

Optimal Capital Structure and Corporate Governance: An Application of Differential Game Theory

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Abstract

The conflicts of interest between the shareholder and the manager, and the debtholder and the manager create complex dynamical problems. These are central issues of corporate governance as the capital structure of a company impacts the value of the company. The conflicts of interest are also complex dynamical problems because the strategic relationship between the shareholder, debtholder and the manager changes over time and the quality of efforts committed by the manager to maximising the company value are unobservable in each period.

To maximise the shareholder, debtholder and company's value, optimal incentive contracts and mechanism designs of corporate governance are required to motivate the manager to select optimal actions in the company. The implementation of corporate governance in designing an optimal capital structure in corporate finance aims at achieving efficient financial management that can eliminate the agency problems existing among shareholders, managers, and debtholders. The use of debt as a mechanism to resolve agency problems can destabilise the optimal capital structure specification of the company, as suboptimal capital structure introduces the risk of financial distress and bankruptcy. This explains the rising demand for best corporate governance practices.

The differential game theory has been proven to be a comprehensive framework in the analysis of non-cooperative games. This study adopts this framework as a dynamic technique to resolve the underlying agency and information asymmetry problems and to establish cooperation and renegotiation proofness over time. The research develops new mathematical models that can embed the static, dynamic principal-agent and information asymmetry issues associated with the choice of an efficient capital structure leading to efficient incentive contracts in a continuous-time setting. The models are solved using Nash open-loop, Nash feedback equilibrium and Pareto optimisation solution concepts to offer robust analysis in a deterministic framework.

In addition, the study examines the impact of historical cost, fair value and creative financial accounting methods on the dynamic mechanism design models

and the optimal capital structure specification. To fill the gap between theoretical dynamic models for mechanism design, capital structure and real-world application of such models, this study simulates the models using data from the 2018 financial statements of Metcash Limited, an Australian ASX 200 company. Verification and validity tests conducted via convergence tests and sensitivity analyses prove the plausibility of the models.

The solutions of the model show that the differential game-based model motivates the manager to provide optimal efforts and select the optimal investment. The model recommends the optimal debt repayment strategy, optimal level of monitoring, and the manager's optimal pecuniary incentives to the shareholder throughout the dynamic contract. The model also recommends the shareholder's optimal strategy if the manager is involved in creative accounting. This leads to higher financial outcomes, indicated by an increasing cash flow, a decrease in debt value, and an increase in productive assets. The model provides the debtholder's optimal cost of debt to enforce cooperation with the manager. The models and results contribute a new approach of solving the underlined problems.

Student Declaration

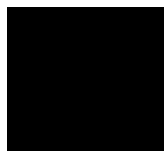


Doctor of Philosophy Declaration

“I, **Olanike Akinwunmi Adeoye**, declare that the PhD thesis entitled “**Optimal Capital Structure and Corporate Governance: An Application of Differential Game Theory**” is no more than 80,000 words in length including quotes and exclusive of tables, figures, appendices, bibliography, references and footnotes. This thesis contains no material that has been submitted previously, in whole or in part, for the award of any other academic degree or diploma. Except where otherwise indicated, this thesis is my own work”.

“I have conducted my research in alignment with the Australian Code for the Responsible Conduct of Research and Victoria University’s Higher Degree by Research Policy and Procedures.”

Signature



Date 14/02/2021

Publications during Candidature

Journal Publication:

Adeoye, OA, Islam, SM & Adekunle, AI 2020, 'Optimal capital structure and the debtholder-manager conflicts of interests: A management decision model', *Journal of Modelling in Management*.

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I dedicate this PhD to the memory of my father.

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List of Symbols/Abbreviations

r	Riskless rate
b	Manager's reservation utility
β	Payout rate of company's productive assets
$E(t)$	Company equity
$Y(t)$	Cash flow
$I(t)$	Pecuniary incentive
$S(t)$	Value of company's productive assets
$D(t)$	Company debt
θ	Effective tax rate
α	Ratio of new debt to existing company debt
α_2	Cost of debt
$u_1(t)$	Debt repayment
$u_2(t)$	Monitoring
$u_3(t)$	Company investment in each period
$u_4(t)$	Manager's effort
μ	Asset depreciation rate
χ	Proportion of management fee
Superscript *	Optimal result
$c(t)$	Cost of debt
$l(t)$	Cost of equity
$M(t)$	Liquid reserve
$V(t)$	Company's total capital
ρ	Discount rate
IFRS	International Financial Reporting Standards
ASX	Australian Securities Exchange
AASB	Australian Accounting Standard Board

Chapter 1 – Introduction

1.1 Introduction

With the consistent failings of public corporations, good corporate governance practices are in crucial demand in the ever-evolving global market. In this context, this study explores optimal corporate governance frameworks and mechanisms designed to create an efficient capital structure. The 1954 Arrow-Debreu general equilibrium model of frictionless markets (which are assumed to be perfectly competitive and absolute, unhindered by taxes, transaction costs, and informational asymmetries) could have been a compelling tool for examining the pricing of claims in financial markets. However, its drawback is that it paid little attention to firms' financial choices and their governance in an imperfect market condition. This led to agency considerations. As a result of this drawback, the presumption that the size of capital is unaffected by how the capital is distributed, or the sources of capital, had to be rejected. This led to several significant publications in the 1970s, such as Jensen and Meckling (1976), Myers (1977) and Ross (1977). This significant shift of interest to agency considerations in corporate finance enjoyed remarkable backing from an enormous amount of empirical literature and from the implementation of institutional design (Tirole 2006). The cost arising from the conflicts of interest prevalent in agency relationships impact the firm value and, therefore, require robust corporate governance mechanisms to minimise the problem. Hence, corporate governance principles are implemented in corporate finance to achieve efficient financial management. This is done by eliminating agency problems that exist in corporate financial management with respect to components of corporate finance, such as dividend policy, capital structure, and capital budgeting, among others.

The financing complexity and agency issues of publicly listed companies has particularly underscored the ties between capital structure and corporate governance. The optimal capital structure is viewed in this study as one of the wealth maximisations goals of the shareholder, as well as a financial management goal of the company. Following the pecking order theory, the order of preference for financing a firm are namely, internally generated returns, debt issue and equity issue. Internally generated returns (retained earnings) are the

most preferred source of finance because they are less information sensitive. However, the ratio of the combination of capital – debt or equity to be used, that is the optimum capital structure, remains an unresolved problem for many companies. Various studies offer different insights and evidence on this issue. The pecking order theory rules against the static debt-equity trade-off theory, which suggests that a firm identifies a static optimal debt-equity ratio and constantly moves towards this. The pecking order theory argues against this because a firm is unable to exhaust the choice of debt before considering the use of equity due to the consequences of excessive indebtedness. This generates an argument for the specification of an optimal capital structure in a dynamic context. Furthermore, both the shareholder and the manager have different perspectives and preferences about the ratio of the combination of capital to be used by the company, but with a general understanding that an optimum capital structure is one that maximises the firm's value while minimising its cost of capital.

In the Miller (1977) model, equity has a personal tax advantage since capital gains are taxed on realisation, while interest deductibility gives the use of debt a corporate tax advantage. A more complex dilemma is how to resolve the issue of optimum capital structure in the presence of agency frictions. Agency problems such as asset substitution, cash diversion and the inappropriate efforts of the manager, destabilise the ideal expectations of company debtholders and shareholders, thereby impacting their forecasts of the company's productivity. This departure from ideal expectations imposes certain costs on the company which impacts the company value. These costs are known as the cost of agency relationships in publicly listed companies due to information asymmetry. A company requires an optimal capital structure to sustain its value proposition for doing business. In the efficient financial management of corporations, the choice of optimal capital structure plays a significant role as this choice affects the financial performance of corporations. A suboptimal capital structure can also increase the company's risk of financial distress. The aim of corporate governance implementation in corporate finance is to achieve efficient financial management for a company by reducing the costs and inefficiency of agency problems discussed above.

The models developed in this research are based on the implications of financial accounting theory rather than how to account. This follows the decision usefulness of financial accounting disclosures for investors, but most importantly the efficient contracting roles of financial reporting. The economics of imperfect information highlight the agency problem found in publicly listed companies based on the theory of rational decision-making by key stakeholders. The framework of guidelines and relationships involved in corporate governance, which are called the corporate governance mechanisms, include the design of contracts such as debt contracts and management compensation contracts to eliminate agency problems in corporate management. To a large extent, the effectiveness of such contracts determines the quality of a corporate governance system (Scott 2015). Contract theory discusses the design of these contracts and it builds on the intuition of the agency theory. In the agency theory, a principal-agent relationship exists between two or more parties in which the agent takes responsibilities and acts on behalf of the principal, in a framework of decision-making (Ross 1973). Therefore, the agency theory is defined as the division of game theory that analyses the design of contracts to spur a rational agent to take actions on behalf of a principal (Scott 2015). The two prominent groups in the discourse of corporate governance mechanisms are those internal to the company and those external to the company (Gillan 2006). This study analyses the two major financing relationships of a leveraged publicly listed company, one from the internal group and another from the external group. The debtholder-manager relationship and the shareholder-manager relationship are the significant agency relationships in finance, with the manager referred to as the agent in both relationships. This follows Black and Scholes' (1973) postulation that corporate securities held by these claimholders (the shareholder and the debtholder) can be viewed as options on the assets of the firm.

This chapter is structured as follows: Section 1.2 presents the problem statement of the research; Sections 1.3 and 1.4 discuss the aims and significance of the study; Section 1.5 discusses the research methodology used; Section 1.6 presents the original contribution of the study; and finally, Section 1.7 presents the organisation of the thesis.

1.2 Problem Statement

Obtaining an optimal capital structure consistent with corporate governance principles for publicly listed companies becomes more complicated in the presence of the underlined agency problem of information asymmetry. Agency problems exacerbate existing problems associated with the choice of companies' efficient capital structure. This highlights a need for optimal mechanism design models for corporate governance that mitigate information asymmetry problems in firms by minimising the conflicts of interest and specifying an optimal capital structure. It is also necessary to develop these optimal mechanism designs in a dynamic framework. Dynamic frameworks offer flexibility in the use of a model, as they provide sophisticated analysis of the problem and its solution over time. In a dynamic framework, the non-cooperation or cooperation of the players is analysed over a finite or infinite period. This study analyses the design of corporate governance mechanisms or contracts that minimise the inherent conflicts of interest between the manager and shareholder, and the manager and debtholder, to obtain an optimal capital structure in a dynamic framework. This provides robust corporate governance models that are efficient in resolving agency problems in firms and optimising its capital structure over time. The study therefore seeks to answer the central research question:

What are the time-dependent optimal corporate governance frameworks and mechanisms for choosing a company's efficient capital structure?

The research question is answered by employing incentive or mechanism designs to develop corporate governance models via the differential game theory framework to optimise the company's capital structure.

1.3 Aim of the Study

The aims of the study are to:

1. Develop an optimal capital structure via an optimal debt-equity ratio and dynamic optimal mechanism design framework to resolve the conflicts of interest between the shareholder and the manager.

2. Resolve the agency problem of asset substitution and minimise the conflicts of interest between the debtholder and the manager via mechanism design, as well as specify the optimal capital structure over time.
3. Develop dynamic corporate governance models based on differential game theory that offer a firm optimal financing and maximise its value in a dynamic framework.
4. Examine the impact of financial accounting theories on the corporate governance models.

1.4 Research Methodology

The study adopts differential game theory as it is suitable for analysing strategic interactions between two or more parties within a standard and specified framework. Games are models of social organisations and their solutions can be specified as stable standards of behaviour (Kunz 2018). Game theory involves the use of mathematical models in analysing conflict and cooperation amongst different utility optimisers whose actions and decisions influence each other's outcomes (Migdalas 2002). The models presented in this thesis were developed as non-zero-sum games using the differential game framework, with each player's interest formalised in an optimal control setting. The models were then solved using the Nash equilibrium open-loop, Nash equilibrium feedback outcomes and the Pareto-efficient outcomes via the Pontryagin's maximum principle. These three solution concepts are the standard solution concepts used in differential game theory. Other concepts that may be used but were not considered in this research include the dominance equilibrium, and Stackelberg concepts, among others. The financial management models developed for efficient contracting in this research were further evaluated to examine the impacts of historical cost, fair value and creative accounting on dynamic mechanism models and the optimal capital structure. To test the models, data from Metcash 2018 financial statements were collected, Metcash is an Australian Securities Exchange (ASX) 200 listed company. Where necessary, the average of five or six-years of financial information was used to provide reliable estimation for certain parameters of the model. Reasonable and justifiable assumptions

were used in deriving data that are impossible to be retrieved from public sources. The model was then computed numerically in the MATLAB2020a(64-bits) programming environment. Verification and validity tests were conducted via convergence tests and sensitivity analysis was carried out to ensure the plausibility of the models.

1.5 Significance of the Study

A positive correlation between corporate governance and a company's value has been well established (Hanjun & Junxi 2006; Harford, Mansi & Maxwell 2008; Naimah & Hamidah 2017). Philippon (2006) observed that poorly governed companies react more to aggregate shocks than well governed companies. Poor corporate governance systems cause lower productivity growth (Heinrich 2002). In addition, investors have greater assurance in organisations with good governance, which consequently improves the ability of such organisations to compete for capital (ASX Corporate Governance Council 2014).

This research included the design of quality corporate governance models with respect to a company's capital structure. Good corporate governance models can generally:

- a. Improve the company's reactions to aggregate shocks
- b. Augment its chances to compete for capital
- c. Enhance the company value.

The dynamic optimal mechanism design models presented in this study resolve the information asymmetry problems between the shareholder and manager, and the debtholder and manager using complementary corporate governance instruments. This will reduce the risk of financial distress and ultimately improve the company value. The differential game-based corporate governance models developed in this study contribute to dynamic mechanism models for solving financial management problems.

1.6 Original Contribution of the Study

This thesis contributes to the existing body of knowledge in corporate finance, corporate governance, dynamic mechanism design theory, differential game

theory, and financial accounting theory. It establishes the advantages of differential game-based framework in modelling and specifying an optimal capital structure which simultaneously resolves underlying agency and information asymmetry problems, hence achieving quality corporate governance through a dynamic integrated analysis. The vital contributions of the thesis are established via the methodical use of a dynamic game theory approach which rigorously analyses the dynamic strategic interaction between the parties involved in the agency relationships. By investigating and integrating the concepts of mechanism design, complementary corporate governance and financial and management accounting into a dynamic game framework, the thesis provides a new dynamic and sophisticated approach to specify an optimal capital structure and resolve dynamic agency problems. The results of the credible and robust analysis obtained in the thesis can minimize the risk of financial distress, establish good corporate governance systems and improve company value.

1.7 The Organisation of the Thesis

This thesis is organised into seven chapters. Chapter 1 has introduced the scope and direction of the research, including the problem statement, aims and significance of the thesis and the research methodology used. The chapter provides a general background to the key concepts and issues analysed in the thesis. This includes dynamic agency relationships, information asymmetry problems, optimal capital structure, and the differential game theory with a focus on improving corporate governance systems and company value.

Chapter 2 provides a comprehensive and critical review of the literature related to existing corporate governance models, capital structure, efficient contracting, mechanism design, accounting policies, dynamic agency problems, differential game applications and other relevant concepts, as well as the interrelationships amongst these key concepts. This situates the study in the existing body of research. The limitations of existing corporate governance models are discussed, and the research motivation and contributions are outlined thereby highlighting the gap to be filled by this new study.

Chapter 3 provides an in-depth outline of the research methodology, examines the conceptual framework and discusses the case study company and financial

market in which it operates. The three main methodology sections of the thesis are Chapters 4, 5 and 6.

Chapter 4 examines the principal-agent relationship between the shareholder and the manager, establishing a dynamic game relationship between them. The chapter develops a differential game-based model which embeds complementary corporate governance mechanisms. This motivates the manager to act in the shareholder's best interests. The model is simulated using a publicly listed company's real-life data. The chapter provides a robust analysis of the implications of the model in a dynamic continuous time setting. It develops a specification of an optimal capital structure, which minimises underlying agency and information asymmetry problems.

Chapter 5 models the debtholder-manager agency relationship as a differential game, as the interest of the players are contradictory with respect to asset substitution. The differential game-based model provides optimal strategies and outcomes using the Nash equilibrium open-loop, feedback and Pareto outcomes via Pontryagin's maximum principle. The model specifies an optimal capital structure that minimises the agency costs of debts. The model is also simulated using a publicly listed company's real-life data. Sensitivity analysis are carried out in three encounters to examine the outcomes of the model in practice.

Chapter 6 examines and analyses the impact of historical, fair value and creative financial accounting measurement and reporting on the dynamic principal-agent game model for optimal capital structure. The impacts of the financial accounting theories are evaluated using the previously developed financial management models presented in Chapters 4 and 5 of the thesis. The results and implications derived from the analysis are discussed accordingly.

Finally, Chapter 7 presents the overall summary of the research, the key concepts of the study and the obtained results. It highlights the major findings, contributions and limitations of the thesis, as well offering suggestions for future research opportunities in this area.

Chapter 2 – Literature Review

2.1 Introduction

In this chapter, the existing literature on the integrated issues of capital structure, corporate governance, efficient contracting, agency problems and differential game theory are critically reviewed. This represents the blocks upon which the research has been built. It situates the research in the existing body of knowledge in the corresponding research area. The Venn diagram below provides a pictorial illustration of the integrated issues to be discussed in this chapter.

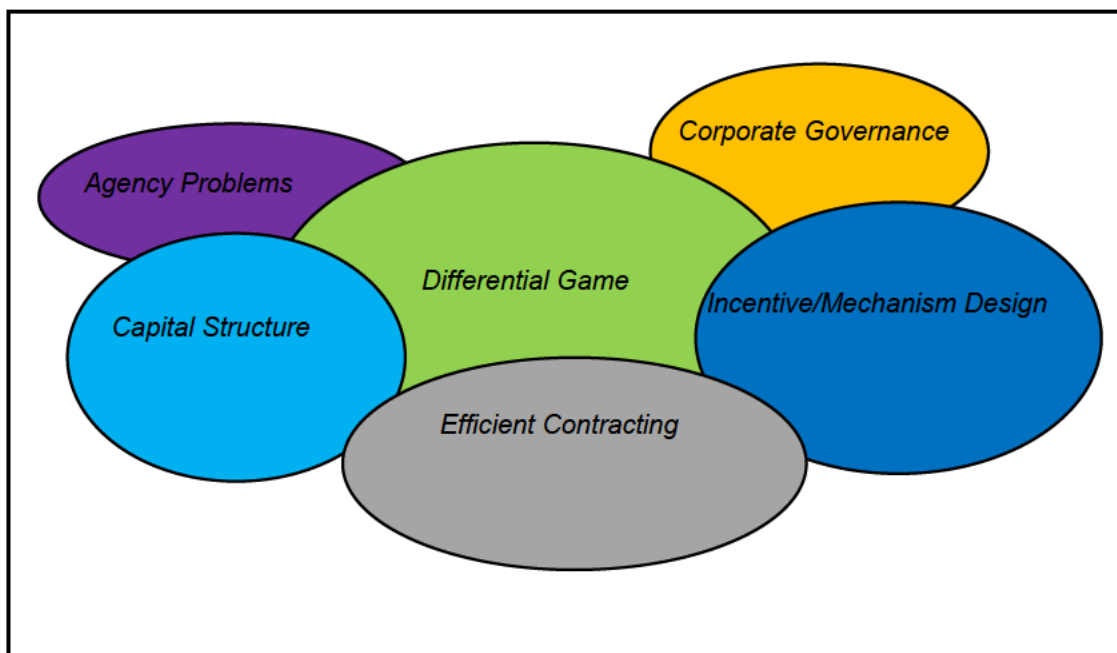


Figure 2.1 Integrated background of the research

Structure of the chapter

This chapter is organised as follows. Section 2.2 describes corporate governance. Section 2.3 establishes the relationship between efficient contracting, agency problems of information asymmetry and corporate governance in capital structure decisions. Sections 2.4 and 2.5 discuss corporate governance as a mechanism design and provides examples of existing corporate governance mechanisms. Section 2.6 examines the application of differential game theory in corporate finance. Finally, Section 2.7 provides the research motivation and contribution.

2.2 Corporate Governance

Naimah and Hamidah (2017) describe corporate governance as a framework of guidelines, relationships, directives, structures and practices by which, leadership and control are exercised in business organisations. Shleifer and Vishny (1997) give a slightly different definition than that given by Naimah and Hamidah (2017), defining corporate governance as the intentional efforts of owners of business corporations to make a return on their investments. The OECD (2012) describes the same concept as the precautionary measures implemented by the owners of a firm to defend their interests against agency problems. These definitions underscore the interests of owners of business organisations, popularly known as shareholders or investors in public companies.

Corporate governance dates from the 1970s when it was developed in the United States of America (USA) (Cheffins 2012). The concept is informed by several ideologies from different fields of study, such as management, finance, accounting, psychology, sociology, political science, law, and philosophy. However, this study analyses corporate governance mainly as a subject in the field of accounting and finance. In the finance literature, the major parties involved in corporate governance are the debtholders, shareholders and managers of business firms. For general modelling purposes, the managers of a publicly listed company are singly referred to as either manager or, interchangeably, as CEO, while shareholders are singly referred to as the shareholder or owner, and debtholders simply as the debtholder. The models are described hypothetically between the debtholder and manager, and the shareholder and manager.

The shareholder-manager relationship is prevalent in public firms because of the separation of management and ownership involved in the complexity of such firms (Thomsen & Conyon 2012). Hence, this study is conducted for publicly listed companies.

In the twenty-first century, the significance of corporate governance has become undeniable, in the wake of corporate downturns emerging from managerial scams, malfeasance, and dereliction that resulted in colossal declines in investor wealth. Examples of companies involved in such corporate scandals include Enron, and Worldcom, among others. Thus, questions such as 'who (if anybody)

is responsible for defending and promoting their investment?’ arose from investors (Baker & Anderson 2010). Therefore, in the finance context, corporate governance can be defined as a mechanism designed to manage the conflict arising from the separation of management and ownership in public firms.

The three major characteristics of principal-agent problems associated with the shareholder-manager relationship are that:

- the shareholder-manager relationship is dynamic (long-term),
- the manager has opportunistic tendencies and abilities to manipulate company profits, and lastly,
- motivating the manager to contribute quality efforts can extend to other financial management decisions or actions such as hubris, adequate payout provisions, risk taking, efficient cost cutting, asset substitution, etc.

This implies that the corporate finance goal of maximising shareholder value may be impeded by agency problems. One main focus of the corporate governance regulation is the contractual protocol in management and debt contracts (Bolton & Dewatripont 2005).

In the design of optimal contracts via the financial management models developed in this study, dynamic issues of strategic interactions over time between the shareholder and manager, and debtholder and manager will be analysed. This includes trigger strategies required to ensure the optimal contract is not renegotiated. This is important because these relevant dynamic issues relate to the dynamic aspects of the agency problem over time.

2.3 The Relationship between Efficient Contracting and Agency Problems in Capital Structure Decisions

Efficient contracting has been underlined as a major component of corporate governance with respect to resolving agency problems. It is defined as the study of the significance of financial accounting information in managing the information asymmetry between contracting parties. Hence, financial accounting information plays a role in improving the quality of contracts, stewardship and corporate governance systems. In the corporate governance context, company

shareholders present a contract to a newly appointed manager or CEO, who decides whether to accept the contract or not. This contract is known as the management contract. Once the contract is accepted and signed by the CEO, the CEO possesses an information advantage over the shareholder. Such information can include key economic facts that may affect the firm's value. Hence, the information advantage often gives the agent incentives to exude opportunistic behaviours rather than act in the interests of the principal (Scott 2015). Similarly, a debtholder presents a debt contract (also known as bonds) to the manager during a company's debt acquisition process. On behalf of the shareholder, the manager considers the contract and may accept or reject it. If the manager accepts the contract, they henceforth possess more detailed information about the debt's productivity and other important company financial statistics. This represents an information advantage over the debtholder. Once the contract is signed, the final contract is autonomous of the preceding bargaining process leading up to the signed contracts (Bolton & Dewatripont 2005).

