = 4). Moving against the direction of flow, pipes 13, 26, 39, 52 and 59 will generate a headloss constraint inequation for node 4, which is terminating at supply node 37 (input source point number 3, see Table 3). The track of pipe 13 contains pipes 13, 26, 39, 52 and 59. Scanning pipe-wise the originating node, the headloss constraint inequations can be generated covering all the pipes. In such a case the total number of constraint equations formulated is $i_L$. This will give rise to the formulation of more than one headloss constraint equation for some of the nodes numbering total to $k_L$. Such headloss constraint equations for the same node will have different set of pipes in the headloss inequation.

Using the cost function and the constraints, a LP problem is formulated to select the pipe link diameters and pumping heads. For the given network in Figure 2, the rate of water supply as 250 litres per person per day, peak factor of 2.5, terminal head 15meters, rate of electricity $0.20/KWh, rate of interest as 0.07 per yr have been adopted. The cost data was obtained from reference rate manual for the valuation of water, sewerage and stormwater assets (Samra and Essery 2003).

As the analysis of a water distribution system is an integral part of the design process, the system is first analysed for pipe discharges and input point discharges for initialized pipe diameters and pumping heads. The system is then designed based on computed pipe discharges and input point discharges giving pipe diameters and input point pumping heads. The system is analysed again for pipe discharges and input point discharges based on computed pipe diameters and input pumping heads. The processes of analysis and design are repeated until two consecutive solutions are close in terms of total capitalised cost. The pipe materials are selected at this stage based on available economic commercial sizes and pressure in pipes. The processes of analysis and design are repeated again till the two solutions are close. For the water distribution network in Figure 2, the pumping head and pumping flow rates are give in Table 2 and the pipe diameters in Figure 1. The variations of capitalised cost over the LP iterations are shown in Figure 3.

**Table 2:** Input point discharges

| Input point | Input point node | Input point head (m) | Input point discharge $m^3/s$ |
|---|---|---|---|
| 1 | 35 | 19 | 0.0374 |
| 2 | 36 | 20 | 0.0367 |
| 3 | 37 | 19 | 0.0196 |

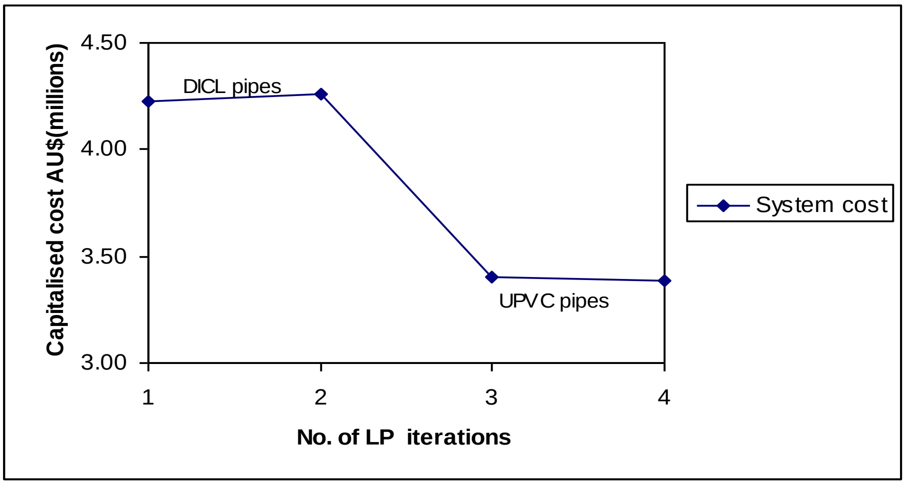


**Figure 3:** Variation of system cost with LP iterations

## CONCLUSIONS

The algorithm for the design of a looped water distribution network has been developed using commercial pipe sizes directly in the optimisation problem formulation. The looped configuration is maintained in the synthesis of the distribution network by bringing all the pipes of the network in one or the other constraint equation of the LP formulation thus preserving the original configuration of the network. The looped network is not converted to branched network in design based on least cost consideration.

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