The problem associated with such an information advantage is known as information asymmetry. This is the major drawback associated with the separation of ownership and management in public firms and, consequently, it is acknowledged as a major corporate governance problem from a theoretical perspective. Such problems are known as agency problems (Thomsen & Conyon 2012). As explained, information asymmetry describes circumstances in which two parties to an intended transaction have unequal information. The conflict of interests emanating from the relationships between the shareholder and manager, and debtholder and manager can cause suboptimal allocation of resources in the firm. Information asymmetry and agency play a key role in corporate finance and are central in the information economics literature (Copeland, Weston & Shastri 2014).

Hart (1995) explains why agency problems are a core of corporate governance. In the absence of agency problems, all stakeholders would be expected and told to optimise the firm's resources. In this supposedly 'perfect' scenario, termed the 'black box' situation, it is presumed that all stakeholders would naturally be committed to the goal, efforts would be generally reimbursed, and there would be

no need for additional motivations in the form of incentives. This neoclassical theory, however, disregards the possibility of opportunistic behaviours amongst stakeholders and assumes that actions and costs are verifiable.

The problem of information asymmetry is commonly referred to as adverse selection (hidden information) and moral hazard (hidden action) (Bonanno 2011). Adverse selection refers to a situation in which the agent possesses more detailed information about the company than the principal who requires the same information to make investment decisions, as well as other information that may signal default to the debtholder. Moral hazard, on the other hand, represents the inability of the company owner to observe and monitor the efforts and actions of the manager. As a result, the owner is left with inadequate information with which to assess and make decisions about executive compensation. Agency theory has been applied to several common economic issues, such as the relationship between a house seller and a real estate agent, or a business firm and a construction firm, as well as sharecropping (Perloff 2017; Sadoulet & De Janvry 1995; Samuelson & Marks 2015). This suggests a wide application of agency theory to different economic situations.

Due to the information advantage available to managers of companies, some may become involved in embezzlement, empire building, excess expenditure, self-dealing, overinvestment, and entrenchment. Hence, the following are highlighted by Copeland, Weston and Shastri (2014), and Thomsen and Conyon (2012) as the main elements of the agency problem:

- the separation between principal and agent,
- the rationality of both parties to promote their own interests,
- conflicting interests - parties involved have different utility functions,
- asymmetric information - the agent is more informed about his actions and the state of the company,
- shirking by the agent
- diversion of resources by the agent for personal consumption,
- differential time of the principal-agent relationship,
- uncertainty (risk) - the presence of other uncontrollable factors,

- risk aversion - some level of risk is involved in the agent's executive compensation, a risk-averse manager will request more compensation for this.

The three commonly discussed agency problems in corporate finance are the unobservable actions of the manager, risk-shifting, and underinvestment. These three agency problems directly impact the optimal capital structure of the company. If the manager's actions are unobservable, their decision to shirk by diverting the company's cash for private consumption and applying an inappropriate level of effort might lead the manager to prefer a suboptimal capital structure. This promotes the manager's opportunistic behaviour at the detriment of the shareholder. Similarly, the manager's information advantage can cause him to shirk, thereby impacting the company's debt value and the debtholder's wealth. The different choices of capital structure preferred by the shareholder and the manager, as well as the manager's opportunistic tendencies, complicates the agency problem.

The agency problem is described as a game of incomplete information played by agents, known as the players of the game. Due to unequal access to information, the strategic interaction between the agents becomes a complex problem, similar to a game situation. Hence, over the years, agency theory has been described as a form of game theory in the design of contracts between agents (Scott 2015). The peculiarities of the agency relationship between the shareholder and the manager, as well as the debtholder and manager, reflect the characteristics of a non-cooperative game. This is because the objectives of the players differ, and each party has enough incentive not to cooperate.

These agency problems can be resolved by designing efficient dynamic contracts, incorporating corporate governance instruments that mitigate against the identified problems.

2.3.1 Different Forms of Managerial Contracts

2.3.1.1 Contract types

Optimal contracts under information asymmetry are described as second-best contracts, as first-best contracts are only obtainable when the actions of each agent are observable in the game (Bolton & Dewatripont 2005). Managerial and debt contracts are designed to share the risk of the company with the manager and align the interests of the manager with the shareholder or debtholder. These therefore serve as forms of corporate governance. In designing contracts that enhance the mutual interests of the players, it is imperative to discuss existing types of contracts. The four common types of contracts are: fixed-fee rental, hire, contingent and implicit contract. Each is discussed in more details below.

Fixed-Fee Rental Contract

In this form of contract, the agent pays the principal a fixed rent (Jørgensen, Mohagheghi & Grimstad 2017) because the principal would not accept a lesser pay than agreed upon. In the shareholder-manager context, the principal may agree to earn a fixed amount from the company profit as a reward for releasing assets to run the company. The residual profit after deducting the fixed rent is earned by the manager, thus providing an incentive for the manager to optimise the firm's capital structure. The disadvantage of using the fixed-fee arrangement, however, is that the manager not only receives the residual profit but also bears all the risk. Otherwise, the principal may agree to pay the manager a fixed salary. Fixed-fee contracts are frequently employed in audit engagements (Copley & Doucet 1993). They are also prevalent in debt contract arrangement as the debtholder only earns the agreed-upon cost of debt (a fixed percentage in most cases) in addition to the debt principal value.

Hire Contract

Here, the principal offers to pay the manager a certain pre-specified amount for each measurable output. Unlike the fixed-fee arrangement, this offers a more flexible arrangement for the agent. However, the agent may have no incentive to overperform once they reach their desired income level. This implies that, without monitoring, the agent may not pursue a joint optimal level of output as desired by the principal. On the other hand, the manager may decide to purchase the

company in a leveraged buyout if they set the pre-specified amount to be paid to the shareholder for each measurable output. The hire contract may be too complicated to use in a comprehensive corporate governance model in large and/or complex companies. Additionally, hire and fixed-fee contracts do not sufficiently reward agent productivity and sales-enhancing actions, causing the agent to shirk (Perloff 2017).

Contingent Contract

One form of contingent contract is the **revenue-sharing contract**, in which the shareholder and the manager agree to share the revenue at a predetermined ratio. This form of contract again permits the manager to understate the revenue of the organisation. Another form of contingent contract is the **profit-sharing** contract in which the two players agree to share the firm's profit by a ratio if the marginal cost of production of each output is clearly observable. Profit-sharing contracts are also known as risk-sharing contracts. However, the manager has the incentive to overstate cost and understate revenue.

Implicit Contract

This is a form of contract that relies on the reputation of the manager. This is tagged as the theory of reputation. The company owner relies strongly on the past performance of the manager in making assumptions about their future ability. The shareholder may reward or promote the manager because of this. In game theory, this is referred to as trust-based game.

Revelation Principle

While implicit contract relies on the reputation of the manager to tell the truth, revelation principle argues differently. The revelation principle argues that if an indirect mechanism design exists, there is as well a direct mechanism with truth-telling as the equilibrium. There at least three conditions to be attained before the revelation principle can be deemed feasible in application. First, the manager must have a guarantee that the truth will not be used by the shareholder to the manager's disadvantage (e.g., if the manager anticipates that the company cannot meet its debt repayment plan without additional borrowing). Second, the management contract must not be restricted by the company's outcomes. For instance, if the compensation schedule requires a maximum debt-equity ratio, the

manager may not be able to tell the whole truth about short-term borrowings and long-term borrowings as this may harm the manager's compensation. Third, the manager's ability to communicate information must not be constrained. For example, a future need or forecast to increase the debt-equity ratio must not impact the manager's current earnings. In addition, if the need required in the next period exceeds what was forecasted by the manager, this may affect their reputation. Hence, the risk of truth-telling introduces a new cost that the shareholder may not be willing to pay (Scott 2015).

In summary, with respect to contract types, the quality of effort that a manager chooses to contribute to the organisation is dependent on the form of contract employed. The form of contract employed provides enough incentive for the manager to shirk or work in committing optimal effort and selecting an optimal capital structure. All forms of the contracts discussed can be translated into games (Scott 2015). These existing forms are incorporated into the developed models in this study. These models are designed to obtain equilibrium strategies that offer the company an optimal capital structure to minimise agency problems.

All of the contracts discussed, except implicit contracts, are regarded as financial contracts in the literature. The focus of the financial contracts is to design incentive schemes that optimally align the interests of the principal and the agent (Tirole 2006).

2.3.1.2 *Contract Rigidity*

Contracts are often regarded as rigid because it can be difficult or costly to modify them. Managerial and debt contracts are reliant on accounting variables and thus it can be expected that accounting standards may change during the contract period. It may be possible to include provisions in the contract to manage unpredicted events. However, it is unrealistic to expect that all future events can be envisaged. A strategy to manage this is to preserve the originally adopted accounting policies used in determining certain variables at the start of the contract. However, this may be costly and have a significant effect on financial statements. This is often applicable to debt covenants developed in debt contracts. Arguably, a more appropriate method of dealing with changes in

accounting standard during the tenure of a contract is to grant the manager some fluidity in the choice of accounting policies (Scott 2015).

2.3.2 The Role of Financial Accounting in Efficient Contracting

Financial accounting information is an output of corporate accounting and external reporting systems. This assesses and discloses audited, quantitative data regarding the financial condition and potential of public companies. Financial accounting information generally offers input directly to corporate control mechanisms and indirectly by supplying part of the information included in share prices (Bushman & Smith 2001). Bushman and Smith (2001) provide evidence that financial accounting information might reduce the agency problems of corporate governance. The Mirrlees/Holmstrom design of the principal-agent model with moral hazard has been thoroughly investigated by accounting researchers. The model establishes a construct for understanding incentive designs and performance measures. Baiman and Demski (1980), Dye (1986), and Lambert (1985) have used principal-agent models to study monitoring problems.

Ball (1989), while advocating for the significant role of accounting in control of organisations, suggests that in setting up and expediting non-market contracts, accounting is employed as the source of information. That is, contracts such as management and debt contracts written by firms use accounting information (Emanuel, Wong & Wong 2003). Accounting is also used in budget systems, regular performance evaluation structures, bonus plans, CEO compensation systems, and cost accounting schemes (Zimmerman & Yahya-Zadeh 2011). Relative performance evaluation also known as 'yardstick competition' has been discussed by Antle and Smith (1986), Holmstrom (1982) and Wolfson (1985). The central idea of their discussion was to compare the performance of firms with similar shocks as an assessment of a manager's performance. Emanuel, Wong and Wong (2003) assert that ample concentration on contract design *ex-ante* will mitigate some of the problems identified with incentive alignment *ex-post*.

Baiman and Evans (1983), Dye (1983), and Penno (1985) have modelled contracting on unverifiable pre-decision and post-decision information communicated by agents.

Accounting information is significant in managerial contracts for investor information and stewardship purposes. Shareholders who constantly make decisions on investments require robust, and error-free information about the organisation. In addition, since managers are employed by shareholders via the board of directors, shareholders need ample and accurate information about the efforts of the manager for accurate compensation purposes. The importance of accounting information for stewardship is therefore heightened because of contract theory. In summary, financial accounting is imperative to oversee compliance with contracts (Scott 2015). As stated earlier, financial accounting information, such as the cost of capital and market value of equity and debt, impacts on efficient contracting. This leads to good corporate governance. An optimal contract in this study is one that minimises the underlying agency and information asymmetry problems and optimises the capital structure in a dynamic framework.

2.3.2.1 *Accounting Policies*

Accounting policies also play major roles in efficient contracting and in propelling managers to act in the best interests of the shareholder or debtholder. In fact, the choice of accounting policies can form part of a solution to addressing a contracting problem (Emanuel, Wong & Wong 2003). The two accounting policies that attend to the exhaustive and reliable information needs of debtholders and shareholders are conservatism and reliability, discussed as follows.

Conservatism

In making financial and investment commitments, debtholders take more interest in information about unrealised losses than in unrealised gains, since the latter is less helpful in forecasting financial hardship. Conditional conservatism, also known as impairment tests, is significant in the efficient contracting theory because it provides debtholders a more realistic financial estimation of the company (Scott 2015).

To reinforce the significance of accounting conservatism in efficient contracting, Beatty, Weber and Yu (2008) prove that accounting conservatism eases a company's opportunity for external capital, while García Lara, García Osma and Penalva (2011) assert that it also reduces its cost of capital.

Reliability

When shareholders consider a company's reported accounting information with respect to its capital structure, realised market transactions take precedence over unrealised market gains. In reporting unrealised transactions, financial parameters may be overstated and may superficially portray an organisation to be in good financial standing. Such superficial depictions of an organisation may lead shareholders into making poor investment decisions, as well as result in an incorrect assessment of the efforts of the manager. Therefore, in mitigating against possible manager opportunistic behaviours, debtholders and shareholders require reliable accounting information. Hence, reliability is a key ingredient in efficient contracting (Scott 2015).

It is clear in the discussion above that while conservatism might be important to the debtholder, reliability is more attractive to the shareholder.

2.4 Corporate Governance Mechanisms

Corporate governance consists of frameworks designed to minimise the total cost resulting from agency conflict in the management of public corporations. Therefore, it requires mechanisms or corporate governance instruments using data from financial accounting to mitigate against agency problems (Heinrich 2002).

A number of mechanisms discussed below have been examined and applied in firms to align the interests of shareholders and managers, as well as debtholders and managers. Since shareholders, debtholders and managers have different objectives and interests in the firm, these need to be synchronised using corporate governance as a mechanism design to ensure a congruity of interests via cooperation. Mechanisms for governing conflicts and achieving a congruity of interests include pecuniary incentives, monitoring, ownership structures, capital structures, intervention threats via hostile takeovers, and bankruptcy, among others. Some of the most prominent mechanisms or governance instruments are discussed below.

2.4.1 Executive Compensation Plans

Of all the existing corporate governance mechanisms developed and discussed, the most popular is the executive compensation system. The aim of this section is to discuss existing incentive compensation plans and their potential to reduce the conflicts of interest between managers and shareholders. This follows the so-called expectancy approach to motivation, which argues that people align their actions to their anticipated rewards. Incentive compensation plans are drafted to achieve congruence between the interests of shareholders and managers, and therefore create a compatibility of manager actions with shareholder objectives (Kunz 2018). They include the manager in the wealth-sharing ratio of the organisation, creating an incentive for the manager to improve the firm's performance. In other words, incentive compensation plans improve the responsiveness of the manager to a loss or increase in the shareholder's value and reduces the opportunistic tendency to take actions that destabilise the firm's optimal capital structure. They are also a means of acknowledging the manager's efforts in achieving the company's success. Therefore, these plans are proposed as a mechanism for incomplete contracting. In addition, incentive systems have the capacity to stimulate innovations among employees of a firm (Azoulay & Lerner 2013). Nyberg et al. (2010) found a positive correlation between incentives and an owner's interest. Executive compensations are functions of the firm's annual performance, as determined by accounting measures such as profit measures and economic value added. A linear relationship exists between dispersed ownership and low leverage. Most firms with dispersed ownership prefer pecuniary incentives associated with firm value in contrast to incentives tied to performance measures observed from monitoring. In contrast, firms with a strong reliance on monitoring, which do not strongly associate pecuniary incentives to firm value, generally have a higher concentrated ownership structure and are thus highly leveraged (Heinrich 2002). Alfred Sloan was particularly enthusiastic about incentive compensations for senior management. This was seen in his formation of the General Motors bonus plan in 1918 (Tirole 2006).

The discussion on executive compensation is also significant because a number of studies have established a positive correlation between company performance

and executive compensation (Bin et al. 2003; Qing & Tong 2010; Wang & Zhu 2016; Youlin 2012). However, Krauter (2013), and Zhengtang (2007) argue against this conclusion, stating instead that large executive compensation may mean better goal congruence, but this is not always the case. Excessively high compensation may signal poor corporate governance systems (Campbell 2006).

Bushman and Smith (2001) found evidence that cash compensation in isolation is not as important in analysing the general pay-performance sensitivities of CEOs. Rather, other forms of compensation, such as executives' stock and stock option portfolios, have overridden the aggregate responsiveness of executive wealth to changes in investors' wealth, and this can lead to an optimal incentive plan (Flor, Frimor & Munk 2014). Accounting information has been proven to provide significant input in executive compensation plans. This is evident in the empirical research, such as in the study of Murphy (1999), which found that 161 of 177 sample public USA firms used at least one measure of accounting profits in their annual bonus plans.

Contemporary executive compensation includes basic salary, bonus awards, employee stock options, non-equity compensation plans, change in pension values, and deferred compensation. Salary is a fixed wage an executive can anticipate taking home irrespective of their risk appetite. Pension benefits are also non-performance based, that is they are not directly associated with a company's current performance but with future performance, when the manager exits the company. Debtholders respond positively to debt-like pay components. Studies such as Kabir, Li and Veld-Merkoulova (2013) show that defined benefits pensions are associated with lower bond yield spread. However, other compensations such as employee stock options, bonus awards, deferred share units, amongst others, are performance-based incentives (Giroux 2015). Therefore, the three primary components of an executive compensation package are salary, profit-related bonus, and stock participation (Constantinides, Harris & Stulz 2003). Existing forms of bonus and incentive contracts are discussed briefly below.

2.4.1.1 *Employee Stock Options (ESOs)*

Employee Stock Options (ESOs) can be part of a financial arrangement in which the manager holds the right to purchase company stock at a later date, but at an earlier agreed price. This form of compensation plan spurs the interest of the manager in the stock performance of the company, thereby influencing the manager's interest in the company's long-term performance. By 1994, the percentage of CEOs in the USA receiving stock options had climbed up to 70% from 30% in 1980 (Rock 2013).

Scott's (2015) arguments against ESOs is that this form of compensation has the capacity to encourage risk aversion in managers. For example, managers can become extremely cautious about making risky investments, hence promoting risk-avoidance, while some of such risks could have been beneficial to the company's growth. In addition, managers may exhibit opportunistic behaviours in order to increase the ESOs value during contract renegotiation. This includes tactics such as 'pump and dump', which means a manager's actions may superficially increase the value of the company before an option exercise, increasing the value of stock which they then sell straight after. A 'spring loading' tactic may also be used by a manager to pressure directors into awarding unscheduled ESOs when they anticipate good earning news for the company. Finally, a manager might use a 'late timing' tactic to backdate ESOs to a preferred date when the share price was lower (Scott 2015). Irrespective of the criticisms against ESOs, Tirole (2006) however argued ESOs have been found to motivate managers to take considerable risks so as to increase the value of their stock options.

2.4.1.2 *Performance Shares*

Performance shares are company shares granted to a manager after they have achieved a pre-set performance objective but still remain with the organisation. One advantage of using a performance shares compensation plan is that it commits the manager to the long-term performance of the organisation, as the compensation is often associated with a manager's contribution over a four-to-six-year period (Giroux 2015).

2.4.1.3 *Stock Appreciation Rights (SAR)*

Stock appreciation rights (SAR) are deferred bonuses directly linked to the growth of the market prices of the company's stock. They are often paid as cash payments, usually alongside employee stock options (Herzel & Perlman 1978). Some forms of deferred bonuses are known as 'Golden parachutes'. These are large compensation packages received by managers at the termination of their employment. The efficiency rationale often associated with golden parachutes is that they can discourage the manager from engaging in earnings manipulation as a desperate means to keep their job. The benefits of golden parachutes lie in their ability to serve as stable incentives for managers as they enhance their payoff in the event of liquidation. Thus, some describes this type of compensation as the cost of incentive compatibility (Tirole 2006). This can be described in a contract design model as the value of pecuniary incentive at a final time T .

The incentive compensation plans discussed above can be structured to induce managers to minimise their cash diversion and inappropriate effort tendencies, as well as mitigate against poor capital structure management. However, Fehrenbacher (2013) emphasises the place of skill-sorting in reducing the moral hazard tendencies of managers. He argues that if attention could be taken away from incentive systems for a moment, effective sorting may help owners select managers who are naturally inclined to work hard rather than shirk. Furthermore, the link between performance and compensation presents some difficulties in the use of compensation plans, namely:

- poor design of compensation packages,
- managers may maintain compensation irrespective of poor performance,
- managers are enriched when they leave a company although their employments have been terminated due to bad performance,
- legally permissible accounting manipulations which may boost their compensation but has a negative impact on the company.

2.4.2 Takeover Models

In addition to executive compensation, another notable mechanism is the market for corporate control. Corporate control mechanisms are ways by which managers are controlled to take actions in the best interests of shareholders

(Bushman & Smith 2001). Hostile takeovers are often used to awaken a manager to their evident slackness in managing the organisation, such as if the manager engages in poor capital structure management. This is also known as regulation by the capital market. In this form of corporate control, the acquirer makes an offer to purchase all or a portion of the company's outstanding shares. The acquirer is successful if they achieve more than 50% of the voting shares. Over time, the acquirer gains a foothold in the board and reserves the right to decide upon the CEO of the company. One advantage of this mechanism is that managers are impelled to perform because of the latent threat of hostile takeover. However, in many countries this mechanism is rarely used because of its prohibitive cost and disruptiveness (Campbell 2006). A promise of reward rather than sanctions should be the preference in mechanism design (Verdier & Woo 2011).

2.4.3 Blockholder Models

In a blockholder model, a partially concentrated ownership framework may be used by an organisation. This means that the primary (largest) shareholder is involved in monitoring the company's management and has the authority to effect modifications to management styles and policies. This corporate governance mechanism is prevalent in continental Europe and other OECD countries, but not so much in the United Kingdom (UK) and the USA. One drawback of this mechanism is that large shareholders, such as pension funds, mutual funds, banks, and insurance companies (which are the commonest forms of primary shareholders) may be passive in monitoring. Lack of incentives can mean that they do not hold enough shares in their company. This counters the expected effectiveness from such institutional ownership. Nevertheless, in the blockholder model, delegated monitoring and large creditors are still identified as adept corporate governance mechanisms (Constantinides, Harris & Stulz 2003).

2.4.4 Board Models

Similarly, in board models, the board of directors may be employed as active monitors of management. The board of directors may decide on the CEO, monitor management, and vote on significant decisions in the organisation. Such decisions include mergers and acquisitions, modifications in the CEO's pay, and

the capital structure of the company, amongst others. One of the pitfalls of the board model is that the board of directors often do not serve as the truly independent monitors they should be. Put differently, the board is said to be captured by management, which causes this type of mechanism to lose its effectiveness. When independence is achieved, boards may prefer the easier role of advisory body rather than being perceived as confrontational, thus converting their active monitoring role to a speculative one.

However, improving the effectiveness of the board, that is board reforms can improve the firm's corporate social responsibility (CSR) performance. This implies that a company's governance structure can determine its environmental and social practices. In addition, if the board can effectively monitor the manager, this may improve the manager's commitment to CSR, particularly if there are lower levels of insider ownership in the organisation. Thus, such improved CSR performance can improve the firm's financial performance (Liao et al. 2021). The improvement in the firm's financial performance is notable for companies with stand-alone CSR reports as complementary financial disclosure. This is because such stand-alone CSR reports reduce analyst forecast errors (Dhaliwal et al. 2012).

2.4.5 Debt

The use of debt by a firm has also been described by Jensen and Meckling (1976) as a mechanism for minimising the conflicts of interest between a shareholder and a manager. Debt liability minimises the ability to convert the company's free cash flow into negative net present value investments or to be used extravagantly. Although the use of debt is the second preferred form of capital in Myers' (1984) pecking order theory, debt causes managers to commit appropriate actions and improve the quality of their efforts in order to generate cash flows to meet future debt repayments and more. Therefore, debt has a disciplining effect on managers. In a financial distress situation without liquidation, debt defaults transfer control of the firm to the debtholder. Furthermore, when a manager holds a significant claim on the company's cash flow, they are motivated to minimise the cost and maximise the residual profit accruable to them (Tirole 2006). Although the use of debt can be considered a complementary governance

instrument, a company cannot use debt excessively in order to discipline its manager. This further elucidates the question of what the optimal debt in the firm's capital structure should be in order to minimise the manager's opportunistic tendencies and the risk of financial distress.

2.4.6 Debt Covenant

The use of debt has been established as a corporate governance mechanism to motivate the manager to focus on the shareholder's wealth maximisation goal. However, the use of debt introduces an additional agency problem known as the agency cost of debt. The debtholder who provides the debt finance may have different objectives and interest in the company in contrast to the manager. This introduces the debtholder-manager agency problem, in addition to the shareholder-manager problem. These are the two significant agency relationships investigated in the study.

Debt covenants are mechanisms or strategies taken by debtholders to protect their interests in a company. While the mechanisms previously discussed are largely strategies to protect the interests of shareholders against the opportunistic tendencies of managers, covenants incorporate the interests and welfare of the debtholder in the company's financial actions. Covenants can limit the transfer of funds to shareholders in the form of cash dividends, and share repurchases, among others. They can be positive covenants or negative covenants. Positive covenants specify actions the borrower (that is the manager on behalf of the shareholder) must take, such as a requirement to maintain the company's assets in good working conditions. On the other hand, negative covenants prohibit the borrower from taking certain actions, such as setting a limitation on its total indebtedness.

2.4.7 Additional corporate governance mechanisms

Other existing corporate governance mechanisms include Say on Pay (SOP); regulation (legal system); product market (competition) (Campbell 2006); truth-telling, which is a dominant strategy (Trost & Heim 2018); dismissal (Kwon 2006); and threat of litigation (Gürtler & Kräkel 2010). The agent's level of conscientiousness is also significant in incentive alignment (Fong & Tosi Jr 2007).

Nonetheless, the effectiveness of these mechanisms is still very debatable. One conclusion from a review of the existing literature is that most of the outlined mechanisms enhance a company until a point when costs start to set in. In addition, agency problems are complex, and cannot be adequately resolved by a single mechanism (Thomsen & Conyon 2012).

2.4.8 The Choice of Optimal Capital Structure

The choice of financing used by a firm remains a significant focus in financial management and corporate governance, as poor financial management can raise questions about the viability of a company.

In the original proposition of Modigliani-Miller (MM1), the market value of any firm is independent of its capital structure (Modigliani & Miller 1958). This original proposition was later refuted due to the tax advantage, bankruptcy costs and other transaction costs associated with the use of debt financing. This highlights the fact that each form of financing, debt or equity comes with different opportunity costs. Modigliani-Miller revised their initial proposition (MM2) by proposing that in the absence of arbitrage opportunities and in the presence of taxes, the cost of equity is directly proportional to the debt-equity ratio. Furthermore, the value of the firm increases linearly with the level of debt. Therefore, determining the optimal capital structure of a firm is a major financial decision.

Debt finances are capital acquired through borrowing of funds to be repaid in the future, as agreed by the debt contract. This form of capital is often described as a cheaper source of funds than equity, or other types of finance, because debt repayment is less consistent with the company's performance in contrast with other financial contracts (i.e., business profits are not shared with the debtholder). In addition, the tax advantage on interest fees and charges increases the attractiveness of debt financing. Other advantages include the maintenance of ownership by shareholders, as debt finance does not give control directly to debtholders. On the other hand, the disadvantages of debt finance include firms' accessibility to a debt facility, the responsibility of interest repayments, the effects of repayment on cash flow and, finally, the risk of bankruptcy (Bolton &

Dewatripont 2005). Use of new debt can also signal the future performance of a company's stock. New debt may signal financial soundness while less use of debt may signal otherwise. This can be seen as lack of confidence by investors. Excess debt can also signal lower internally generated funds (Amaro de Matos 2001). However, examining this issue in more detail is beyond the scope of this research.

Equity capital is raised from the sale of company stocks. The advantage of this form of capital is that companies do not have an obligation to repay funds. However, for companies to retain a healthy stock valuation in the capital market, it must improve its ability to pay dividends from business profits. Equity capital is often regarded as a form of capital with high costs. In addition, the sale of stock spreads the control of the company to an increasing number of shareholders, thereby diluting its ownership. Shareholders also expect a high rate of return from their investments due to their responsibility for bearing the company's risks.

These different advantages and disadvantages associated with the two forms of capital make it difficult for companies to decide which is the most appropriate. Therefore, most companies resolve to use a mix of both forms of finance. This leads to the problem of what is the optimal mix of these financial forms. In addition, optimal capital structure is a value-added and strategic variable in the optimisation of a company's investment decisions (Amaro de Matos 2001), and optimal leverage has the potential to minimise agency problems (Jensen & Meckling 1976).

Schorr and Lips (2019) considered an empirical approach rather than a game-theoretic modelling approach to obtain the optimal capital structure of Swiss dairy farms. This was achieved by minimising the correlation between economic performance indicators and a distance function of the debt-to-asset ratio distribution to its quantiles. Like Schorr and Lips, a number of other studies have considered how to optimise a business' capital structure, but without considering the possible agency relationship involved. These studies include that of Myint, Lupi and Tsomocos (2017), which examined the impact of investment opportunities on capital structure, and Palazzo (2019), which investigated the interaction of other financial variables, such as cash flow risks and capital

structure. Other perspectives from which the optimal capital structure has been explored include the optimal allocation to different forms of debt instruments or debt management choices (Martellini, Milhau & Tarelli 2018), and optimal capital structure and bankruptcy preferences (Antill & Grenadier 2019).

Grigore and Gurau (2019) concluded that the prominent factors affecting the capital structure of a firm are: firm profitability, asset specificity, size of the company, age of the company, growth prospects, corporate income tax, dividend policies, institutional, legal and financial factors. The traditional problem of the optimal capital structure becomes even more obscure with the introduction of the agency relationships and agency problems inherent in public companies. The optimal capital structure has been well discussed as a significant financial management goal of most public companies and as a key corporate governance mechanism to mitigate agency problems (Jensen & Meckling 1976).

Distinct from the second Modigliani-Miller theorem (MM2), Craven and Islam (2013) considered a case in which the borrowing rate is set to be increasing as the firm's borrowing increases. They concluded that this departure from an assumed constant borrowing rate in MM2 leads to a unique optimal debt-equity ratio and further enunciates the need for an optimal capital structure in which flexible parameters are introduced. Craven and Islam (2013) obtained this result from their design of an optimal financing model. However, higher debt does not necessarily imply a higher borrowing rate when the agency theory is incorporated. This is because more debt facility may be provided by the debtholder as a reward for the manager's reduced opportunistic behaviours, and consequently, a lower borrowing rate may be demanded from the firm as compensation. This further suggests that the introduction of information economics may provide further insights into the analysis of the optimal capital structure. A company's capital structure is usually informed by its compliance to the code of corporate governance best practices (Jensen 1988; Kaplan 1989).

Following Jensen and Meckling's (1976) argument of agency relationships and the need for efficient corporate governance mechanisms, including an optimal capital structure, Lewis and Sappington (1995) modelled the problem between an external discretionary principal capable of deciding the agent's (firm) capital

structure by choosing how much equity is given to outside financiers. As could be expected, this model is limited in its representativeness of a modern company as managers are significantly involved in the choice and size of equity to be sold.

Other elements of the optimal capital structure theory include whether the ratio of the firm's total debt should be short-term or long-term, that is, the maturity structure of the firm's debt. Should the firm use variable-rate or fixed-rate debt? Should long-term bonds pay annual coupons with a balloon payment or stick with a full amortisation (equal periodic payments) (Copeland, Weston & Shastri 2014)?

As a corporate governance mechanism, the choice of an optimal capital structure specifies the optimal use of debt. This should minimise the agency problem between the shareholder and the manager by stimulating the manager to commit appropriate effort to meet the company's debt obligation. The optimal capital structure also serves as a corporate governance mechanism that minimises the marginal agency cost of debt arising from the provision of debt finance by the debtholder. Corporate governance mechanisms can be expressed as managerial incentives, as illustrated in the following chart.

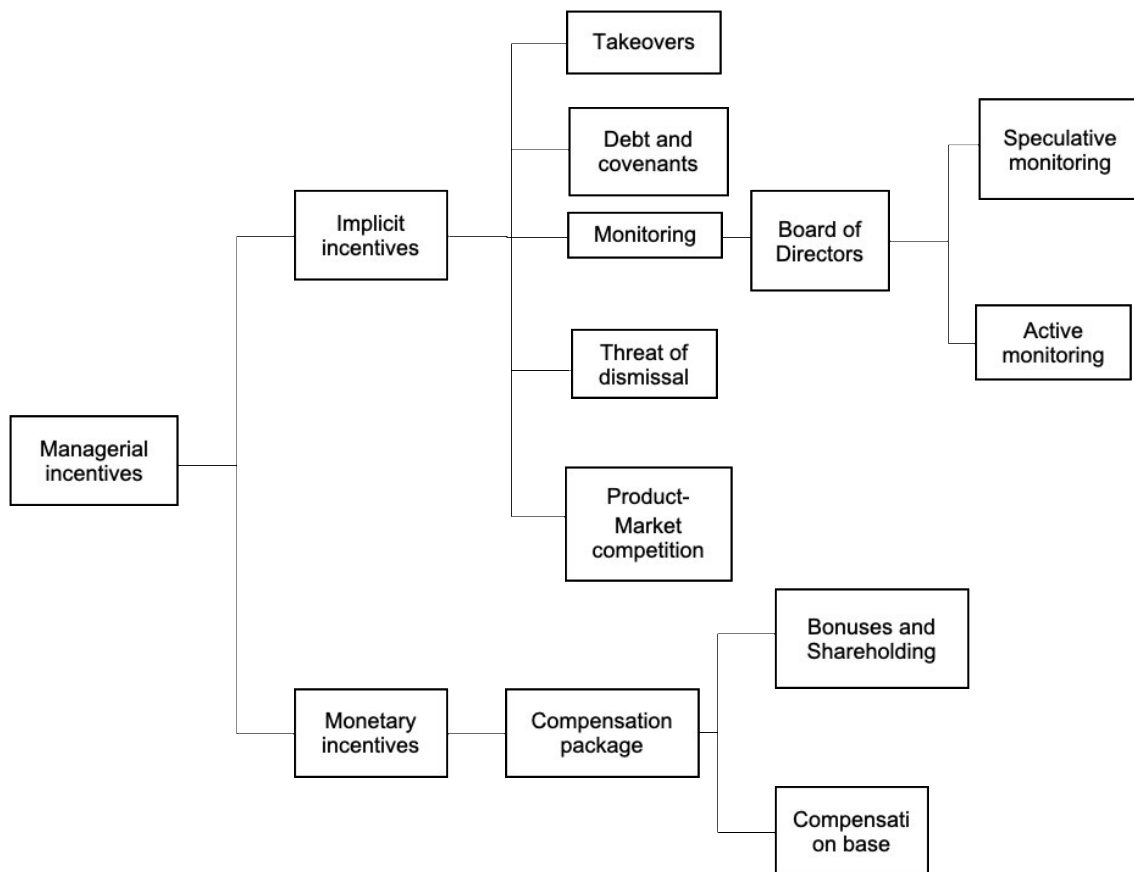


Figure 2.2 Managerial incentives

Heinrich (2002) discussed the theory of complementarities in corporate governance. He investigated the interaction between corporate governance instruments and the opportunity costs associated with that use. Governance instruments mitigating one agency problem raises an opportunity cost if it simultaneously raises other agency problems in other agency relationships in the corporation, e.g., between controlling and minority shareholders. This leads to the complementarity and substitution relationships between various governance instruments. An instrument is complementary if it minimises the opportunity cost or increases the marginal benefit of the other, but substitutive if it increases the opportunity costs or minimises the gains of the other.

Comparison of Corporate Governance Practices

A number of studies in the corporate governance literature compare the corporate governance practice in different countries, such as Allen and Gale (2000), and Heinrich (2002). Allen and Gale (2000) compared financial systems in the US,

UK, France, Germany, and Japan. They determined that, in the US and the UK, which have some of the strongest corporate governance mechanisms, the most common form of internal governance is the board of directors (monitoring) and the main form of external governance is the market for corporate control (i.e., takeovers and mergers). In Anglo-American countries, large banks, which are the providers of company funds, take on the role of monitoring by overseeing the financial activities of the company and its investment decisions, disciplining it when necessary. In the US, the *Glass-Steagall Act* restricts institutional holdings of shares, thus prohibiting banks from holding equity in firms except in rare circumstances, such as when the firm is reported bankrupt. New York regulations traditionally prohibited insurers from owning equity stakes. However, recently, this restriction has been relaxed for life insurers, giving them an opportunity to hold a limited amount of equity stakes. The most common forms of shareholders in the US are investment funds and householders, while the prevalent shareholders in other countries are other firms and wealthy families (Heinrich 2002).

In Japan, managers seek employment stability for employees at the expense of enriching shareholders. Similar to Japan, while Germany and France prioritise the interests of shareholders, they still retain a focus on other stakeholders such as employees (Allen & Gale 2000). Large numbers of shareholders and bank monitoring in Japan are argued to be superior corporate governance mechanisms because of their lower cost of capital effects (Stulz 1996).

While many studies have focused on the issue of selecting a company's optimal capital structure as a corporate governance mechanism, most have only focused on the interests of shareholders without taking into consideration the interests of managers or debtholders in choosing the optimum capital structure of a firm. This includes Elton and Gruber (1974), and Titman and Tsyplakov (2007).

In this study, differential game theory is adopted to provide additional insights into choosing an optimal capital structure over time. Furthermore, there has been a limited application of the differential game framework in the design of incentive contracts in the selection of an optimal capital structure. Other studies that have analysed the dynamism of an optimal capital structure include that of Elliott and

Shen (2015), which emphasises the evolution of the growth rate and volatility while incorporating tax benefits and bankruptcy costs.

In this study, a combination of existing mechanisms rather than a single mechanism is used to develop a unique model to improve the affinity of interests between the debtholder and manager, and shareholder and manager. This gives a different analysis for mitigating the agency problem in choosing the capital structure of a company. To resolve the complexity of agency problems and mechanism design, a combination of tools in mathematics and economics have become prevalent in recent literature such as (Heinrich 2002).

2.5 The Application of Differential Game Theory in Corporate Finance

Standard finance theories have been inadequate in supplying suitable explanations for observed phenomena. This has created a need for exploration of theories using new methodologies, particularly in corporate finance. Game theory has been found to be reliable because of its ability to provide insights into unexplained phenomena by combining asymmetric information and strategic interaction into its analysis (Allen & Morris 2014). Game theory models are suitable tools for aiding management operations and, therefore, become a key tool in management accounting (Kunz 2018).

The use of game theory as a methodology in corporate finance began with early studies, such as Bhattacharya's (1979) model of dividends as a signal of a company's profitability. This built on Miller and Modigliani's (1961) suggestions that dividends might convey significant information about the corporation's prospects. Therefore, Bhattacharya assumed that, by giving out a generous level of dividends, a firm may be revealing the profitability of its investments. However, John and Williams (1985), and Miller and Rock (1985) suggested other methods of signalling without committing to a specific level of dividends and while avoiding the significant costs associated with the credibility of such signals. The effect of the signalling theory discussed here is that it matches every dividend payment to some sort of new information. In addition, every level of dividend implies some elements of information and, therefore, makes it challenging to match dividend signal models with smoothing.

The trade-off theory of capital structure had proved unsatisfactory due to the empirical effect of bankruptcy costs, plus interest tax shields do not particularly reconcile with observed capital structures. Again, this led to the use of game-theoretic techniques in this field. This has improved the field significantly with the development of signalling models as the first contributions of game-theoretic applications to capital structure (Allen & Morris 2014). Ross (1977) developed a model in which the prospects of the firm are signalled to the financial markets by selecting a suitable level of debt. This is similar to Myers and Majluf's (1984) contribution to capital structure models of asymmetric information. In their analysis, an informed manager may have opportunistic tendencies to prefer the use of debt rather than equity to finance new investments. Private information about the net present value of investments available to managers may reveal that equity may be undervalued when compared to the dilution incurred by existing shareholders.

A second contribution of game-theoretic applications to capital structure is identified within the study of agency costs, following the discussion of the agency problems of risk-shifting, unobservable actions and underinvestment. This contribution with respect to agency costs has led to extensive literature on financial contracts, as well as the applications of a game-theoretic approach in understanding the role of incomplete contracting tendencies in establishing financial contracts, especially the use of debt contracts (Allen & Morris 2014). The contributions of Jensen and Meckling (1976) show that, at equilibrium, the informed agent bears the agency costs because the uninformed agent (i.e., the principal) can pre-empt the moral hazard problem and can revise the contract appropriately. Fama (1980) investigated the theory of reputation as a mechanism design to mitigate the agency effects.

This thesis builds upon the second contributions of game theory in understanding capital structure with respect to agency costs, particularly unobservable actions and risk-shifting.

Extensions of game-theoretic methods that involve comprehensive informational models are particularly significant for finance (Allen & Morris 2014). This rising demand and the need for models that can analyse finance theories in a dynamic framework have led to the introduction of dynamic game theories, notably

differential game theory. Dynamic games are suitable because employment contracts usually span multiple years. Differential games are continuous time-infinite games in which the evolution of the state variable(s) is regulated by *differential equations* and players act over the entire time interval (Cesar 1994). Differential game theory is a comprehensive way of analysing dynamic situations of conflict. A significant part of the method is its use of the descriptive capability of differential equations as a modelling tool (Mehlmann 1988). The theory of dynamic game models, of which differential game is a form, can be applied to both infinite time horizon and finite time horizon games (Miranda & Fackler 2004). Differential game problems may be considered the generalised form of optimal control problems, for which there is more than one planner. Each player has their separate objective function that they seek to optimise, tempered by a set of differential equations (Bauso 2016; Sethi & Thompson 2000).

Differential game models are highly suitable for analysing issues concerning conflicts of interest between two or more parties over time. It has been established that conflicts of interest exist in the selection of the optimal financing of a company. Therefore, it is anticipated that the differential game framework will offer a robust analysis of the highlighted conflict of interest and proffer a model that minimises this conflict over a period of time. Solution concepts for a differential game problem include minimax, open-loop and feedback Nash equilibrium, Pareto-optimal, and Stackelberg methods (Shoham & Leyton-Brown 2008).

Jørgensen and Gromova (2016) applied the differential game model over an infinite horizon to an advertising competition scenario among similar firms. They concluded that to sustain cooperation, each member of the grand coalition must receive an incentive that makes it unprofitable for them to unilaterally renege on the coalition. This is closely related to the outcome of the work of Cesar (1994), who employed the model to optimise/reduce the emission of Greenhouse gases.

The differential game model has also been prominently applied in international macroeconomics problems, air combat manoeuvring, research and development, transportation systems, optimal investment, and reinsurance strategies (Bondarev & Greiner 2018; Mao et al. 2016; Öner, Başar & Günel 2017; Xu, Wei & Zhang 2017). Many of these applications of the differential game

model returned reliable results, albeit with the possibility of improvement. The agency problems discussed in this research are analysed as non-cooperative games and further established in cooperative Pareto solution concepts. While non-cooperation suggests that players have dissimilar goals, it may be possible to ensure that players have identical targets. This raises questions such as what is a fair sharing ratio or what is a definition of fairness, among other matters (Kunz 2018).

A mathematical model is fundamental for modelling the governance controlling mechanisms to generate an optimal capital structure and optimal financial management strategies.

2.6 Research Motivation and Contribution

Elton and Gruber's (1974) foremost application of the control theory to an optimum cost of capital, financing and valuation of a firm provides helpful insight into optimum firm behaviour over time. The authors developed a simple model that integrates the intertemporal interconnection between financing and investment decisions. The model presents the optimum cost of capital for debt and equity as well as the cut-off rate for new investment and financing standards. The worth of the firm following this pattern of action is also specified. They argued that the Modigliani and Miller (MM) proposition pertaining to the 'no consequences of dividends' and the suitable cut-off rate for new investment proven to hold in a discrete single-period would also hold in a continuous time horizon. The model developed was presented as optimal firm behaviour in the interests of a corporation. However, the model did not take into consideration the realistic expectations of the opportunistic tendencies of the manager, or the agency problem, as discussed earlier.

In addition, more recent studies such as Mu, Wang and Yang (2017) that have disregarded the 'no consequences of dividend and tax' rule of the MM, but which incorporated the agency problem of information asymmetry and conflicting interests of parties in the wealth maximisation of claimholders in a firm, have been limited in their analysis. In the design of optimal contracts and corporate governance that align the interests of the manager with the goal of the claimholder, most studies have designed models around the theory of incentive

pay. Yet, these have been found to be incomplete as there may be an opportunity to use additional governance instruments to further minimise the agency problem (Heinrich 2002). These studies on incentive pay, which are limited in reducing problems, suggest a need for more research that incorporates other governance instruments in its analysis. Therefore, this study improves upon the existing research on incentive compensation by incorporating complementary corporate governance mechanisms in mathematical models. These can help resolve the agency problems identified in corporate finance.

Moreover, most of the literature, such as the work of Kunz (2018), and Thomsen and Conyon (2012), develops an optimal contract using incentive pay or compensation systems to maximise the claimholder's (shareholder/debtholder) wealth. But there has been limited research on seeking the optimal capital structure as the focus of the shareholder's goal and to optimise the financial management systems of the company.

Furthermore, although there has been an impressive analysis of the dynamic issues of agency theory and corporate governance using game theory, this area has not been well developed using real life data in a numerical analysis. In addition, the informational structure of the strategic interaction and game in a long-term period has not been well explored.

These limitations and gaps provided the motivation for this current research, which examines the integration of key issues in corporate finance and information economics (i.e., the agency theory, optimal contracting, corporate governance, and capital structure) by using a dynamic game tool. This provides a comprehensive analysis of the non-cooperation of players and an analysis of cooperation. Thus, in this study, a game-theoretic analysis of the agency problem is presented. It is argued that the use of debt as a mechanism by the shareholder may produce a suboptimal capital structure. Therefore, it is important to optimise the capital structure and the mechanism design jointly to motivate the actions of the manager in a dynamic framework.

Thus, the salient contribution of this research is to design a combination of incentive/mechanisms or corporate governance instruments that can resolve

agency problems and specify an optimal capital structure for a firm using a comprehensive dynamic game tool, the differential game theory, as a mathematical modelling tool.

This is done by incorporating the interests of each player in the game and obtaining the Nash sets of strategies that produce an efficient combination of optimal mechanisms (complementary corporate governance mechanisms). This minimises managerial volition and specifies an optimal capital structure for the company in the two agency relationships. The resulting corporate governance models minimise the agency problem with respect to a firm's optimal capital structure and maximise shareholder and debtholder value. Another contribution of this study is that it analyses the conflicts of interest as they evolve over time, using the differential game theory with a thorough analyses of the game over time in a dynamic framework. According to Bolton and Dewatripont (2005), optimal contracting outcomes can be represented as solutions to constrained optimisation problems. It follows, therefore, that dynamic optimal contracting outcomes can be represented as solutions to differential game problems.

Reduced agency costs can reduce the risk management responsibilities of corporations (Schroeck 2002). Thus, efficient corporate governance models that resolve the underlying agency and information asymmetry problems in a dynamic framework can minimise the risk associated with modern corporations.

Limitations of the Existing Literature

Generally, game theory models or contract theory models of asymmetric information agency problems have been formulated as optimisation problems. This proves that adequate contract-theoretic frameworks can be used as solutions to agency problems by reducing the divergence of interests between agents. Lukas (2014) modelled the agency problem between employer/employee with respect to the employees' labour services as an optimisation problem, as well as the information asymmetry between a house seller and a buyer. Lukas veered into corporate financing by highlighting that the new issuance of capital by a company provides a lot of information about the company's stock performance. This emphasises the role of capital structure. Furthermore, the study by Lukas developed an optimal contract to compensate and motivate a

manager to work for the principal effectively. However, this optimal contracting model was not developed in the context of an optimal capital structure. This is similar to the approach taken in other studies, such as Bolton and Dewatripont (2005), which investigated the optimal contracting between a typical agent and a principal.

Other corporate finance studies had investigated the principal-agent problem between the manager and shareholder and analyse the agency costs associated with the use of debt and equity (i.e., monitoring costs, risk-shifting, and underinvestment) using fee schedule design. The optimal fee schedule for the manager was developed to compel them to provide optimal efforts to the firm (Amaro de Matos 2001; Moreno-Bromberg & Rochet 2018), this limits the study on corporate governance to optimal fee design.

Similarly, Thomsen and Conyon (2012) designed a corporate governance model by setting the principal utility maximisation as the objective function. This is because the principal has the power to set the rules of the agent's remuneration, while the agent's utility maximisation is represented as a constraint in the model. However, using differential game theory, which requires an objective functional for each player, the objective of each player is set as their wealth maximisation function.

Most of the existing literature has examined the design of optimal contracts to optimise the shareholder value and minimise the agency problem. However, limited attention has been given to designing dynamic optimal mechanisms that jointly minimise information asymmetry and specify an optimal capital structure in a dynamic framework that can improve firm value over time.

Chapter 3 – Conceptual Framework, Research Methodology and Case Study Background

3.1 Introduction

This chapter explores the conceptual framework adopted in this study for the development of a financial management model based on a mechanism design approach. This was designed to address the main research question: *What are the time-dependent optimal corporate governance frameworks and mechanisms for choosing a company's efficient capital structure?* Establishing an integrated mechanism design model as a financial management problem, this study conceptualises and integrates all factors relevant to devising optimal capital structure strategies. It also presents the design of an incentive contract, including accounting and corporate financial management policies and corporate governance mechanisms.

The objectives of the study are outlined in detail and elements of the study, such as strategies, actions and time period, are analysed. These are identified by Lynn (2015) as the features of a prescriptive theory. To simulate the model, this study employed a case study approach using secondary data from a relevant company (a publicly owned consumer staple company listed on the ASX). Further, to analyse and test the model, Matrix Laboratory (MATLAB) software was used for solving the mathematical programming problem.

3.1.1 Structure of the Chapter

The structure of this chapter is as follows. Section 3.2. presents the conceptual framework of this study. Section 3.3 discusses mechanism design as a dynamic game problem for the new integrated financial management model. Section 3.4 discusses the research data and computer program used in the study. Section 3.5 provides the context of the case study and, finally, Section 3.6 concludes the chapter.

3.2 Conceptual Framework of the Thesis

The existing literature on accounting and corporate finance reveals that an optimal capital structure has interrelationships with the concepts of agency theory, corporate governance and company valuation. Previous research studies have proven that capital structure is a crucial mechanism in mitigating agency conflicts between managers and shareholders (Brown, Beekes & Verhoeven 2011; Ghosh 2007; Williamson 1988). Furthermore, achieving the optimal capital structure is a primary goal of the financing function of a business. Obtaining the optimal capital structure, also referred to as the capital equilibrium, has additional benefits, namely: greater efficiency in the use of capital, enhanced ownership retention, and fewer risks. Therefore, an optimal capital structure and good corporate governance in corporations should be considered essential elements in developing a sound corporate financial strategy.

Furthermore, the separation of ownership and control in companies often create agency problems between principal and agent, largely caused by information asymmetry when one party (i.e. managers/agents) has an information advantage over the other party (i.e. shareholders/debtholders/principal) (Berle, Means & William 1932; Jensen, Michael & Meckling, William 1976). Of all the mechanisms that can be imposed to mitigate the underlying problem, such as external regulations and internal mechanisms, an incentive-compatible contract is the most notable for addressing the agency problem, aligning conflicts of interest between related parties, and minimising the agency costs associated with the separation of ownership and control (Shleifer & Vishny 1997). The incentive-compatible contract is optimal for addressing the agency conflict as it develops a mechanism that aligns the utility of shareholder/debtholder with the interest of management. Hence, this study adopts this mechanism design approach to incorporate the research issues into a corporate financial management model. This will minimise agency problems, lead to good corporate governance, ensure an optimal capital structure, and achieve optimal company performance.

This study aims to investigate the extent to which an optimal capital structure and other designed governance mechanisms can have positive economic benefits within the context of mechanism design and game theory. These benefits include

achieving optimal financial management strategies leading to good corporate governance, efficient allocation of resources and enhanced company value. As the methodology, this study designs an integrated mechanism design model as a financial dynamic game problem to examine the research issues. In a dynamic game problem, using the differential game theory, each party's utility function is specified as an objective functional. The shareholder or debtholder's value maximisation is reflected as the principal's utility function, and the manager's as the agent's utility function. The principal's problem is to assign their responsibility by sharing rules with the agents in order to maximise their expected utility at the end of the period. Each player selects a utility function governed by the systems of the game. These describe elements of the financial relationship and incentives and are expected to interrelate throughout the dynamic process following their different objectives in the non-cooperative framework. In the cooperative framework, the two parties come to an agreement to foster each player's utility function under certain conditions.

Generally, the model design is based on accounting and finance principles. It fuses basic accounting equations with crucial financial management elements, incentive-compatible contracts, and corporate governance mechanisms as the states and constraints of the model. Via the specification of the objective functionals, state equations, constraints, variables and parameters of the model, this model reflects the incentive-compatible mechanism design problems, efficient resource allocation, corporate governance and finance, and accounting issues of a company. The outcome of this study is an integrated financial management model for examining the implications of an optimal capital structure and mechanism design on realising good corporate governance and improving company value. Figure 3.1 presents the structure of the conceptual framework in this study and the relationship and integration of the three modelling chapters that analyse the relationships between the shareholder and the manager, and the debtholder and the manager.

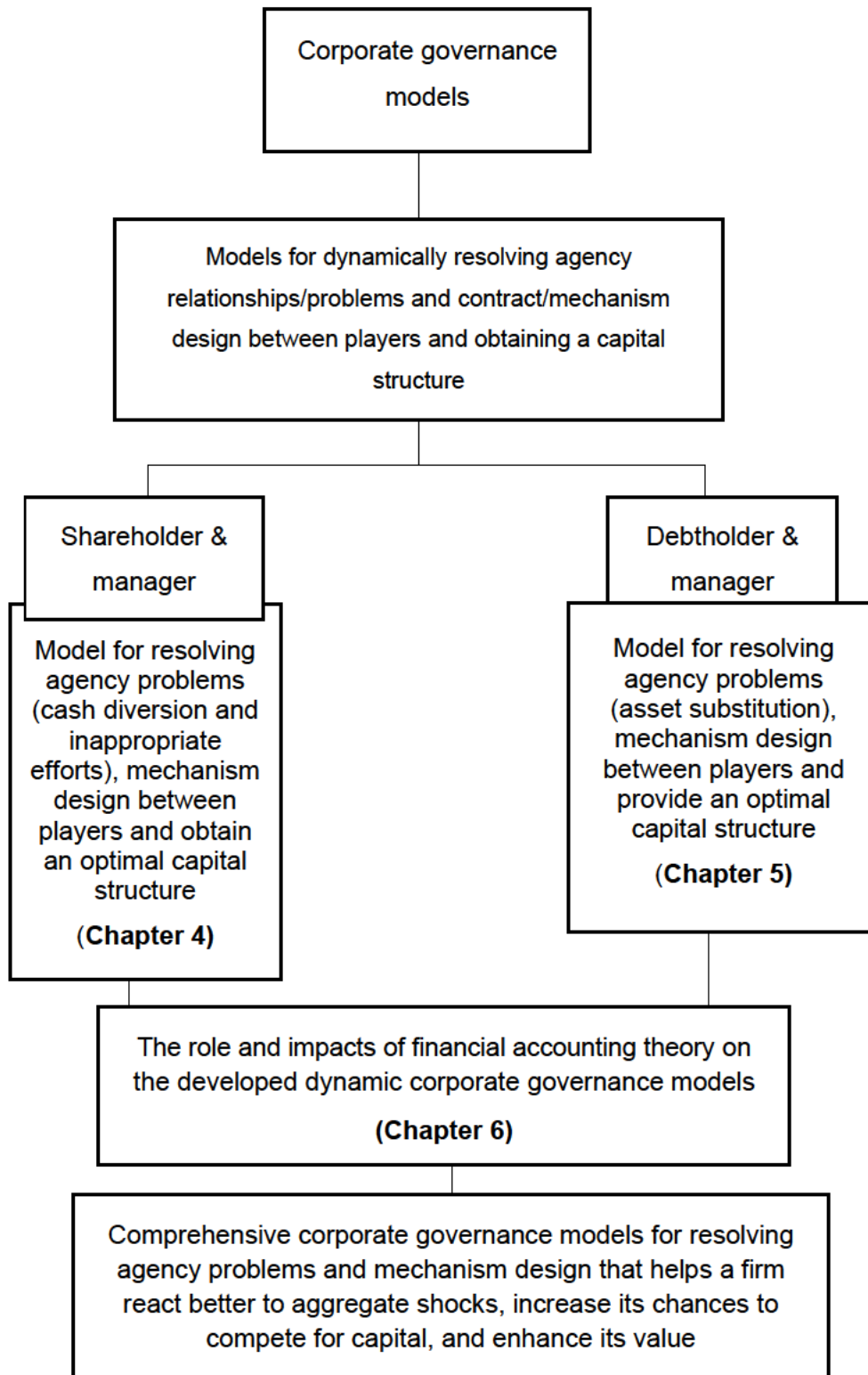


Figure 3.1 Conceptual framework, and interrelationships between the developed corporate governance models

3.3 Mechanism Design as a Dynamic Game Problem

As previously discussed, modern corporations are marked by the separation of ownership and control. This can cause asymmetric information leading to moral hazard and adverse selection problems. These agency problems have been compared to a dynamic game of information exchange between the relevant parties, thus fitting into the construct of a dynamic game problem. In each period, each party takes suboptimal decisions based on the limited available information in the game or relationship. These suboptimal decisions are capable of harming the utility functions of the other party, and essentially the growth of the business. This creates a challenge when the business seeks to develop an organisational design in an efficient and productive way. Therefore, this study examines the benefits of an optimal capital structure and the use of other specified mechanisms for achieving good corporate governance and enhancing company value. It does so by applying the principal-agent relationship and mechanism design as fundamental to creating a corporate financial management model. The development of the model is explained further below.

3.3.1 Features and Descriptions of the Financial Management Model

The integrated financial management model developed in this study is based on current financial models. These include those of Nuryanah and Islam (2015), Brodt (1978), Carleton, Dick and Downes (1973), Hamilton and Moses (1973), Ijiri, Levy and Lyon (1963), and Amaro de Matos (2001), as well as an exemplification of a corporate model from Ho and Lee (2004). The financial management model developed in this current study extends the previous models based on their identified limitations. The main attributes of the developed financial model are as follows:

1. *Game theory and mechanism design: a principal-agent relationship and mechanism design framework for incorporating the research issues*

In this study, a financial management model is designed to incorporate the research issues by employing a principal-agent relationship and a mechanism design framework. In the model, each player's utility function is specified as a maximisation or minimisation problem subject to state equations and other incentive contract constraints to motivate the agent

(i.e., managers) or compel that agent to manage the resources of the company efficiently and effectively. The model integrates governance mechanisms and financial management policies in its elements. This is to ensure an excellent organisational design and good corporate governance, mitigate agency problems and, ultimately, improve company performance.

2. Dynamic and integrated

First, the relationship and contract design between shareholders, debtholders and managers span across a number of years and evolves with time. Second, in the design of a business plan, there are observable time lags between an output obtained in a current business period and its use as an input for the next business period. Therefore, it is impractical to assume this fits a static model; a dynamic feature in the model is required. This study proposes a prescriptive financial management model that can reflect the dynamic real financing conditions faced by a real business. The differential game theory adopted in this research employs a dynamic multi-period optimisation approach in designing the financial management model. Finally, the use of this approach is advantageous for modelling the financing strategies of a company as a dynamic process and provides robust understanding about company behaviour over time (Elton & Gruber 1974).

3. A numerical application based on a case study approach

There are few illustrations of financial management strategies and mechanism design models applied to real-case business problems. In this study, a numerical case study is applied to show the practicality of the financial management model developed in a real-life business context. The case study approach is valuable as it helps to illustrate how specific governance mechanisms and the optimal capital structure can be incorporated in formulating financial management strategies for obtaining optimal organisational design and improving firm value.

4. A reliable model for decision-making processes

The model developed in this study was tested thoroughly for validity and verification, as well as parameter sensitivity. This highlights the credibility and scope of the model as a managerial decision-making tool.

3.3.2 Elements of the Model

3.3.2.1 *Objective Functional of the Model*

While optimisation and optimal control is suitable for the optimisation of one decision maker's goal, differential game theory offers a robust analysis of the strategic interactions and relationships between multi-players over time, representing each player in an optimal control framework. In the classical optimal control theory, the objective is either to minimise a certain cost or maximise a certain value (profit). As the model is developed within the framework of capital structure and the agency theory, the objective functions should reflect the player's interests and relate to the concepts and parameters that are crucial for the stakeholders to assess the management's performance (Stern 1972; Van Horne & Wachowicz 2005). In Chapter 4, the first modelling chapter, the objective of the shareholder is to select the optimal debt repayment and monitoring strategies and the corresponding pecuniary incentives that maximise the company's cash flow in each period.

In Chapter 5, the second modelling chapter, the goal is to model the debtholder-manager agency relationship as a dynamic game. Here, the objective of the manager, who represents the shareholder, is to minimise the company's weighted average cost of capital while maximising the financial income of the firm from asset substitution. On the other hand, the objective of the debtholder is to maximise the value of debt face value and cost of debt given to the company at a minimal monitoring cost. Finally, in Chapter 6, the impact of historical cost, fair value and creative accounting theories on the models presented in Chapter 4 and Chapter 5 are evaluated.

3.3.2.2 *State Equations for the Integrated Financial Management Model*

In a differential game model, state equations are elements of the game that give some observable information about the evolution of the game and, to a certain extent, the outcome of the game. The state equations governing the dynamic game process of the financial management model includes value of debt, a firm's liquid reserve and the value of productive assets. These state equations incorporate financial accounting principles via variables that represent the

financial outcome of the game for the company and provide investor-informing data to stakeholders. Ultimately, they also determine the capital structure of a firm.

3.3.2.3 Constraints for the Integrated Financial Management Model

Where necessary, in addition to the state equations, the objective functionals are subjected to certain constraints that represent the incentive mechanisms integrated in the model, such as a participatory constraint. These reflect incentive-compatible mechanisms, corporate governance and financial management policies that minimise the agency problems, and conflicts of interest.

3.3.2.4 Mathematical Model of a Differential Game Problem

In a differential game with two players with different objectives, each player selects at least one control variable to achieve their objective. The choice of a player's control variable is a function of the player's strategy. A player's strategy relies on the information observable from the evolution process up to time t only (Cesar 1994). Each player i ($i = 1, 2$) selects a control $u_i(t)$ to optimise their objective function:

$$J_i(u_1(t), u_2(t), t) = \int_0^T f_i(x(t), u_i(t), u_2(t), t) dt + W_i(x(T))$$

$$\text{Subject to: } \dot{x}(t) = g(x(t), u_1(t), u_2(t), t) \quad x(0) = x_0$$

This is the equation of motion (differential equation) that tempers their utility function, x is the state variable. $x, u, g(\cdot), f(\cdot)$ are all assumed as vectors. It is also assumed that the control variables are compact continuous functions of time and the functions $g(\cdot)$ are continuously differentiable.

An information structure is defined as a set-valued function $\varrho_i(t) = [x(s), s \in S(t)]$ where $S(t)$ is non-decreasing in T and $S(t)$ is a subset of $[0, T]$. Given an information structure, the strategy for player i can then be defined as a vector valued function $\Sigma^i(\varrho_i(t), t)$. Hence, the choice of the player's control variable is: $u_i(t) = \Sigma^i(\varrho_i(t), t)$.

Tree models are specified as unsuitable in a differential game framework (Mehlmann 1988). The loop decision rule for a non-cooperative game, which is identified as an alternative to the tree model, can be specified as:

$$\left. \begin{array}{l} \text{an open – loop} \\ \text{a memoryless} \\ \text{a (state)feedback} \end{array} \right\} \text{decision rule}$$

The open-loop and feedback Nash equilibrium concepts are non-cooperative solution concepts, while the Pareto outcome analyses the game as cooperative.

3.3.3 Non-cooperative Solution Concept

Non-cooperative game theory assumes players in the game have contrasting objectives and are unable to make binding agreements, thus pre-play communication is neglected (Mehlmann 1988). The manager has the ability to neglect the originally signed management contract. This leads to the question of strategic behaviour and player decisions and the need to find an equilibrium when the manager does not commit to the management contract.

3.3.3.1 Nash Equilibrium

Equilibrium implies that each player selects the strategy that is the optimal response to the other player's strategy (Dixit & Skeath 1999). Equilibria can be obtained via different approaches depending on the game context. The most popular and important solution techniques for a simultaneous game are Nash equilibrium and dominance. A Nash equilibrium developed by John Nash, is a design of strategies, such that each player's strategy is their optimal response, provided the other player sticks to their optimal/equilibrium strategies. Pure equilibrium strategies are further defined as strategies specifying a defined course of action, void of uncertainty (Dixit & Skeath 1999). However, as many games do not have dominant strategies, Nash equilibrium analysis is often considered. A Nash equilibrium solution technique can be analysed in an open-loop form or feedback form.

3.3.3.2 Open-loop Nash Equilibrium

If $(\beta_1^*(x_0, t) = u_1^*(t), \beta_2^*(x_0, t) = u_2^*(t))$ is an open-loop Nash equilibrium, and $[x^*(t), 0 \leq t \leq T]$ is the corresponding trajectory, then a costate function exists $\varpi_i^*(t)$ such that $i = 1, 2; j \neq i$:

$$\dot{x}(t) = g(x^*(t), u_1^*(t), u_2^*(t), t) \quad \text{with } x^*(0) = x_0$$

$$u_i^*(t) = \arg \max \mathcal{H}_i(x^*(t), u_i^*(t), u_j^*(t), \varpi_i^*(t), t) \quad \text{with } u_i \in \Gamma_i$$

$$\dot{\varpi}_i^*(t) = -\frac{\partial \mathcal{H}_i}{\partial x}(x^*(t), u_1^*(t), u_2^*(t), \varpi_i^*(t), t)$$

$$\varpi_i^*(T) = -\frac{\partial W_i}{\partial x} W_i(x(T))$$

Where the Hamiltonian $\mathcal{H}_i$ for player i is defined as:

$$\mathcal{H}_i(x, u_1, u_2, \varpi_i, t) = f_i(x, u_1, u_2, t) + \varpi_i g(x, u_1, u_2, t)$$

A Nash solution is a pair of strategies (β_1^*, β_2^*) such that:

$$J_1(\beta_1^*(t), \beta_2^*(t)) \geq J_1(\beta_1(t), \beta_2^*(t)) \quad \forall \beta_1 \in \Gamma_1$$

$$J_2(\beta_1^*(t), \beta_2^*(t)) \geq J_2(\beta_1^*(t), \beta_2(t)) \quad \forall \beta_2 \in \Gamma_2$$

The pair of equations for the Nash solution shown above imply that a player cannot improve their reward (i.e., is not better off) if they adopt any other strategy β_i other than β_i^* if the other player sticks to their optimal strategy.

The open-loop Nash solution assumes that the only available information for action at time t is that of the initial state $x(t_0)$ or $x(0)$; the current state of the game is unknown.

3.3.3.3 Feedback Nash Equilibrium

If $[\beta_1^*(x(t), x_0, t) = u_1^*(t), \beta_2^*(x(t), x_0, t) = u_2^*(t)]$ is a feedback Nash equilibrium, such that $\beta_i(x(t), x_0, t)$ is continuously differentiable, and $[x^*(t), 0 \leq t \leq T]$ is the corresponding trajectory, then a costate variable exists $\varpi_i^*(t)$ such that $i = 1, 2; j \neq i$:

$$\dot{x}(t) = g(x^*(t), u_1^*(t), u_2^*(t), t) \quad \text{with } x^*(0) = x_0$$

$$u_i^*(t) = \arg \max \mathcal{H}_i(x^*(t), u_i^*(t), u_j^*(t), \varpi_i^*(t), t) \quad \text{with } u_i \in \Gamma_i$$

$$\begin{aligned} \dot{\varpi}_i^*(t) = & -\frac{\partial \mathcal{H}_i}{\partial x}(x^*(t), u_1^*(t), u_2^*(t), \varpi_i^*(t), t) - \dots \\ & \dots - \frac{\partial \mathcal{H}_i}{\partial u_j} \frac{\partial u_j^*}{\partial x}(x^*(t), u_1^*(t), u_2^*(t), \varpi_i^*(t), t) \end{aligned}$$

$$\varpi_i^*(T) = -\frac{\partial W_i}{\partial x} W_i(x(T))$$

The feedback Nash solution assumes that the information available for action at time t is that of the current state expressed as $\varrho_i(t) = [x(s), 0 \leq s \leq t]$. The feedback strategy is also referred to as a Markovian strategy, or the equilibria satisfying a Markov property because of the ‘no memory beyond the current state’ information restriction placed on the solution concept (Bauso 2016). The feedback Nash equilibrium is sometimes referred to as the subgame perfect Nash equilibrium because of the property of strong time consistency, otherwise known as subgame perfectness. This means that an equilibrium remains so regardless of the state from which the game began, not the state reached along the equilibrium trajectory (Cesar 1994).

3.3.4 Pareto Outcome

The Pareto solution concept jointly optimises the utility function of all players over the time interval. This presents a form of corporate governance mechanism, as players agree to cooperate.

Such that:

$$\alpha J_1 + (1 - \alpha) J_2$$

In assessing the efficiency of the solution concepts, more insights could be obtained by comparing the two non-cooperative cases and the cooperative case.

Other forms of popularly examined differential games include the linear-quadratic game. In a linear-quadratic differential game, the equations of motion are linear in the state and control variables, while these variables are quadratic in the objective functionals. A linear-quadratic form of a differential game is expressed as:

$$J_i(u_i(t), u_j(t)) = \int_0^\infty [\frac{1}{2}u_i' C_i u_i + \frac{1}{2}x' D_i x + x' E_i u_i + c_i' u_i + d_i' x + h_i] e^{-\rho t} dt$$

$$\text{Subject to} \quad \dot{x} = Ax + B_1 u_1 + B_2 u_2 \quad x(0) = x_0$$

Where A, B, C, D, E are matrices assumed to be constant over time.

Cesar (1994) considered self-enforcing agreements using trigger (if a robust penalty can be applied to a defaulting player) and renegotiation proof strategies as mechanisms to sustain cooperation. These mechanisms incorporate threats and punishment to players who renege on cooperation over the time interval. Other incentives considered by Cesar are technology transfer and issue linkage.

3.3.5 Expected Results and Their Uses

The financial management model proposed in this study integrates all the relevant factors for developing an optimal incentive contract and optimal capital structure with respect to agency theory, mechanism design theory and corporate finance theory. It incorporates internal and external governance mechanisms that propel the management to supply reasonable efforts in the claimholder's interests. The model also accommodates incentive-compatible mechanisms, which establish optimal contracts that can resolve agency conflicts among the firm's stakeholders. Furthermore, the model is designed as a time-dependent model to integrate the dynamic business conditions peculiar to modern corporations. The outcomes of the model can improve a firm's financial strategies, specifically in the area of operating and financing activities, incentive-compatible mechanisms and capital structure decisions. The effectiveness of the financial management model can be judged based on the implications of the results for the examined research issues. This includes assessing the optimal capital structure model and most efficient contract design that can be adopted to

enhance company value, and how incentive mechanisms can achieve goal congruence between the shareholder and manager, and the debtholder and manager. Therefore, this study aimed to develop models that can enhance organisational designs, quality corporate governance practices and viable firm growth.

3.3.6 Verification and Validation of the Model

Model validation is key in any empirical research to enhance the model's implementation and insight into problems (McCarl & Apland 1986). Additionally, in a predictive and prescriptive model, validation provides insights into the practicality of the developed model in a real-world setting. The main objective of a validation test is to investigate how precisely a model performs with respect to its design objectives (McCarl & Spreen 2011). The process of model validation is often subjective because the validity test methods, the test setting, the criteria and the data to be used are dependent on the modeller. Nonetheless, it is essential to conduct the model's verification and validation to identify the soundness and plausibility of the model, and supply a reliable reasoning for its decision-making processes (Williams 2013). This can reveal a model's strengths and inadequacies, which is beneficial to the model's users and other interested parties who apply information from the model's results (McCarl & Spreen 2011).

The two approaches for validating a model are: validation by construct and validation by results. Validation by construct discloses whether the model design is sufficient and thus valid. On the other hand, validation by results aims at examining whether the outputs of the model are comparable against real-world observations. A discussion of the approaches to validation is given below.

3.3.6.1 Validation by Construct

Validation by construct is substantiated by one of the many assertions about modelling:

- 1) Correct procedures are adopted by the modeller in developing the model. This step involves the assertion that the techniques and approaches adopted in the model design are congruous with industry, previous studies, research, and/or theory; and the data are determined using

reasonable scientific approximations or retrieved from sound accounting procedures.

- 2) Trial results suggest the model is behaving competently, that is the model's results do not repudiate the modeller's, user's and/or expert's perceptions of reality.
- 3) The model is tempered with constraints that restrict the model to realistic solutions.
- 4) The modeller ensures that the data setup for the model makes it possible for the real-world outcome to be replicated.

Although validation by construct is necessary in modelling research, this type of validation is inadequate because it is assumed rather than tested. This leads to the need for a secondary form of validation (i.e., validation by results).

3.3.6.2 *Validation by Results*

Validation by results involves a comparison of the model's solutions and outcomes with real-world observations. The five steps for this test are:

- 1) Parameter outcome sets. This step involves the determination of a set of real-world outcomes and the generated data responsible for these outcomes. Examples of these data are the values of the model input parameters and output measures.
- 2) Validation experiments. These involve the internal consistency of the model, and include the feasibility experiment, the quantity experiment, the price experiment, the prediction experiment and the change experiment.
- 3) Use of appropriate data in the model setup.
- 4) Degree of association test. The objective of the association test is to measure how comparable a set of model results are to observed results.
- 5) Decisions based on the model's validity. A model validated by construct and results can be deemed satisfactory by decision-makers. However, since a model development involves many assumptions, the model may be unable to pass the validation test. Hence the modeller can improve on

the model by adjusting its assumptions, changing its programming techniques or applying structural corrections.

3.3.7 Justification of the Methodology Applied in the Thesis

This study aim was to develop a financial management model for integrating the inter-relationships between corporate governance, optimal capital structure, and agency theory. In designing the model, the mechanism design framework was adopted for formulating an efficient organisational design that can resolve agency problems, achieve good corporate governance and enhance company value.

This study employed a mathematical differential game modelling approach, which follows the popularly used mathematical optimisation approach. One advantage of this approach for developing a financial management model is that it provides a highly flexible apparatus for a decision-making process. Examples of studies in which the mathematical optimisation or game-theoretic approach have been deployed in financial modelling include Demski (1967), Ho and Lee (2004), Ijiri, Levy and Lyon (1963), Lee, Lee and Lee (2009), and Nuryanah and Islam (2015). These studies used the approach to relate the managerial strategies formulation with precise and general mathematical theory fundamental to the basic accounting relationships. The dynamic game model also has several remarkable features for managerial decision-making, as discussed previously. Therefore, differential game theory was chosen to achieve three goals, discussed as follows.

First, the model was designed within the principal-agent relationship. In this relationship, the principal cannot fully observe nor evaluate the agent's strategies or determine whether they chose the optimal course of action to maximise their utility, or achieve the systematic level of resource allocation, and thus enhance company value. Hence, the proposed model should provide optimal strategies and efficient allocation of resources to minimise the conflicts of interest between the principal and agent and achieve congruity of goals. The differential game approach can achieve these goals simultaneously by incorporating the interests of the two parties and analysing their strategic interactions.

Second, as long-term incentive contracts generally bring more advantages than short-term contracts, the model should mirror the optimal sequence of contracts as viable long-term financial management strategies. The model can then deliver

the optimal strategies and systematic resources allocation over time. This goal can be achieved through the dynamic behaviour of the differential game method adopted in this study.

Finally, the financial management model should be applicable at the inception of the long-term strategic period of the company to provide long-term benefit. Thus, the development of the financial management model requires the methodology to be comprehensive and stable. Based on the methodical stages discussed earlier, including the model's verification and validation, the mathematical programming and dynamic game approach employed in this study provides an effective method to achieve this objective. Additionally, the financial management model developed in this study was simulated and tested through a numerical application based on a case study approach to investigate the application of the model in a real-life modern business problem. Thus, the proposed methodology can be justified as sufficient and suitable for this study.

3.4 Research Data and Computer Programs

3.4.1 Justification of Data Collection

As previously discussed, this study conducted a case study of a non-financial company to develop the numerical financial management model. The developed financial management model was designed to be relevant to global non-financial companies, particularly consumer staple firms with agency problems. The developed financial management model seeks to optimise the financial management strategies from a managerial and accounting context with the integration of appropriate accounting principles, incentive-compatible mechanisms, governance instruments, and corporate finance concepts. Possible outcomes of the developed model include optimal financial management strategies, based on optimal capital structure design, leading to improved company performance and optimal incentive contracts for aiding good corporate governance practices and systematic organisational design.

The study applies details of operational and financial issues of a non-financial company to simulate and to obtain practical solutions for the proposed model. Secondary data was gathered from the historical financial information of the chosen company and publicly available reports provided by national financial

institutions, including the capital markets, central bank, and the ASX. If data were unavailable in public repositories, figures were simulated based on reasonable assumptions from historical data.

3.4.2 Software Specifications for Modelling Dynamic Game Theory

The financial management model in this study was developed as an integrated corporate financial differential game model. To solve the differential game problem, several computer programs could be used, including MATLAB, GAMS and the Solver application for Microsoft. This study employed MATLAB (2020a), developed by MathWorks, to simulate and analyse the game model. MATLAB was developed as a multi-paradigm numerical computing environment and a proprietary programming language. The program permits matrix manoeuvring, plotting of functions and data, actualisation of algorithms, and formation of user interfaces. This software has robust technical support for conventional optimisation and includes numerous modelling built-in solvers with its embedded toolbox, such as the optimisation toolbox, Simulink. The differential game model is specified in the optimal control framework and incorporates the strategic interaction of multi-players, thus the numerical algorithm for the model is specified as such in the MATLAB environment.

3.4.3 Numerical Algorithm

The numerical analyses for the three solution concepts – Nash open-loop, Nash feedback equilibrium and Pareto optimisation were computed by devising an algorithm that creates an approximation of the optimal piecewise continuous controls/strategies for instance, $l(t)$ and $c(t)$. This is done by splitting the time interval $[t_0, T]$ mapped as $[0,10]$ in this dynamic process into pieces with certain points of interest, which are equally spaced. The optimal controls/strategies $l(t), c(t)$ obtained from the optimality conditions are imputed into the ordinary differential equations for $M(t), D(t), \lambda_1(t), \lambda_2(t), \gamma_1(t), \gamma_2(t)$, which then form two-point boundary point value problems.

The Runge-Kutta numerical method was then used to solve the optimality system. The model provides initial conditions for the two states variables as

$M(t_0) = M_0$ and $D(t_0) = D_0$ and final time conditions for the adjoint variables $\lambda_1(T) = \lambda_2(T) = \gamma_1(T) = \gamma_2(T) = 0$. Using the forward-backward sweep method, the Runge-Kutta 4 (RK4) approach was applied via the following outlined algorithm:

Step 1: Provide an initial guess for $l(t), c(t)$ over the interval, this is set as zero.

Step 2: Using the initial conditions $M_0 = 2.4, D_0 = 0.27$ and the values for $l(t), c(t)$, solve $M(t), D(t)$ forward in time following the differential equations in the optimality system.

Step 3: Using the transversality conditions $\lambda_1(T) = \lambda_2(T) = \gamma_1(T) = \gamma_2(T) = 0$ and the values for $l(t), c(t), M(t), D(t)$, solve for λ, γ backward in time following the differentials in the optimality system.

Step 4: $l(t), c(t)$ are then updated by entering the new values obtained for $M(t), D(t), \lambda_1(t), \lambda_2(t), \gamma_1(t), \gamma_2(t)$ into the characterisation of the optimal controls/strategies.

Step 5: Check for convergence. The solutions are outputted if values of the variables in the current iteration is negligibly close to those from the last iteration, otherwise return to Step 2. The convergence test used in this study is $\delta ||l(t)|| - ||l(t) - oldl(t)|| \geq 0$ and $\delta ||c(t)|| - ||c(t) - oldc(t)|| \geq 0$.

In a standard RK4 routine using a step size h and an ordinary differential equation (ODE) $\dot{x}(t) = f(t, x(t))$, the approximation of $x(t + h)$ given $x(t)$ is specified as:

$$x(t + h) \approx x(t) + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where

$$k_1 = f(t, x(t))$$

$$k_2 = f(t + \frac{h}{2}, x(t) + \frac{h}{2}k_1)$$

$$k_3 = f\left(t + \frac{h}{2}, x(t) + \frac{h}{2}k_2\right)$$

$$k_4 = f(t + h, x(t) + hk_3)$$

For more details, see Lenhart and Workman (2007).

3.4.3.1 Numerical Results

The control variables $l(t), c(t)$ are called outcomes of the game if a continuous and piecewise smooth function $M(t), D(t)$ on $[t_0, T]$ exists, which is a unique solution to the initial value problem,

$$\dot{M}(t) = f(t, M(t), l(t), c(t)); \quad M(t_0) = M_0 \quad (3.1)$$

$$\dot{D}(t) = f(t, D(t), c(t)); \quad D(t_0) = D_0 \quad (3.2)$$

And satisfies the state constraints $M(t), D(t)$ for all $t \in [t_0, T]$. Functions $M(t), D(t)$ are called the state trajectory corresponding to the outcomes $l(t), c(t)$ (Lenhart & Workman 2007). The optimal strategies $l(t), c(t)$ are obtained using the RK4 numerical method of solving ordinary differential equations outlined in the previous section. The optimal debt-equity ratio (optimal capital structure) is derived from the optimal strategies and corresponding state trajectories $M(t), D(t)$.

3.4.3.2 Global Optimality

In a differential game typically formulated as an optimal control problem, characterisations of global solutions are derived (i.e., the necessary conditions for optimal solutions). The neighbourhood considered in a standard optimisation problem is referred to as the time horizon in a dynamic problem. In considering the global optimality, therefore, the time path considered for the game is said to be the whole space. Consequently, since the characterisations derived from the solution concepts and computed numerically provide optimal solutions within the time path investigated, then the optimal solutions are said to be global optimal in the time path if only the objective functional and constraints are convex.

3.4.3.3 Existence and Uniqueness Theorem

The existence theorem was used in this study, as specified by Fleming and Rishel (1975).

Theorem:

Consider the objective functional

$$J(l, c) = \int_{t_0}^T f(t, M, D, l, c) dt \quad (3.3)$$

$$\text{Subject to } \dot{M}, \dot{D} = g(t, M, D, l, c), \quad M(t_0) = M_0, D(t_0) = D_0. \quad (3.4)$$

We leave out the t subscript from the vector variables $M, D, l, c, \lambda, \gamma$. Suppose that $f(t, M, D, l, c)$ and $g(t, M, D, l, c)$ are both continuously differentiable functions in their five arguments and concave in M, D, l, c for the manager's objective functional (minimisation problem) and convex for the debtholder's objective functional (maximisation problem). l^*, c^* are controls with associated states M^*, D^* , and λ, γ are piecewise differentiable functions, such that $l^*, c^*, M^*, D^*, \lambda, \gamma$ together satisfy on $t_0 \leq t \leq T$:

$$f_{l,c} + \lambda g_{l,c} = 0, \quad (3.5)$$

$$\dot{\lambda} = -(f_{M,D} + \lambda g_{M,D}), \quad (3.6)$$

$$\lambda(T) = 0, \lambda(t) \geq 0 \quad (3.7)$$

Then for both controls l, c , we have

$$J(l^*, c^*) \geq J(l, c).$$

Proof: l, c are controls and M, D the associated states, by the tangent line property

$$\begin{aligned} f(t, M^*, D^*, l^*, c^*) - f(t, M, D, l, c) &\geq (M^* - M), (D^* - D) f_{M,D}(t, M^*, D^*, l^*, c^*) + \\ &+ (l^* - l), (c^* - c) f_{l,c}(t, M^*, D^*, l^*, c^*). \end{aligned} \quad (3.8)$$

This gives

$$\begin{aligned} J(l^*, c^*) - J(l, c) &= \int_{t_0}^T f(t, M^*, D^*, l^*, c^*) - f(t, M, D, l, c) dt \geq \int_{t_0}^T (M^* - M, D^* - \\ &D) f_{M,D}(t, M^*, D^*, l^*, c^*) (l^* - l), (c^* - c) + f_{l,c}(t, M^*, D^*, l^*, c^*) dt. \end{aligned} \quad (3.9)$$

Substituting

$$f_{M,D}(t, M^*, D^*, l^*, c^*) = -\dot{\lambda} - \lambda g_{M,D}(t, M^*, D^*, l^*, c^*) \quad \text{and}$$

$$f_{l,c}(t, M^*, D^*, l^*, c^*) = -\lambda g_{l,c}(t, M^*, D^*, l^*, c^*)$$

The last term in Equation (3.9) becomes

$$(M^* - M), (D^* - D)(-\lambda g_{M,D}(t, M^*, D^*, l^*, c^*) - \dot{\lambda}) + (l^* - l), (c^* - c) - \lambda g_{M,D}(t, M^*, D^*, l^*, c^*) dt \quad (3.10)$$

Recall $\lambda(T) = 0$ and $M_0 = M^*_0, D_0 = D^*_0$, and using integration by parts,

$$\int_{t_0}^T -\dot{\lambda} (M^* - M), (D^* - D) dt = \int_{t_0}^T \lambda (M^* - M)', (D^* - D)' dt = \int_{t_0}^T \lambda g(t, M^*, D^*, l^*, c^*) - g(t, M, D, l, c) dt.$$

Making the substitution,

$$J(l^*, c^*) - J(l, c) \geq \int_{t_0}^T \lambda [g(t, M^*, D^*, l^*, c^*) - g(t, M, D, l, c) - (M^* - M), (D^* - D)g_{M,D}(t, M^*, D^*, l^*, c^*) - (l^* - l), (c^* - c)g_{l,c}(t, M^*, D^*, l^*, c^*)] dt. \quad (3.11)$$

The desired result then satisfies $J(l^*, c^*) \geq J(l, c)$ for the minimisation problem and $J(l^*, c^*) \leq J(l, c)$ for the maximisation problem.

Similarly, for **uniqueness**, l^*, c^* are such that $J(l, c) \leq J(l^*, c^*) < \infty$ for both controls l, c in the minimisation problem, and $J(l, c) \geq J(l^*, c^*)$ for a maximisation problem. Therefore, l^*, c^* are said to be unique whenever $J(l^*, c^*) = J(l, c)$, then $l^*, c^* = l, c$ at all but finitely many points. The associated states M^*, D^* are called the unique optimal states.

3.5 Context of the Case Study

To achieve the goal of the study, a case study investigation was carried out with a definite sample, chosen using purposive sampling with specific criteria. The first criterion required the company to be a publicly owned or listed company, as the financial data would be publicly available. Second, a non-financial company in a

developed country was selected to minimise the extreme financial regulation and peculiar parameters associated with financial firms. Lastly, the company had to be operational in a country with well advanced adoption of the International Accounting Standards (IAS) and International Financial Reporting Standards (IFRS). The company selected was based in a consumer staple sector of Australia.

The sections that follow provide a broad background of the case study company, including the country (i.e., Australian) context. The first subsection briefly reviews the Australian capital market and regulatory environments. Following this, the specific characteristics of the case study company, Metcash Limited, are highlighted. The representation of the sample chosen is important to understand the underlying framework in developing the corporate financial game model. It also provides a foundation for how the analysis and implications of the proposed model are further conducted.

3.5.1 Australian Economy

3.5.1.1 *Australian Capital Market*

Established on the 1 April 1987, under the initial name Australian Stock Exchange, the Australian financial market exchange is now registered under the trading name, the Australia Securities Exchange (ASX). Currently, the ASX is one of the largest listed exchanges in the world, ranking 16th worldwide as of February 2020. It has a current total market capitalisation of approximately \$1.9 trillion and is ranked amongst the top eight in the Asia-Pacific region.

The movement of the ASX all ordinaries (XAO) share price index over a ten-year period is presented in Figure 3.2, including the 500 largest Australian listed companies with respect to market capitalisation. The XAO index is identified as the indicator of the Australian capital market. Figure 3.2 illustrates that, with the exception of 2011 which experienced a significant dip, the Australian capital market share prices progressed positively across the financial period. This indicates that the ASX is a reliable capital market to explore.

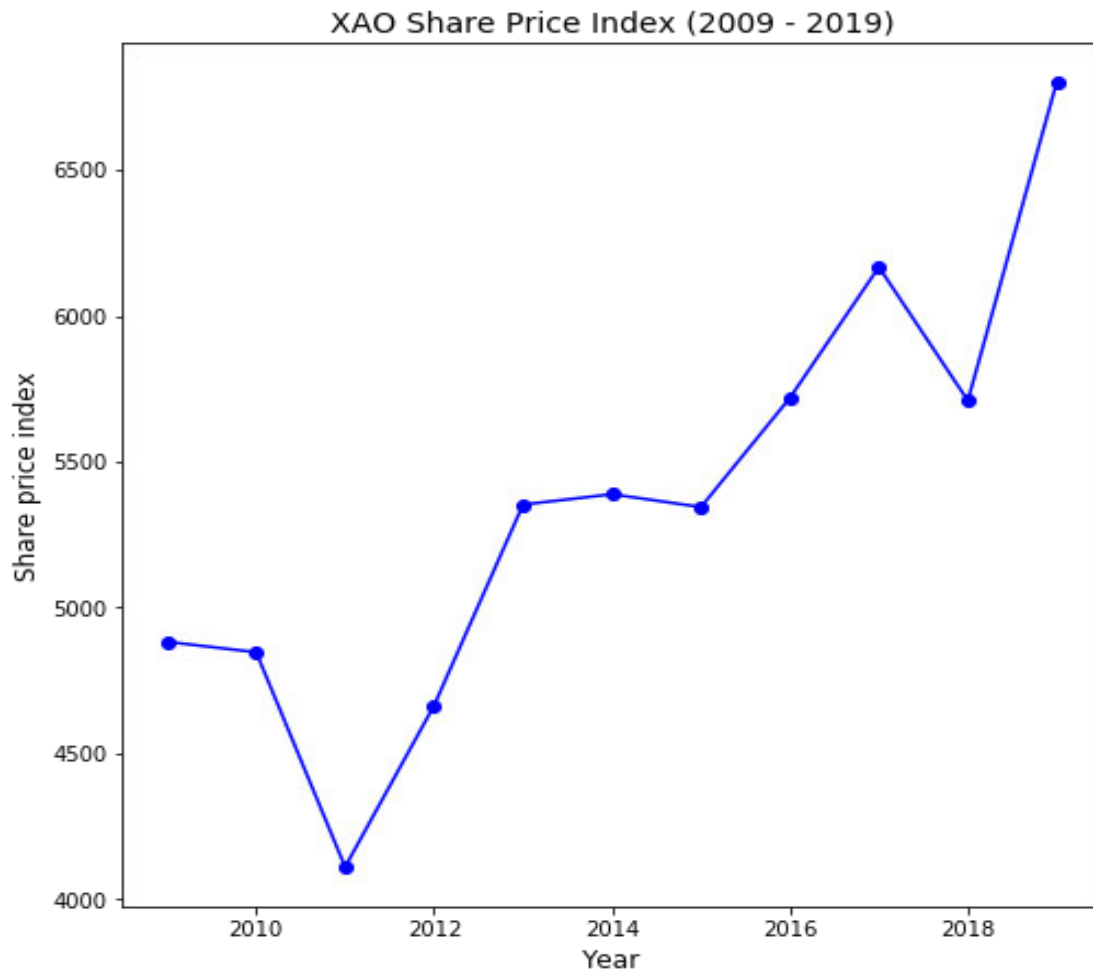


Figure 3.2 Graph created based on data from the ASX (www.asx.com.au)

Similarly, Australia has a well-developed economy with a gross domestic product valued just under \$2 trillion. The ASX is a highly regulated environment, monitored by law enforcement agencies, namely the Australian Securities and Investments Commission (ASIC) and the Reserve Bank of Australia (RBA). These agencies are empowered to oversee the activities of the ASX, such as listings, trading, clearing and settlement across various asset classes.

3.5.1.2 Australian Accounting Standards

In Australia, accounting standards are enforced by the Australian Accounting Standards Board (AASB), established under the *Corporations Act 2001*. The AASB enforces accounting standards in order to ensure that financial statements are compliant and accurately represent the financial position of a firm to its stakeholders and other interested parties. The IFRS were adopted in Australia

on 1 January 2005, following a strategic direction from the Financial Reporting Council (FRC). This guarantees that financial statements prepared in accordance with AASB standards also comply with the IFRS.

The transaction neutrality policy imposed by the AASB ensures that similar financial statement items and events are accounted in a similar manner by all corporate entities. This ensures uniformity in reporting, particularly for companies in the same sector. Importantly, this policy also means that the financial management model developed in this study has the flexibility to be used by firms similar to the case study company.

3.5.2 Company Profile: Metcash Limited

Metcash Limited is a publicly listed company in the consumer staple sector of the Australian economy. It is known for distributing and marketing groceries, fresh produce, alcoholic beverages, hardware, and other consumer goods. The company is recognised as one of the leading wholesale distribution and marketing companies in the country. Metcash Limited generally focuses on supplying independent retail companies in Australia. It owns notable retailers such as IGA, Mitre 10, and Home Timber and Hardware. Having completed a thorough investigation of the company's year-to-year financial statements, it was concluded that Metcash has all the major features that make it feasible to replicate the model developed in this study to any global (public) non-financial company.

The state equations and other constraints of the model were obtained from quality corporate governance practices and management and financial accounting principles. The figure below is an example of a non-financial company's balance sheet statement, from which most elements of the developed model were taken.

XYZ Ltd.

Balance Sheet as of 31 December 20xx

<i>Symbol</i>	<i>Name of Accounts</i>
	ASSETS
	Current Assets:
	Cash and cash equivalents 556,648
	Accounts receivable 20,634
	Inventory 80,125
	Total current assets/liquid reserve 657,407
	Non-current Assets:
	Property & equipment 51,473
	Goodwill 37,252
	Total non-current productive assets 88,725
	TOTAL ASSETS 746,132
	LIABILITIES
	Current Liabilities:
	Accounts payable 120,000
	Accrued expenses 45,000
	Unearned revenue 25,000
	Total current liabilities 190,000
	Non-current Liabilities:
	Interest bearing borrowings 105,000
	Derivative financial instruments 75,000
	Other financial liabilities 250,000
	Total non-current liabilities 430,000
	TOTAL LIABILITIES 620,000
	EQUITY
	Share capital 27,262
	Retained earnings 1,333

Other reserves	16,360
Parent interest	80,640
Non-controlling interest	537
TOTAL EQUITY	126,132

Figure 3.3 Example structure of a non-financial firm's balance sheet

3.5.2.1 Corporate Governance Practices in Metcash Limited

Metcash has established corporate governance mechanisms. This includes monitoring by an eight-person board of directors, and incentive packages. It has also disclosed most of its corporate governance policies via the following charters, rules and reports: code of conduct, market disclosure, Metcash constitution, Metcash diversity policy, Metcash risk management policy, Metcash securities trading policy, Metcash speak-up policy, Metcash workplace gender equality agency (WGEA) public report, minimum shareholding policy, shareholder communication policy, and tax transparency report.

In its current financing structure, Metcash uses a mix of equity-holding and debt, currently using more equity than long-term debt.

3.6 Conclusion

This chapter has presented the conceptual framework of this study, conceptualising and integrating all the factors relevant for addressing the research problem. Interconnections between optimal capital structure, mechanism design, corporate governance, and company performance have been established as the basis of the conceptual framework.

Chapter 4 – Shareholder-Manager Conflicts of Interest and the Optimal Capital Structure

4.1 Introduction

4.1.1 Context

This chapter examines the optimal capital structure specification problem when complicated by the principal-agent problem between the shareholder and the manager. This can be solved by employing corporate governance mechanisms to give incentives, thus integrating mechanism design into the choice of optimal capital structure. To achieve this, a game-theoretic model was used to analyse non-cooperative strategic interactions. As agency problems are dynamic problems, it was important to adopt a dynamic game theory approach to analyse the non-cooperative strategic relationship between the shareholder and the manager over time, as well as the resulting optimal capital structure.

The inherent agency conflict discussed in this chapter is information asymmetry. In this case, the shareholder is unable to observe all the actions of the manager to determine the quality of effort they are committing to optimise the firm. The shareholder wealth maximisation is a function of the manager's quality of effort. This is evident in their decision whether to divert cash or not and in the choice of capital structure. The shareholder presumes the manager's opportunistic tendencies; therefore, the shareholder must solve the problem by designing a mechanism or incentive contract that induces the manager to take optimal actions and consistently select the optimal capital structure.

In Chapter 2, the different forms of corporate governance instruments available to minimise the agency problem between the shareholder and the manager were discussed in detail. This included incentive pay, debt, hostile takeover, product-market competition, and monitoring, among others. From these previously discussed mechanisms, in this chapter, the following corporate governance mechanisms are used: debt mechanisms, compensation packages (pecuniary incentive), and monitoring. These three mechanisms are used because they can be efficient in resolving the agency problem complementarily. The use of debt as

a corporate governance mechanism was well documented by Jensen (1986), as debt reduces the available free cash. The use of pecuniary incentives, such as stock option and fixed pay, has also been well established in the literature, such as Bolton and Dewatripont (2005), Copeland, Weston and Shastri (2014), and Thomsen and Conyon (2012). Myers (2001) also developed a survey of compensation contracts. The manager requires a sufficient incentive package to encourage their participation in the management contract. Similarly, the incentive must be adequate enough to cause them to act in the best interests of the shareholder. Boly (2011) and Kaplan (1989) found monitoring to be effective in propelling the manager's effort, and minimising the agency problem of free cash flow (Karpavičius & Yu 2017). The higher the cost the company incurs for monitoring, the less free cash flow is available to the manager. Therefore, a debt mechanism, incentive pay, and monitoring were set as complementary corporate governance mechanisms used to align the actions of the manager with the goal of the shareholder. This would minimise the agency problems of cash diversion and the inappropriate actions of the manager, while simultaneously selecting an optimal capital structure in a framework to be applied over time.

Due to the imperfect alignment of the shareholder and manager's interests, the shareholder introduces debt as a disciplining measure for the manager. This commits the manager to improve cash flow, repay debts, and improve the company value. The use of debt induces debtholders to serve as additional monitors for the firm. Debt finance induces the manager to focus on debt repayment and hence minimises the available free cash flow. The wealth maximisation goal of the shareholder becomes important to the manager when they are at risk of bankruptcy (Grossman & Hart 1982; Jensen 1986). The shareholder may also prefer the use of debt because it minimises the dilution of the company's shares and the value derived from the use of debt can be added to the shareholder's wealth. The use of more debt has the added benefit of reducing the company's weighted average cost of capital (Ni, Chu & Li 2017). However, managers may avoid debt because of its disciplining effect on them, and issue equity instead when it is overvalued due to information asymmetry.

Since debt serves as a disciplining measure on the manager, the manager may prefer less debt than is optimal (Liu et al. 2017; Mu, Wang & Yang 2017). Thus, there is a need for the shareholder to induce the manager to select the optimal use of debt rather than divert free cash. Moreover, although the shareholder prefers the use of debt as a corporate governance instrument, excessive debt usage poses the risk of financial distress and bankruptcy costs (Brisker & Wang 2017). The findings of Mu, Wang and Yang (2017), and Shibata and Nishihara (2010), emphasise the need to develop a corporate governance/incentive framework to minimise the agency problem of moral hazard with respect to an optimal capital structure. From their results, it is evident that firms issue more debt with higher coupons and default earlier with manager moral hazard problems. Thus, the shareholder's appetite for debt as an incentive mechanism must be constrained to the optimal capital structure. This will avoid a debt overhang problem.

4.1.2 Gaps in the Existing Literature

According to modern agency theory, there is a well-defined relationship between the firm's capital structure and the shareholder-manager agency relationship. However, most studies of incentive design ignore the role of the financial structure of the firm (Mu, Wang & Yang 2017). To minimise the agency cost caused by managerial inclination and misaligned incentives, it is economically important to jointly optimise the firm's capital structure and compensation contracts (Martellini, Milhau & Tarelli 2018). Since debt is used to align the interests of the manager, it thus proves the relationship between compensation contracts and capital structure (Ortiz-Molina 2007). Most of the literature has addressed one of the two problems, either developing an optimal mechanism design or designing an optimal capital structure. While there are a number of studies that have sought to address the two problems simultaneously, such as Heinrich (2002), none have done so using a differential game theory framework. Thus, this serves as a limitation of the existing literature and the contribution of this thesis.

4.1.3 Aims of the Chapter

As indicated earlier, the aim of this chapter is to find the optimal capital structure when a situation is characterised by the existence of agency problems between

the shareholder and the manager. This can be resolved via corporate governance mechanisms based on differential game theory. The issues generated by the interconnection between the capital structure, dynamic game theory and the agency conflicts of interest or corporate governance raise the following questions: What is the optimal mechanism design required to minimise the manager's opportunistic tendencies that can be best represented by a game theory model? What is the optimal capital structure required to reduce the agency problem sufficiently and simultaneously maximise the firm value? What are the outcomes for the shareholder and manager in a dynamic framework? Hence, the objective, as evident in this chapter was to develop a differential game theory model that selects the optimal capital structure as an incentive mechanism, in addition to other complementary corporate governance instruments, to resolve the agency problem of information asymmetry.

4.1.4 Structure of the Chapter

This chapter is organised as follows: Sections 4.1 has provided a general introduction, while Section 4.2 provides more detailed background information about the focus of the chapter. Section 4.3 presents the model setup with information asymmetry. Section 4.4 analyses the model as a non-cooperative dynamic game model using Nash equilibrium concepts and a Pareto-efficient outcome as a cooperative outcome. Section 4.5 provides the computational results. Section 4.6 presents the implications and insights from the study. Finally, Section 4.7 summarises and concludes the chapter.

4.2 Background Literature: Motivation for this Study

There is a large number of literature in corporate finance in the area of capital structure. This includes Elton and Gruber (1974), Fischer, Heinkel and Zechner (1989), Goldstein, Ju and Leland (2001), Karpavičius (2014), Leland (1994), Leland and Toft (1996), Titman and Tsyplakov (2007), among others. These studies examined the optimal static or dynamic capital structure strategy for a company. However, in these models, the mathematical optimisation approach focused on the results for the shareholder. Thus, they are aggregate models for a firm's choice of the optimal capital structure that do not include agency theory.

These studies therefore imply that the principal-agent relationship does not impact on the optimal capital structure. Hence, they are inadequate in reflecting the true optimal capital structure required by a firm. The new model presented in this chapter embeds the agency problem explicitly. This has been described as a determinant of capital structure.

The literature on the principal-agent problem, particularly in the context of management contracts includes the following. DeMarzo and Sannikov (2006) examined optimal contracts in a cash-flow diversion model. Cvitanic, Wan and Zhang (2009) designed optimal contracts for one-off payments to the agent. Ai and Li (2012) considered the integration of moral hazard and restricted enforcement into a neoclassical economy. DeMarzo et al. (2012) integrated dynamic investment with optimal dynamic incentive contracting to develop an analytical principal-agent model. The major difference between these studies and the new study is that debt financing, and the theory of optimal capital structure was left out of the design of their models and analysis.

There is a large body of literature on the design of optimal contracts, including Amaro de Matos (2001), Bolton and Dewatripont (2005), Campbell (2006), DeMarzo and Sannikov (2006), Fama and Jensen (1983), Jensen and Meckling, (1976), Nyberg et al. (2010), Scott (2015), and Copeland, Weston and Shastri (2014). These studies focused on the use of pecuniary incentives in the design of an optimal contract. The use of a specified compensation structure and equity ownership has been developed and adopted in aligning the shareholder-manager interest. However, contracts that solely associate the manager's pay with company performance incur a cost of pecuniary incentives, which causes a suboptimal allocation of risk. The use of additional corporate governance instruments can reduce the cost of such pecuniary incentives (Heinrich 2002). Studies such as Agrawal and Knoeber (1996), Dalton et al. (2003), Heinrich (2002), and Tosi et al. (2000) have highlighted the drawbacks of such pecuniary-focused contract designs. Accounting measures are short-term, instead, shareholders should focus on the long-term. Thresholds introduce non-linearities by giving managers who are able to predict company outcomes the opportunistic tendencies to withhold effort if the company's current performance exceeds the

pre-specified caps. Particularly if additional efforts which can improve company performance may not improve the manager's pre-specified incentives. Or the manager may become less optimistic if the expected performance becomes unattainable. In addition, manager's may delay the reporting of profits until the next financial period in order to boost their bonus in that period. In addition, argue Jensen and Murphy (1990), executive compensation is not sufficiently connected to company profits. Thus, the use of alternative or multiple or complementary corporate governance mechanisms to control the shareholder-manager conflict is encouraged.

Agrawal and Knoeber (1996) examined the interdependence of seven mechanisms using ordinary least squares (OLS) regression. The model was designed in a static framework, and largely investigated the relationship between the mechanisms. Bebchuk and Fried (2003) also argued that incentive pay was a major part of the agency problem. They stated that managers hold too much power over their incentive pay and thus focus on reducing the relationship between their performance and their pay. Olarewaju, Migiro and Sibanda (2017) suggested that managerial compensation should consider managerial power not only the theory of optimal contracting. These studies discouraged the over-reliance on incentive pay to minimise the shareholder-manager agency problem. Dalton et al. (2007) highlighted the following as the three major ways to minimise agency problems: agent equity ownership, board independence (monitoring), and the market for corporate control (active mergers and acquisitions or takeovers). Thus, pecuniary incentives remain significant in reducing the agency problem but cannot eliminate the problem. It is therefore efficient if companies use multiple governance mechanisms, such that the optimal pecuniary incentive is dependent on how well the company uses other governance mechanisms (Heinrich 2002).

Different forms of corporate governance instruments designed to minimise the agency problem between shareholders and the managers have been discussed extensively. These include incentive pay, hostile takeovers, product-market competition, and monitoring, among others. As already stated, this study uses three mechanisms (debt, monitoring and pecuniary incentives) to minimise the

agency problem and obtain an optimal capital structure in a dynamic framework. Bertrand and Mullainathan (1998) modelled takeover as a complementary mechanism to a pecuniary incentive. However, Agrawal and Knoeber (1998) suggested that these two complementary mechanisms may not lead to optimal mechanism design. This is because shareholders may interpret takeover threats as a requirement for higher (but not necessarily optimal) pecuniary incentives due to the higher risk imposed on the manager. Mehran (1995) used monitoring and financial leverage as complements to a pecuniary incentive in a static model. Similarly, Heinrich (2002) modelled a pecuniary incentive, monitoring and debt as complementary governance mechanisms in a three-period discrete-time model. The use of monitoring as a corporate governance mechanism to complement a pecuniary incentive is important because the Pareto optimal fee cannot maximise the shareholder's expected profit sufficiently without incorporating the cost of monitoring (Amaro de Matos 2001). Similarly, to derive the optimal capital structure that minimises the agency problem, debt is a significant instrument. Thus, this study builds on these existing models by considering these three complementary corporate governance mechanisms in a continuous-time framework using differential game theory.

This study is also relevant to previous studies on free cash flow, with an emphasis on the role of debt as a mechanism for reducing agency conflicts between shareholders and managers. These previous studies include Dewatripont and Tirole (1994), Fluck (1999), Hart and Moore (1995), Jensen (1986), Stulz (1990), and Ortiz-Molina (2007), among others. These studies demonstrate the significance of debt financing in improving efficiency by discouraging the manager from taking on unprofitable investments due to their debt repayment responsibilities. In this chapter, it is argued that the manager's efforts and actions have reasonable effects on the company's profitability; the use of debt can minimise managerial discretion on efforts that impact the cash flow growth rate. The current study builds on the intuition of existing studies to develop an optimal capital structure for the firm.

In addition, this study links to existing research on the interconnection between capital structure and principal-agent problems or corporate governance in

different forms. This research includes the work of John and John (1993), He (2011), Novaes and Zingales (1995), Morellec (2004), and Mu, Wang and Yang (2017). John and John (1993) observed the correlation between the optimal managerial compensation and the design and mix of external financing by a company. Novaes and Zingales (1995) considered a framework that permits comparison of the shareholder's value-maximising and the managerial approaches to each party's capital structure choice. This proposes the optimal capital structure from both a manager's and the shareholders' perspective. This considers the non-cooperative analysis of the shareholder-manager game analysis. In this current study, the model was developed with the objective of inducing the manager to select the optimal capital structure that is beneficial to the company. Morellec (2004) demonstrated the consequences of managerial discretion and the impact of corporate control mechanisms on debt issuance and firm value. He (2011) found evidence that agency problems can cause smaller firms to use a suboptimal level of debt. He observed this by comparing results from the capital structure framework by Leland (1994) with his calibrated results. The studies by He and Leland further justify a need to resolve the agency problem as it impacts the firm's capital structure, a gap the current research aims to fill. In addition, He (2011) considered the interaction between the capital structure problem and the principal-agent framework. He considered an optimal capital structure alongside the design of an optimal contract design, using pecuniary incentives in a dynamic framework. This new study is different because it integrates monitoring as a complementary corporate governance instrument and analyses the outcomes for the players in a dynamic framework using a differential game approach. Hence, although some of the existing literature has integrated optimal mechanism design and the optimal capital structure, no study has done this using the differential game theory framework and complementary corporate governance mechanisms.

In dynamic agency theory, the manager selects different actions throughout the management contract, and these actions affect the output in each period (DeMarzo et al. 2012). This results in different optimal outcomes for the company. In most optimal mechanism designs, the renegotiation proofness of the contract is also ignored. Further, most models of capital structure, such as that of Mu,

Wang and Yang (2017), assume that debt is issued only at an initial time $t = 0$, and remains so throughout the lifecycle of the firm, thus specifying the capital structure as static. However, this is inadequate as a company has the ability to attract or use additional debt finance, depending on its relationship with debtholders and the capital market generally. Moreover, depending on the company's output in a period of assessment, shareholders may recommend greater use of debt to induce the manager's effort in the following period.

These dynamic issues associated with the interconnection between the optimal mechanism design and the optimal capital structure have not been studied comprehensively in the existing contract theory literature. Addressing this gap thus forms part of the objectives of this chapter. Such issues and the strategic interactions over time require a dynamic game theory in the form of differential game theory.

A key component of differential games is that they maximise the descriptive ability of differential equations as a modelling tool. Differential games can explain dynamic situations of conflict comprehensively (Mehlmann 1988). Thus, this approach has been used in many situations, such as to analyse a model of negotiation between management and a union during a strike negotiation (Leitmann & Liu 1974), and in the problem of wealth allocation between workers and capitalists in a Keynesian context (Lancaster 1973). The loop model of differential games allows the formulation of interaction between the actions of players. Contrary to the Kuhn tree, in which simultaneous play is not modelled, the loop formulation in differential games assumes players make simultaneous decisions by selecting optimal strategies dependent on the information available in the game (Mehlmann 1988). This extensive loop formulation of the simultaneous strategic interactions between shareholder and manager makes it appropriate to apply differential game theory rather than a discrete time game theory model.

In addition, there is significant evidence that cooperative behaviours should be considered as continuous strategies that vary over time (Harrald & Fogel 1996; Killingback 2002). A continuous-time framework enhances the reciprocity and

strategic interactions between players (Zhong, Kokubo & Tanimoto 2012). Continuous strategies are also more realistic in a management contract as actual provisions might be continuous rather than discrete. A manager's optimal strategy is a combination of strategies adequately represented as continuous values, such as decreased investments and less use of debt, rather than a discrete decision to use debt or not.

Thus, the differential game theory approach comprehensively analyses the strategic interaction between the shareholder and the manager over time. It also provides outcomes for the problem as a non-cooperative game following some scheme of rational and stable behaviour, and as a cooperative game following the Pareto-efficient incentive design framework.

Hence, the significant contribution of this study, as outlined in this chapter, is that it designs a dynamic optimal mechanism and an optimal capital structure specification that minimises the shareholder-manager agency problem. This is achieved via complementary corporate governance mechanisms using a differential game theory approach.

4.3 Model Setup: A Dynamic Game Framework

In this section, a dynamic game framework is developed between the shareholder as the principal and the manager as the agent. This is developed via the principal-agent model, along with the firm's capital structure, under a continuous-time framework based on a differential game theory model. First, some necessary assumptions about the tax environment, optimisation problems, compensation structure and the dynamic game are provided.

4.3.1 Model Assumptions

It is presumed that the shareholder owns the company. However, since the shareholder lacks the managerial skills and needs time for leisure and other activities, they delegate the management of the company to a skilled manager via a contract. Managing the company requires costly effort from the manager. The shareholder, via the board of directors, offers the manager a compensation package $I(t)$ that is a function of the company's outcomes $Y(t), D(t), S(t)$ such

that $I(t)(Y(t), D(t), S(t))$. The manager has an alternative contract that can provide them with at least b , their reservation utility, by employing their skills elsewhere. Hence the compensation schedule must provide the manager with at least b , such that $I(t) \geq b$. Once the exogenous management contract has been signed, the manager has an information advantage that causes them to divert free cash and ignore the optimal debt level of the company, hence the information asymmetry problem. Doing otherwise requires a costly effort $u_4(t)$ from the manager, this is their quality of effort which is unobservable to the shareholder. When the manager commits high quality effort, this can be interpreted as a lower cash or resources diversion rate, and when the manager commits low quality effort, this is interpreted as a higher cash or resources diversion rate. As the shareholder cannot fully observe the level of effort provided by the manager, and to achieve the wealth maximising goal of the shareholder, the shareholder must select the optimal $u_1(t)$ and $u_2(t)$ that indirectly or directly induces the manager to maximise $(Y(t), D(t), S(t))$.

Assumption 1. $Y(t), I(t), S(t)$ are all assumed to be strictly increasing functions with respect to $u_4(t)$; $Y'(t) > 0$, $I'(t) > 0$, $S'(t) > 0$. This means that with a greater quality of effort from the manager, the cash flow, productive assets and pecuniary incentive will be higher.

Assumption 2. Unlike other capital structure models, such as Leland (1994), this model assumes that debt can be issued throughout the period and thus changes over the lifecycle of the company. This means that the debt issued at the initial time $D(0)$ may not be the same as the final time $D(T)$ except the company's ratio of new debt to existing value of company debt $\alpha = 0$ throughout the period.

1. Cash flow dynamic with information asymmetry

Following Goldstein, Ju and Leland (2001), Moreno-Bromberg and Rochet (2018), and Mu, Wang and Yang (2017), the model presented in this chapter is set up in a continuous-time framework. In order to set up the differential game analysis of the model, it is set up via ordinary differential equations (ODEs) in a deterministic framework. The following are the benefits of a continuous-time

framework. It provides a tidier characterisation of the optimal contract via an ODE, and allows for cleaner comparative statics and the possibility to hypothesise the model in broader contexts (DeMarzo & Sannikov 2006). Thus, it is assumed that a firm's cash flow before tax, also known as earnings before tax (EBT) denoted by $Y(t)$, is defined as an ODE. In Mu, Wang and Yang (2017), the firm's cash flow is defined by the company's growth rate, which is dependent on the quality of the manager's effort $u_4(t)$. The quality of the manager's effort can also be described as their response to the incentive at any time t . Improving on Mu, Wang and Yang's (2017) definition of the cash flow, in this study the impact of mechanism design is specified as a deduction from the cash flow. Thus, the cash flow evolves as a function of financial income $\beta S(t)$, where β is the payout rate of company's productive assets $S(t)$, plus the operating income defined by the riskless rate r on $Y(t)$ set as a function of the manager's effort. This is less the portion of the cash flow invested by the company $u_3(t)Y(t)$, less the cost of pecuniary incentive $u_4(t)I(t)$ depending on the manager's quality of effort, less the portion of the company's income paid back as debt repayment $u_1(t)Y(t)$, plus the value derived from the use of debt $u_2u_4\alpha_2D(t)$ depending on the manager's quality of effort and enforced by monitoring $u_2(t) \geq 0$, where $u_3(t)$ is the company's level of investment decision by the manager, and $u_1(t)$ is the portion debt repaid in each period, and α_2 is the cost of debt.

Such that:

$$\dot{Y}(t) = \beta S(t) + (u_4(t) - u_3(t))rY(t) - u_4(t)I(t) - u_1(t)Y(t) + u_2u_4\alpha_2D(t) \quad (4.1)$$

Where $u_4(t) \geq 0$ is the quality of the manager's effort, specified as one of his control variables or strategies. Generally, $u_4'(t) > 0$ and $u_4''(t) \leq 0$. The first condition implies that a higher quality effort from the manager improves the firm's profitability and the second condition reflects the decreasing marginal impact of the manager's effort. This study differs from other works in the capital structure literature, in which the change in cash flow is exogenous and constant, but is consistent with Mu, Wang and Yang (2017), in which the firm's change in cash flow is dependent on the quality of the manager's effort over time. The expected utility for the shareholder will be lowered by their ignorance since they must

consider a costly way to induce the manager to an optimal action. The first-best solution is attainable only if actions are observable, thus the shareholder can lower the manager's incentive pay until their actions are optimal. However, the first-best solution is not always attainable, leading to the need for second or third-best solutions.

II. Tax and debt financing

A simple tax environment was considered. Taxes are payable on the company's profit at the effective tax rate θ . When $\theta > 0$, the company enjoys a tax benefit on its profit. Let α_2 specify the constant cost of debt or coupon payment on debt, and $D(t)$ denote the face value of debt. Price of debt is competitively decided by diversified debtholders. The average change in the debt issue of the case study company over a five-year period was calculated and α was set to this in the analysis, thus $\alpha > 0$ is applicable in the model. Thus, the change in the value of debt $\dot{D}(t)$ is described as the addition of the company's ratio of new debt $\alpha D(t)$ and the cost of debt incurred depending on the manager's effort $u_4(t)\alpha_2 D(t)$, less the portion of debt repaid $u_1(t)Y(t)$ in each period. Therefore, the capital structure is dynamic.

$$\dot{D}(t) = (\alpha + u_4(t)\alpha_2)D(t) - u_1(t)Y(t) \quad (4.2)$$

III. Productive asset

The value of a company in any period is impacted by the value of its productive asset. The value of the company's productive asset is also significant in the mechanism design literature because the manager may have incentive to trade some of the company's assets if such assets have a market value significantly greater than its historical or book value. This boosts the firm's measured net worth or reduces its leverage (Tirole 2006). The value of the company's productive assets $S(t)$ is specified as the value derivable from the company's investment $ru_3(t)Y(t)$ which is the riskless rate r on investment $u_3(t)$ on the company's cash flow $Y(t)$, plus the value derivable from the use of debt depending on the manager's effort without the shareholder's monitoring $(1 - u_2)\alpha_2 u_4 D(t)$ less the depreciation of the assets $\mu S(t)$ where μ represents the rate of asset depreciation

due to production use. The specification of the productive asset implies that the manager has the option to enhance the quality of the firm's productive asset via $u_3(t)$ their quality of investment. Thus, the company's productive asset is impacted by the optimal mechanism design in each period. The specification of the productive asset in this study is closely related to that of Goetz et al. (2019). The company is unable to improve the value of its productive asset sufficiently if the value of its investment does not improve or the manager does not commit productive inputs. Such that:

$$S(t) = ru_3(t)Y(t) + (1 - u_2)\alpha_2 u_4 D(t) - \mu S(t) \quad (4.3)$$

IV. The contract

In contract design, the output processes, $(Y(t), D(t), S(t))$ are publicly observable by major stakeholders, including the shareholder and the manager. However, the manager's choice of effort $u_4(t) \geq 0$ remains unobservable to the shareholder. To encourage the manager to provide costly but optimal efforts, the shareholder uses pecuniary incentives in the contract design to requite the manager's efforts based on observable realised output. In this new model, it was also presumed that the output processes $(Y(t), D(t), S(t))$ are publicly observable. Once the management contract has been signed, the strategic relationship between the shareholder and the manager evolves as a dynamic game that lasts the period of the contract. In addition to the output process, the manager focuses on the value of their pecuniary incentive $I(t)$ which is a function of $u_4(t)$ in addition to the minimum rate χ they earn from the company's profitability. The manager's pecuniary incentive is determined by the company's profitability $Y(t)$ (DeMarzo et al. 2012), thus $I(t)$ should be set as a state variable. In practice, the proportion of management fees is usually set around 2% of the business profit (Mu, Wang & Yang 2017), such that $\chi = 0.02$. Therefore, the manager's remuneration specification or pecuniary incentive is a fixed proportion of the business profit plus a variable depending on the manager's quality of effort, which may be observed via pre-specified indices by the firm, such as construction workers' fatality rates, employee turnover rates, or risk management, among others.

$$\dot{I}(t) = u_4(t)I(t) + \chi Y(t) \quad (4.4)$$

V. Value of company equity

The value of company equity is specified as the cash flow plus the value of a productive asset, less the value of debt. This provides an estimated value of the company in any period t .

$$E(t) = Y(t) + S(t) - D(t) \quad (4.5)$$

From Equations (4.2) and (4.5) above, the trajectory of the optimal capital structure is calculated as:

$$\frac{\dot{D}(t)}{\dot{E}(t)} = \frac{(\alpha + u_4(t)\alpha_2)D(t) - u_1(t)Y(t)}{\beta S(t) + (u_4(t) - u_3(t))rY(t) - u_4(t)I(t) - u_1(t)Y(t) + u_2u_4\alpha_2D(t) + ru_3(t)Y(t) + (1 - u_2)\alpha_2u_4D(t) - \mu S(t) - (\alpha + u_4(t)\alpha_2)D(t) + u_1(t)Y(t)}$$

VI. Contract design constraints

To optimise the utility functions of the shareholder and the manager, it is necessary to provide constraints relevant to the dynamic process of contract design.

The manager will only participate in the management contract if their pecuniary incentive $I(t)$ is at least the value of their reservation utility b , the value obtainable for an outside contract (Thomsen & Conyon 2012). It can be assumed that $b = \chi Y(t)$ The **participatory constraint** is specified as:

$$I(t) \geq \chi Y(t) \quad (4.6)$$

In addition, the manager requires an **incentive-compatible constraint**. The contract is incentive-compatible if the manager earns more when they provide a higher quality of effort and select an optimal capital structure (Scharfstein & Stein 2000; Thomsen & Conyon 2012), such that the pecuniary incentive $I(t)$ evolves with the value of $u_4(t)$. The value of pecuniary incentive when the manager engages a high quality of effort $u_4(t)$ must be greater than when they employ a lower quality of effort, assumed as $1 - u_4(t)$, such that:

$$I(t) \cdot u_4(t) \geq I(t) \cdot (1 - u_4(t))$$

To better represent this in the model, the dynamic motion of pecuniary incentive $I(t)$ is specified as a function of the quality of the manager's effort, $\dot{I}(t) = u_4(t)I(t)$, such that $I(t)$ grows or decline with respect to the value of the $u_4(t)$ in each period t . $I(t)$ increases if the manager inputs a higher quality of effort $u_4(t)$ and vice versa.

Finally, the controls must be non-negative, $u_1(t), u_2(t), u_3(t), u_4(t) \geq 0$ (4.7)

4.3.2 The Dynamic Game Model and the Optimisation Problem

The dynamic and strategic relationship between the shareholder and the manager after the contract is signed represents the problem as a dynamic game problem, played simultaneously. It is assumed that the flow of profit is discounted at the rate ρ .

The formulation of the shareholder's problem: The shareholder delegates the management of the business to the manager, with a certain proportion of the firm's finance already financed with debt $D(0)$. Depending on the quality of effort $u_4(t)$ committed by the manager, the shareholder selects a portion of debt in each period to be repaid $u_1(t)$ to the debtholder to induce the manager in their quality of effort. A greater level of debt repayment reduces the available cash flow to the manager. Leverage keeps the manager in a position to make a profit for the company consistently, so as to assure the repayment of debts, and restrain the manager from diverting resources (Stulz 1990). Therefore, the shareholder's objective is to maximise the firm's value $E(t)$, by selecting $u_1(t)$, the optimal monitoring $u_2(t)$ and optimal capital structure $\frac{D(t)}{E(t)}$ required to maximise the manager's effort in the firm.

The shareholder's objective functional is specified as:

Shareholder: J_1

$$= \max \int_0^T e^{-\rho t} (1 - \theta) (Y(t) + u_1(t)^2 - I(t) - u_2(t)^2) dt \quad (4.8)$$

Where $(1 - \theta)(Y(t) + u_1(t)^2 - I(t) - u_2(t)^2)$ is the net cash flow after incentive cost, monitoring and tax accruing.

The formulation of the manager's problem: Considering that the manager like the shareholder is a utility maximiser, Jensen and Meckling (1976) established that a manager will constantly select actions that will lower the value of the firm compared to what it would have been if the firm belonged solely to the manager, irrespective of whether the firm operated as a monopoly or otherwise. For a given strategy chosen by the shareholder, the manager's objective is to select the optimal quality of effort $u_4(t)$ and optimal investment policy $u_3(t)$ required to maximise the profit from the contract $I(t)$. $u_3(t)$ describes the improvement of the company's asset quality by its investment. These two choices of strategy are consistent with Goetz et al. (2019) and Morellec (2004). The manager's objective functional is specified as follows:

$$\text{Manager: } J_2 = \max \int_0^T e^{-\rho t} (ru_3(t)^2 + I(t) - u_4(t)^2) dt \quad (4.9)$$

The parameters used in the model are presented in Table 4.1 below.

Table 4.1 Parameters used in the model

S/N	Parameters	Definition
1	r	Riskless rate $r > 0$, assumed to be constant
2	b	Manager's reservation utility
3	β	Payout rate of company's productive assets, assumed to be constant
4	$E(t)$	Company equity
5	$Y(t)$	Cash flow
6	$I(t)$	Cost of pecuniary incentive
7	$S(t)$	Value of company's productive assets
8	$D(t)$	Company debt
9	θ	Effective tax rate
10	α	Ratio of new debt to existing company debt
11	α_2	Cost of debt

12	$u_1(t)$	Debt repayment
13	$u_2(t)$	Monitoring cost
14	$u_3(t)$	Company investment in each period
15	$u_4(t)$	Manager's effort
17	μ	Rate of asset depreciation
18	χ	Proportion of management fee
19	Superscript *	Optimal result

4.3.3 Discussion of the model

The model specified above is a differential game theory model, based on a principal-agent problem, for designing corporate governance mechanisms to give an optimal capital structure specification. The shareholder's objective functional specified in Equation (4.8) represents the shareholder's wealth maximisation goal, their payoff in the game. Equation (4.8) maximises the net cash flow of the company, after the monitoring cost, pecuniary incentive cost and tax have been deducted. Debt, in addition to monitoring cost, and pecuniary incentive are corporate governance mechanisms used in the model. Due to the conflicts of interest between the rational shareholder and the rational manager, as described in an agency framework that specifies the game as a non-cooperative game, the manager prefers a conflicting payoff. The payoff of the manager in Equation (4.9) is to maximise the pecuniary incentives $I(t)$. To achieve this, the manager's strategy is to select the choice of company investment $u_3(t)$ and their quality of effort $u_4(t)$. The objective functionals are governed by state equations and constraints (4.1) – (4.7). In each period $t = (1, 2, \dots, T)$ the shareholder selects the optimal debt repayment strategy and optimal monitoring cost, $u_1(t), u_2(t)$. Selecting the optimal mechanisms will reduce the agency problem by minimising the opportunistic tendencies of the manager as they reduce the resources available to the manager without excessive negative impact on the company's cash flow. The model stated below therefore represents the interests of the shareholder specified as the objective functional J_1 and the interests of the manager specified as J_2 . The model maximises the after-tax wealth of the shareholder and the company by selecting the optimal debt-equity ratio following $\frac{\dot{D}(t)}{\dot{E}(t)}$ which provides an optimal mechanism design for the company. The optimal mechanisms used in the game, in addition to the optimal state variables, produces the optimal debt-equity ratio, resulting in the optimal capital structure for the firm. Since the model is a dynamic game model, the model will provide outcomes of the strategic interactions between the two players, such as trigger strategies required to ensure the optimal contract is not renegotiated. Moreover, these relevant dynamic issues relate to the dynamic aspects of agency problems over time.

In summary, the model is, set out as follows.

Shareholder-manager game model:

$$\text{Shareholder: } J_1 = \max \int_0^T e^{-\rho t} (1 - \theta) (Y(t) + u_1(t)^2 - I(t) - u_2(t)^2)$$

Subject to:

$$\dot{Y}(t) = \beta S(t) + (u_4(t) - u_3(t))rY(t) - u_4(t)I(t) - u_1(t)Y(t) + u_2u_4\alpha_2D(t), Y(0) = Y_0; I(0) = I_0$$

$$\dot{D}(t) = (\alpha + u_4(t)\alpha_2)D(t) - u_1(t)Y(t), D(0) = D_0$$

$$\dot{I}(t) = u_4(t)I(t) + \chi Y(t), Y(0) = Y_0; I(0) = I_0$$

$$\dot{S}(t) = ru_3(t)Y(t) + (1 - u_2)\alpha_2u_4(t)D(t) - \mu S(t), S(0) = S_0$$

$$E(t) = Y(t) + S(t) - D(t)$$

$$\frac{\dot{D}(t)}{\dot{E}(t)} = \frac{(\alpha + u_4(t)\alpha_2)D(t) - u_1(t)Y(t)}{\beta S(t) + (u_4(t) - u_3(t))rY(t) - u_4(t)I(t) - u_1(t)Y(t) + u_2u_4\alpha_2D(t) + ru_3(t)Y(t) + (1 - u_2)\alpha_2u_4D(t) - \mu S(t) - (\alpha + u_4(t)\alpha_2)D(t) + u_1(t)Y(t)}$$

$$I(t) \geq \chi Y(t)$$

$$u_1(t), u_2(t), u_3(t), u_4(t) \geq 0$$

$$\text{Manager: } J_2 = \max \int_0^T e^{-\rho t} (ru_3(t)^2 + I(t) - u_4(t)^2) dt$$

The dynamic game is summarised in Table 4.2, as follows.

Table 4.2 A summary of the game

	Description and features of the game	
Game	Two-person non-zero-sum game	
Players	Player 1: Shareholder (Principal) Player 2: Manager (Agent)	
Period of the game	The game is played over time, and a single period in the over time horizon is reckoned as a year, that is $t = 1$. This is because significant decisions, such as issuance of new debts, rarely happen in shorter periods (e.g., monthly). The final time T is taken as $T = 10$.	
Strategies	Player 1: Debt repayment and monitoring Player 2: New investment and quality of effort	
Actions	Player 1 - Shareholder (Principal)	Decrease or Increase incentive $I(t)$
	Player 2 - Manager (Agent)	Commit appropriate or inappropriate effort $u_4(t)$
Payoffs	Player 1: $J_{shareholder} = \max \int_0^T e^{-\rho t} (1 - \theta) (Y(t) + u_1(t)^2 - I(t) - u_2(t)^2) dt$	
	Player 2: $J_{manager} = \max \int_0^T e^{-\rho t} (ru_3(t)^2 + I(t) - u_4(t)^2) dt$	
Information available in the game	The game is said to be one with perfect information because the actions of the manager are somewhat observed through the output processes or state variables of the game. On the other hand, the game is said to be one with incomplete information because the preferences or actions of one player are unobservable to the other player.	

The dynamic game between the shareholder and the manager was specified as a differential game. Following this, the game model was analysed and solved via adequate equilibrium concepts, first as a non-cooperative game using the Nash open-loop and Nash feedback; and second, as a cooperative game using the Pareto solution concepts to obtain the equilibrium strategies, optimal state outcomes and the optimal capital structure for the company.

4.4 Model Solutions

4.4.1 Non-cooperative Game Analysis - Nash Equilibrium

4.4.1.1 *Optimal decisions with open-loop Nash equilibrium concept*

The conflict of interest between the shareholder and the manager, as described above, is specified as a non-cooperative game in which each player has a different utility function described in an objective functional. The manager does not commit to a pre-specified management contract but displays opportunistic tendencies to divert the company's resources and select a suboptimal capital structure. An adequate equilibrium concept to analyse the non-cooperative game is the Nash solution. This concept implies that a player cannot improve their payoff if they adopt a different strategy to the optimal strategy, when the other player sticks to their optimal strategy (Cesar 1994). There are two forms of information structure in the Nash solution concepts. The first is known as the open-loop Nash equilibrium solution concept and the second is known as the feedback Nash equilibrium solution concept discusses in the next subsection. The open-loop Nash equilibrium implies that the shareholder and manager ignore the current information available in the game, such as the change in management fee, and other matters.

The model was first solved using the open-loop Nash equilibrium solution concept. The open-loop Nash solution assumes that the only available information for action at time t is that of the initial states $Y(0)$, $D(0)$, $I(0)$, $S(0)$. The players in the game sought to optimise their interests at time 0 and do not revise their strategies afterwards by the revelation in the game. The information scheme does not give the players knowledge about the changes in state

variables, this is known as pre-commitment (Bressan 2011). In this model, this means that the shareholder and manager do not revise their optimal strategies following the updated value of cash flow, debt value, remuneration packages or productive assets, but rather select strategies regardless of the company's outcomes. The open-loop information system is illustrated in the figure below.

Open-loop System of the Game

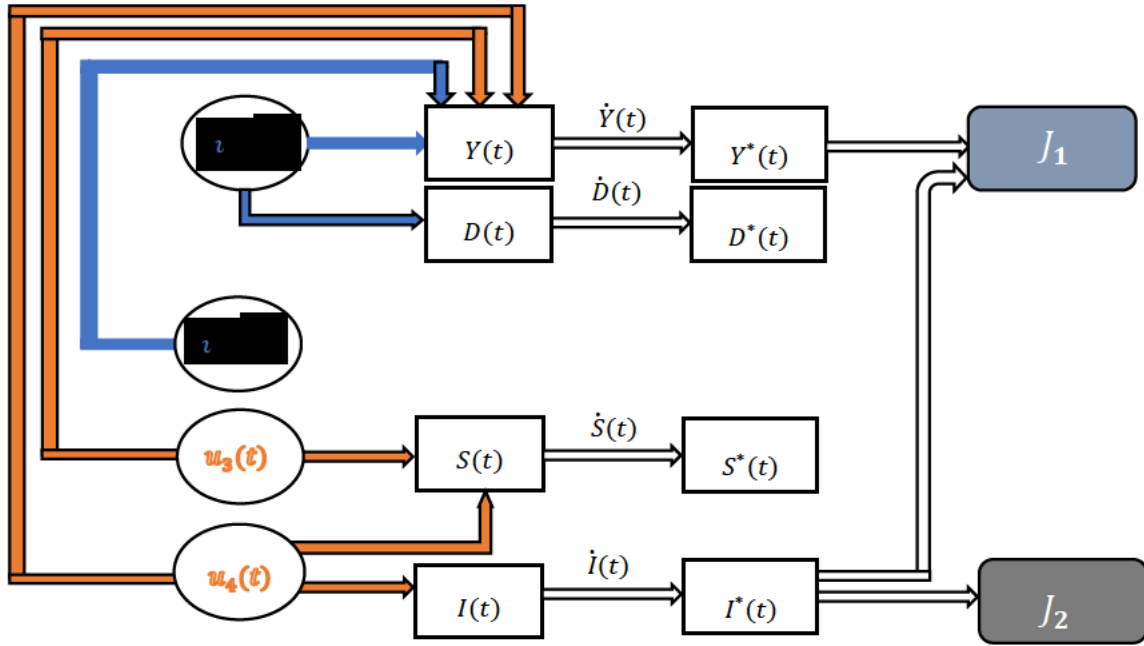


Figure 4.1 Open-loop system of the game

As shown in Figure 4.1, the players' strategies $u_1(t)$, $u_2(t)$, $u_3(t)$, $u_4(t)$ are specified as inputs into the states of the game $Y(t)$, $D(t)$, $I(t)$, $S(t)$. The systems of the game, $\dot{Y}(t)$, $\dot{D}(t)$, $\dot{I}(t)$, $\dot{S}(t)$ react to the information from the strategies. They produce equilibrium state trajectories $Y^*(t)$, $D^*(t)$, $I^*(t)$, $S^*(t)$ at the Nash pair of strategies for which a player cannot improve their outcome (J_1 , J_2) if the player moves from this strategy while the other player sticks to theirs.

The necessary conditions developed by Pontryagin and his co-workers in Boltyanskii, Gamkrelidze and Pontryagin (1956) were derived by generating the Hamiltonian obtained by adjoining the state equations to the objective functional for each player with adjoint or co-state functions, $\lambda_j, j = 1, 2, 3, 4$ for player 1 (the

shareholder) and $\phi_j, j = 1, 2, 3, 4$ for player 2 (the manager), where j represents the number of state variables. Therefore, Hamiltonians for players 1 and 2 are defined as:

$$\begin{aligned}\mathcal{H}_1(Y, D, I, S, \lambda_j, u_1, u_2, u_3, u_4, t) \\ = f_i(Y, I, u_1, u_2, u_3, u_4, t) + \lambda_j g(Y, D, I, S, u_1, u_2, u_3, u_4, t)\end{aligned}$$

$$\begin{aligned}\mathcal{H}_2(Y, D, I, S, \phi_j, u_1, u_2, u_3, u_4, t) \\ = f_i(I, u_1, u_2, u_3, u_4, t) + \phi_j g(Y, D, I, S, u_1, u_2, u_3, u_4, t)\end{aligned}$$

This method is known as an indirect method of solving two-point boundary value problems, which are known to have higher precision than direct methods. This is because the solutions satisfy the required necessary conditions for optimality (Nikooeinejad, Delavarkhalafi & Heydari 2016).

For the purpose of brevity, the subscript t is taken off the vectors.

$$\begin{aligned}H_1 = (1 - \theta)(Y + u_1^2 - I - u_2^2) + \lambda_1(\beta S + (u_4 - u_3)rY - u_4I - u_1Y + \\ u_2u_4\alpha_2D) + \lambda_2((\alpha + u_4\alpha_2)D - u_1Y) + \lambda_3(u_4I + \chi Y) + \lambda_4(ru_3Y + (1 - \\ u_2)\alpha_2u_4D - \mu S)\end{aligned}\tag{4.10}$$

$$\begin{aligned}H_2 = ru_3^2 + I(t) - u_4(t)^2 + \phi_1(\beta S + (u_4 - u_3)rY - u_4I - u_1Y + u_2u_4\alpha_2D) + \\ \phi_2((\alpha + u_4\alpha_2)D(t) - u_1Y) + \phi_3(u_4I + \chi Y) + \phi_4(ru_3Y + (1 - u_2)\alpha_2u_4D - \mu S)\end{aligned}\tag{4.11}$$

Theorem 4.1.

The open-loop Nash equilibrium solutions are:

$$u_1^* = \frac{Y}{2(1-\theta)} (\lambda_1 + \lambda_2)\tag{4.12}$$

This calculation means that the amount of debt repayable by the company will increase with a decrease in the effective tax rate and an increase in the company's cash flow. In other words, with a greater value of cash flow, the company is likely to repay a greater value of debt.

$$u_2^* = \frac{\alpha_2 u_4 D (\lambda_1 - \lambda_4)}{2(1-\theta)} \quad (4.13)$$

This calculation means that the level of monitoring adopted by the shareholder will increase with greater use of debt, depending on the difference in the marginal rate of change of cash flow and productive assets. This implies that since a lower effective tax rate may increase the use of debt, this will heighten the monitoring adopted by the shareholder, provided that the marginal rate of change of cash flow is higher than the productive assets. In addition, with an increase in the cost of the debt associated with the use of debt, the shareholder is likely to require a higher monitoring level to enforce debt repayment.

$$u_3^* = \frac{Y}{2} (\phi_1 - \phi_4) \quad (4.14)$$

This calculation means that the company's level of investment is a function of the current cash flow of the company and the productive asset. This means that the higher the difference in the marginal rate of change of cash flow and productive assets, the higher the company's value of an investment. Thus, from this formula, it is required that the rate of change of cash flow is constantly higher than the productive assets to improve the company's investment.

$$u_4^* = \frac{1}{2} (\phi_1 (\alpha_2 u_2 D + rY - I) + \phi_2 \alpha_2 D + \phi_3 I + \phi_4 (1 - u_2) \alpha_2 D) \quad (4.15)$$

This calculation implies that the optimal quality of effort required is mainly determined by the marginal rate of change of debt, cash flow and the manager's pecuniary incentive. The manager responds positively to a higher pecuniary incentive and supplies more effort when the company uses more debt with a greater level of monitoring.

Proof of Theorem 4.1.

The set of necessary conditions makes it possible to identify the equilibrium time path for the variables, which has implications for the ideal financial management policies. The **first** part of the principle states that each control variable selected at any moment in time must have an effect that maximises or minimises the Hamiltonian. This implies:

$$\frac{\partial H_i(t)}{\partial u_{1...4}(t)} = 0 \quad \text{for all } i = 1, 2 \text{ and all } t$$

$$\frac{\partial H_1}{\partial u_1} = 2u_1(1 - \theta) - (\lambda_1 + \lambda_2)Y = 0$$

$$u_1^* = \frac{Y}{2(1-\theta)} (\lambda_1 + \lambda_2)$$

$$\frac{\partial H_1}{\partial u_2} = -2u_2(1 - \theta) + (\lambda_1 - \lambda_4)\alpha_2 u_4 D = 0$$

$$u_2^* = \frac{\alpha_2 u_4 D (\lambda_1 - \lambda_4)}{2(1-\theta)}$$

$$\frac{\partial H_2}{\partial u_3} = 2ru_3 + (\phi_4 - \phi_1)rY = 0$$

$$u_3^* = \frac{Y}{2} (\phi_1 - \phi_4)$$

$$\frac{\partial H_2}{\partial u_4} = -2u_4 + \phi_1(\alpha_2 u_2 D + rY - I) + \phi_2 \alpha_2 D + \phi_3 I + \phi_4(1 - u_2)\alpha_2 D = 0$$

$$u_4^* = \frac{1}{2} (\phi_1(\alpha_2 u_2 D + rY - I) + \phi_2 \alpha_2 D + \phi_3 I + \phi_4(1 - u_2)\alpha_2 D)$$

End of Proof.

Equations (4.12) - (4.15) are the characterisations of the Open-loop Nash strategies.

The **second** condition necessitates the rate of change with respect to time of each co-state variable to be equivalent to the negative of the partial derivative of the Hamiltonian with respect to the associated state variable. The adjoint variable can be defined as a marginal change associated with an increment in each state variable for each player. In other words, if one dollar of cash flow is added to the company's finance portfolio, this corresponds to the effect on the optimal wealth accruable to the shareholder or the manager.

The adjoint equations determine the values of the optimal strategies adopted by the shareholder or the manager in each period. From the Hamiltonian functions (4.10) and (4.11), the adjoint equations are:

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H_1}{\partial Y} = \lambda_1(\rho + (u_3 - u_4)r + u_1) - (1 - \theta) + \lambda_2 u_1 - \lambda_3 \chi - \lambda_4 r u_3 \quad (4.16)$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H_1}{\partial D} = \lambda_2(\rho - \alpha - u_4 \alpha_2) - \lambda_1 u_2 u_4 \alpha_2 - \lambda_4 (1 - u_2) \alpha_2 u_4 \quad (4.17)$$

$$\dot{\lambda}_3 = \rho\lambda_3 - \frac{\partial H_1}{\partial I} = \lambda_3(\rho - u_4) + \lambda_1 u_4 + (1 - \theta) \quad (4.18)$$

$$\dot{\lambda}_4 = \rho\lambda_4 - \frac{\partial H_1}{\partial S} = \lambda_4(\rho + \mu) - \lambda_1 \beta \quad (4.19)$$

$$\dot{\phi}_1 = \rho\phi_1 - \frac{\partial H_2}{\partial Y} = \phi_1(\rho + (u_3 - u_4)r + u_1) + \phi_1 u_2 - \phi_3 \chi - \phi_4 r u_3 \quad (4.20)$$

$$\dot{\phi}_2 = \rho\phi_2 - \frac{\partial H_2}{\partial D} = \phi_2(\rho - \alpha - u_4 \alpha_2) - \phi_1 u_2 u_4 \alpha_2 - \phi_4 (1 - u_2) \alpha_2 u_4 \quad (4.21)$$

$$\dot{\phi}_3 = \rho\phi_3 - \frac{\partial H_2}{\partial I} = \phi_3(\rho - u_4) + u_4 \phi_1 - 1 \quad (4.22)$$

$$\dot{\phi}_4 = \rho\phi_4 - \frac{\partial H_2}{\partial S} = \phi_4(\rho + \mu) - \phi_1 \beta \quad (4.23)$$

Also, these co-states functions satisfy the terminal conditions:

$$\begin{aligned} \lambda_1(T) = 0 \quad \lambda_2(T) = 0 \quad \lambda_3(T) = 0 \quad \lambda_4(T) = 0 \quad \phi_1(T) = 0 \quad \phi_2(T) = 0 \\ \phi_3(T) = 0 \quad \phi_4(T) = 0 \end{aligned}$$

The terminal conditions for the adjoint variables can be logically deduced from the structure of the problem. For example, the present value of a dollar earned in the infinite future is zero (Elton & Gruber 1974).

The **third** condition requires the state equations to be achieved:

$$\dot{Y}(t) = \beta S + (u_4 - u_3)rY - u_4I - u_1Y + u_2u_4\alpha_2D, Y(0) = Y_0; I(0) = I_0 \quad (4.24)$$

$$\dot{D}(t) = (\alpha + u_4\alpha_2)D - u_1(t)Y, D(0) = D_0 \quad (4.25)$$

$$\dot{I} = u_4I + \chi Y, Y(0) = Y_0; I(0) = I_0 \quad (4.26)$$

$$\dot{S} = ru_3Y + (1 - u_2)\alpha_2u_4D - \mu S; S(0) = S_0 \quad (4.27)$$

Equations (4.12 - 4.27) above are identified as the optimality system. The optimality system, which generates the equilibrium outcomes, is a forward-backward system of differential equations stated as follows:

$$\dot{Y}(t) = \beta S + (u_4 - u_3)rY - u_4I - u_1Y + u_2u_4\alpha_2D, Y(0) = Y_0; I(0) = I_0 \quad (4.28)$$

$$\dot{D}(t) = (\alpha + u_4\alpha_2)D - u_1(t)Y, D(0) = D_0 \quad (4.29)$$

$$\dot{I} = u_4I + \chi Y, Y(0) = Y_0; I(0) = I_0 \quad (4.30)$$

$$\dot{S} = ru_3Y + (1 - u_2)\alpha_2u_4D - \mu S; S(0) = S_0 \quad (4.31)$$

$$\begin{aligned} \dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H_1}{\partial Y} = \lambda_1(\rho + (u_3 - u_4)r + u_1) - (1 - \theta) + \lambda_2u_1 - \lambda_3\chi - \\ \lambda_4ru_3 \end{aligned} \quad (4.32)$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H_1}{\partial D} = \lambda_2(\rho - \alpha - u_4\alpha_2) - \lambda_1u_2u_4\alpha_2 - \lambda_4(1 - u_2)\alpha_2u_4 \quad (4.33)$$

$$\dot{\lambda}_3 = \rho\lambda_3 - \frac{\partial H_1}{\partial I} = \lambda_3(\rho - u_4) + \lambda_1u_4 + (1 - \theta) \quad (4.34)$$

$$\dot{\lambda}_4 = \rho\lambda_4 - \frac{\partial H_1}{\partial S} = \lambda_4(\rho + \mu) - \lambda_1\beta \quad (4.35)$$

$$\begin{aligned} \dot{\phi}_1 = \rho\phi_1 - \frac{\partial H_2}{\partial Y} = \phi_1(\rho + (u_3 - u_4)r + u_1) + \phi_1u_2 - \phi_3\chi - \phi_4ru_3 \\ \end{aligned} \quad (4.36)$$

$$\dot{\phi}_2 = \rho\phi_2 - \frac{\partial H_2}{\partial D} = \phi_2(\rho - \alpha - u_4\alpha_2) - \phi_1u_2u_4\alpha_2 - \phi_4(1 - u_2)\alpha_2u_4 \quad (4.37)$$

$$\dot{\phi}_3 = \rho\phi_3 - \frac{\partial H_2}{\partial I} = \phi_3(\rho - u_4) + u_4\phi_1 - 1 \quad (4.38)$$

$$\dot{\phi}_4 = \rho\phi_4 - \frac{\partial H_2}{\partial S} = \phi_4(\rho + \mu) - \phi_1\beta \quad (4.39)$$

$$\begin{aligned} \lambda_1(T) = 0 \quad \lambda_2(T) = 0 \quad \lambda_3(T) = 0 \quad \lambda_4(T) = 0 \quad \phi_1(T) = 0 \quad \phi_2(T) = 0 \\ \phi_3(T) = 0 \quad \phi_4(T) = 0 \end{aligned}$$

with Nash equilibrium strategies:

$$u_1^* = \frac{Y}{2(1-\theta)} (\lambda_1 + \lambda_2) \quad (4.40)$$

$$u_2^* = \frac{\alpha_2 u_4 D (\lambda_1 - \lambda_4)}{2(1-\theta)} \quad (4.41)$$

$$u_3^* = \frac{Y}{2} (\phi_1 - \phi_4) \quad (4.42)$$

$$u_4^* = \frac{1}{2} (\phi_1(\alpha_2 u_2 D + rY - I) + \phi_2 \alpha_2 D + \phi_3 I + \phi_4(1 - u_2)\alpha_2 D) \quad (4.43)$$

4.4.1.2 Optimal decisions with feedback Nash equilibrium concept

In the feedback Nash solution concept, the players' strategies are a function of the current value of the state variables (Cesar 1994). This means that the shareholder and the manager adapt their strategies according to changes in the state variables, thus relying on the current state of the game to make informed decisions. This eliminates the problem of information non-uniqueness from the equilibria (Yeung & Petrosjan 2006), and ensures that the equilibria are time consistent (Mehlmann 1988). Figure 4.2 illustrates the feedback system of the game.

Feedback System of the Game

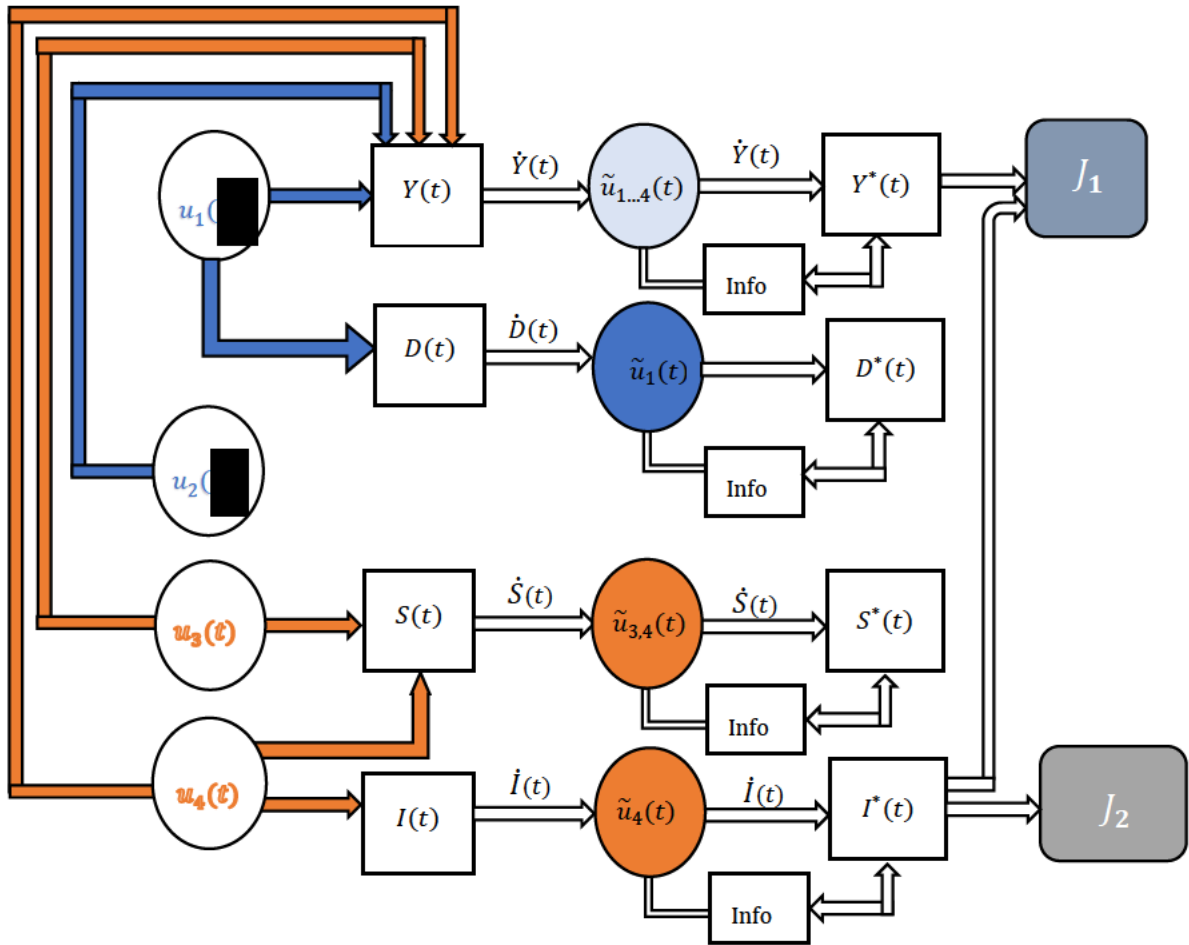


Figure 4.2 Feedback system of the game

In the open-loop outcome of the game, the shareholder and manager pre-commits to the game, such that they act upon the initial strategies, $u_1(t) \dots u_4(t)$. In the feedback system of the game, players continuously revise their optimal strategies as a function of the updated states of the game. This means that the updated optimal strategies $\tilde{u}_1(t) \dots \tilde{u}_4(t)$ are functions of the current information in the game from $Y(t), D(t), I(t), S(t)$ at any time t . This feedback information system is reflected in the solution method as a cross-derivative of the Nash strategy of one player in the Hamiltonian of the other.

The set of necessary conditions to be satisfied in the feedback Nash equilibrium case is similar to those of the open-loop Nash equilibrium case.

The Hamiltonians are presented as before:

$$H_1 = (1 - \theta)(Y + u_1^2 - I - u_2^2) + \lambda_1(\beta S + (u_4 - u_3)rY - u_4I - u_1Y + u_2u_4\alpha_2D) + \lambda_2((\alpha + u_4\alpha_2)D - u_1Y) + \lambda_3(u_4I + \chi Y) + \lambda_4(ru_3Y + (1 - u_2)\alpha_2u_4D - \mu S)$$

$$H_2 = ru_3^2 + I(t) - u_4(t)^2 + \phi_1(\beta S + (u_4 - u_3)rY - u_4I - u_1Y + u_2u_4\alpha_2D) + \phi_2((\alpha + u_4\alpha_2)D(t) - u_1Y) + \phi_3(u_4I + \chi Y) + \phi_4(ru_3Y + (1 - u_2)\alpha_2u_4D - \mu S)$$

The derivations for the feedback Nash equilibrium solution are similar to Theorem 4.1 specified for the open-loop case:

$$\frac{\partial H_1}{\partial u_1} = 2u_1(1 - \theta) - Y(\lambda_1 + \lambda_2) = 0$$

$$u_1^* = \frac{Y}{2(1-\theta)} (\lambda_1 + \lambda_2)$$

$$\frac{\partial H_1}{\partial u_2} = -2u_2(1 - \theta) + (\lambda_1 - \lambda_4)\alpha_2u_4D = 0$$

$$u_2^* = \frac{\alpha_2u_4D}{2(1-\theta)} (\lambda_1 - \lambda_4)$$

$$\frac{\partial H_2}{\partial u_3} = 2ru_3 + (\phi_4 - \phi_1)rY = 0$$

$$u_3^* = \frac{Y}{2} (\phi_1 - \phi_4)$$

$$\frac{\partial H_2}{\partial u_4} = -2u_4 + \phi_1(\alpha_2u_2D + rY - I) + \phi_2\alpha_2D + \phi_3I + \phi_4(1 - u_2)\alpha_2D = 0$$

$$u_4^* = \frac{1}{2} (\phi_1(\alpha_2u_2D + rY - I) + \phi_2\alpha_2D + \phi_3I + \phi_4(1 - u_2)\alpha_2D)$$

While the optimal strategies for the feedback case have the same necessary conditions as the open-loop case, the adjoint equations reflect the updated information available in the game. The adjoint equations include a cross-

derivative of the change in the Hamiltonian with respect to the optimal strategy, multiplied by the partial derivative of the optimal strategy with respect to the current state value. This, therefore, updates the Nash pair of strategies of both players, as necessary.

From the Hamiltonian functions (4.10) and (4.11), the adjoint equations are:

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H_1}{\partial Y} - \frac{\partial H_1}{\partial u_1} \frac{\partial u_1^*}{\partial Y} - \frac{\partial H_1}{\partial u_2} \frac{\partial u_2^*}{\partial Y}$$

$$\therefore \dot{\lambda}_1 = \lambda_1(\rho + u_1 + (u_3 - u_4)r) - (1 - \theta) + \lambda_2 u_1 - \lambda_3 \chi - \lambda_4 r u_3 - \frac{(\lambda_1 + \lambda_2)}{2(1 - \theta)} (2u_1(1 - \theta) - Y(\lambda_1 + \lambda_2)) \quad (4.44)$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H_1}{\partial D} - \frac{\partial H_1}{\partial u_1} \frac{\partial u_1^*}{\partial D} - \frac{\partial H_1}{\partial u_2} \frac{\partial u_2^*}{\partial D}$$

$$\therefore \dot{\lambda}_2 = \lambda_2(\rho - \alpha - u_4\alpha_2) - \lambda_1 u_2 u_4 \alpha_2 - \lambda_4(1 - u_2)\alpha_2 u_4 - \frac{u_4\alpha_2(\lambda_1 - \lambda_4)}{2(1 - \theta)} ((\lambda_1 - \lambda_4)\alpha_2 u_4 D - 2u_2(1 - \theta)) \quad (4.45)$$

$$\dot{\lambda}_3 = \rho\lambda_3 - \frac{\partial H_1}{\partial I} - \frac{\partial H_1}{\partial u_1} \frac{\partial u_1^*}{\partial I} - \frac{\partial H_1}{\partial u_2} \frac{\partial u_2^*}{\partial I}$$

$$\therefore \dot{\lambda}_3 = \lambda_3(\rho - u_4) + \lambda_1 u_4 + (1 - \theta) \quad (4.46)$$

$$\dot{\lambda}_4 = \rho\lambda_4 - \frac{\partial H_1}{\partial S} - \frac{\partial H_1}{\partial u_1} \frac{\partial u_1^*}{\partial S} - \frac{\partial H_1}{\partial u_2} \frac{\partial u_2^*}{\partial S}$$

$$\therefore \dot{\lambda}_4 = \lambda_4(\rho + \mu) - \lambda_1 \beta \quad (4.47)$$

$$\dot{\phi}_1 = \rho\phi_1 - \frac{\partial H_2}{\partial Y} - \frac{\partial H_2}{\partial u_3} \frac{\partial u_3^*}{\partial Y} - \frac{\partial H_2}{\partial u_4} \frac{\partial u_4^*}{\partial Y}$$

$$\therefore \dot{\phi}_1 = \phi_1(\rho + (u_3 - u_4)r + u_1) + \phi_1 u_2 - \phi_3 \chi - \phi_4 r u_3 - \frac{(\phi_1 - \phi_4)}{2} (2r u_3 + (\phi_4 - \phi_1)rY) - \frac{\phi_1 r}{2} (\phi_1(\alpha_2 u_2 D + rY - I) + \phi_2 \alpha_2 D + \phi_3 I + \phi_4(1 - u_2)\alpha_2 D - 2u_4) \quad (4.48)$$

$$\dot{\phi}_2 = \rho\phi_2 - \frac{\partial H_2}{\partial D} - \frac{\partial H_2}{\partial u_3} \frac{\partial u_3^*}{\partial D} - \frac{\partial H_2}{\partial u_4} \frac{\partial u_4^*}{\partial D}$$

$$\begin{aligned} \therefore \dot{\phi}_2 &= \phi_2(\rho - \alpha - u_4\alpha_2) - \phi_1 u_2 u_4 \alpha_2 - \phi_4(1 - u_2)\alpha_2 u_4 - \frac{1}{2}(\phi_1 \alpha_2 u_2 + \phi_2 \alpha_2 + \\ &\phi_4(1 - u_2)\alpha_2) \cdot (\phi_1(\alpha_2 u_2 D + rY - I) + \phi_2 \alpha_2 D + \phi_3 I + \phi_4(1 - u_2)\alpha_2 D - 2u_4) \end{aligned} \quad (4.49)$$

$$\dot{\phi}_3 = \rho\phi_3 - \frac{\partial H_2}{\partial I} - \frac{\partial H_2}{\partial u_3} \frac{\partial u_3^*}{\partial I} - \frac{\partial H_2}{\partial u_4} \frac{\partial u_4^*}{\partial I}$$

$$\begin{aligned} \therefore \dot{\phi}_3 &= \phi_3(\rho - u_4) + u_4\phi_1 - 1 - \frac{(\phi_3 - \phi_1)}{2}(\phi_1(\alpha_2 u_2 D + rY - I) + \phi_2 \alpha_2 D + \phi_3 I + \\ &\phi_4(1 - u_2)\alpha_2 D - 2u_4) \end{aligned} \quad (4.50)$$

$$\dot{\phi}_4 = \rho\phi_4 - \frac{\partial H_2}{\partial S} - \frac{\partial H_2}{\partial u_3} \frac{\partial u_3^*}{\partial S} - \frac{\partial H_2}{\partial u_4} \frac{\partial u_4^*}{\partial S} = \phi_4(\rho + \mu) - \phi_1\beta \quad (4.51)$$

The calculations in Equations (4.44) – (4.51) imply that the marginal rate of change of the state variables that specify the feedback Nash equilibrium solutions include an updated strategy of each player that has a maximising effect on the Hamiltonian with respect to the current states of the game.

The third condition remains that the state equations are achieved as follows:

$$\dot{Y}(t) = \beta S + (u_4 - u_3)rY - u_4 I - u_1 Y + u_2 u_4 \alpha_2 D, Y(0) = Y_0; I(0) = I_0$$

$$\dot{D}(t) = (\alpha + u_4 \alpha_2)D - u_1(t)Y, D(0) = D_0$$

$$\dot{I} = u_4 I + \chi Y, Y(0) = Y_0; I(0) = I_0$$

$$\dot{S} = r u_3 Y + (1 - u_2)\alpha_2 u_4 D - \mu S; S(0) = S_0$$

Also, the co-states functions satisfy the terminal conditions:

$$\begin{aligned} \lambda_1(T) &= 0 & \lambda_2(T) &= 0 & \lambda_3(T) &= 0 & \lambda_4(T) &= 0 & \phi_1(T) &= 0 & \phi_2(T) &= 0 \\ \phi_3(T) &= 0 & \phi_4(T) &= \end{aligned}$$

In general, the feedback Nash equilibrium solution concept is difficult to solve analytically, but they can be simulated using some form of dynamic programming (Cesar 1994).

4.4.2 Cooperative Game Analysis – Pareto Outcome

4.4.2.1 *Optimal decisions with a Pareto-efficient outcome*

The non-cooperative game analysis provided above via Nash equilibrium elucidates the incongruity of interests between the shareholder and the manager. Consequently, this does not fully resolve the agency problem but provides optimal strategies to minimise the marginal agency problem. Congruity of interests between the shareholder and the manager may be established as a corporate governance mechanism in the selection of an optimal capital structure via a Pareto-efficient outcome. The Pareto solution concept, also represented as the cooperative form of the game, jointly optimises all players' utility functions over the time interval. It is therefore presumed that the equilibrium of a cooperative game will be Pareto optimal. This implies that it is impossible to allocate resources in a way that makes a player better off without leaving the other player at least worse off (Yeung & Petrosjan 2006). Although the plausibility of cooperation in a typical non-cooperative game may be argued, due to the difficulty of ensuring congruity, it may be otherwise argued by the so-called Coase Theorem. This states, in part, that when one player is affected by the externality of the other player's actions, both players (if rational) will transact to reach a Pareto optimal solution (Coase 1960). That is, if a rational shareholder observes the acute effect of the manager's opportunistic actions on the company's performance, they will readily negotiate on a Pareto optimal outcome.

Pareto System of the Game

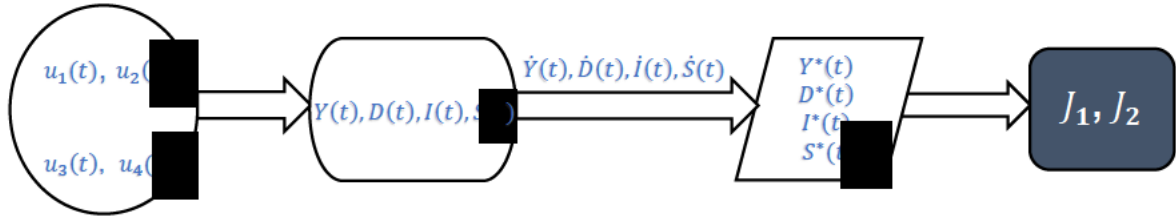


Figure 4.3 Pareto system of the game

Figure 4.3 illustrates the Pareto system of the game. The game is represented as a multi-objective optimal control problem in which both players agree to optimise their objectives with respect to a weight assignment as a corporate governance mechanism. The shareholder's strategies $u_1(t)$, $u_2(t)$ and the manager's strategies $u_3(t)$, $u_4(t)$ jointly influence the states of the game $Y(t), D(t), I(t), S(t)$. These are processed by the systems of the game $\dot{Y}(t), \dot{D}(t), \dot{I}(t), \dot{S}(t)$, and result in optimal state trajectories at the optimal strategies. The optimal states and strategies are then implemented in each players' utility function to derive a Pareto frontier for the players' payoffs.

In the solution concept, the interests of both players are prioritised with respect to the assigned φ .

Such that:

$$\varphi J_1 + (1 - \varphi) J_2$$

However, a controversial question in most multi-objective literature is focused on the basis for weight assignment; one way out of this dilemma is to create a Pareto front consisting of possible weight assignments.

Recall:

$$\text{Shareholder: } J_1 = \max \int_0^T e^{-\rho t} (1 - \theta) (Y(t) + u_1(t)^2 - I(t) - u_2(t)^2) dt$$

$$\text{Manager: } J_2 = \max \int_0^T e^{-\rho t} (ru_3(t)^2 + I(t) - u_4(t)^2) dt$$

The joint objective functional of the game model becomes:

$$\begin{aligned} \max \int_0^T & \varphi e^{-\rho t} (1 - \theta) (Y(t) + u_1(t)^2 - I(t) - u_2(t)^2) \\ & + e^{-\rho t} (1 - \varphi) (ru_3(t)^2 + I(t) - u_4(t)^2) dt \end{aligned} \quad (4.52)$$

Subject to:

$$\dot{Y}(t) = \beta S(t) + (u_4(t) - u_3(t))rY(t) - u_4(t)I(t) - u_1(t)Y(t) + u_2u_4\alpha_2 D(t), Y(0) = Y_0; I(0) = I_0$$

$$\dot{D}(t) = (\alpha + u_4(t)\alpha_2)D(t) - u_1(t)Y(t), D(0) = D_0$$

$$\dot{I}(t) = u_4(t)I(t) + \chi Y(t), Y(0) = Y_0; I(0) = I_0$$

$$\dot{S}(t) = ru_3(t)Y(t) + (1 - u_2)\alpha_2 u_4(t)D(t) - \mu S(t), S(0) = S_0$$

$$E(t) = Y(t) + S(t) - D(t)$$

$$\frac{\dot{D}(t)}{\dot{E}(t)} = \frac{(\alpha + u_4(t)\alpha_2)D(t) - u_1(t)Y(t)}{\beta S(t) + (u_4(t) - u_3(t))rY(t) - u_4(t)I(t) - u_1(t)Y(t) + u_2u_4\alpha_2 D(t) + ru_3(t)Y(t) + (1 - u_2)\alpha_2 u_4(t)D(t) - \mu S(t) - (\alpha + u_4(t)\alpha_2)D(t) + u_1(t)Y(t)}$$

$$I(t) \geq \chi Y(t)$$

$$u_1(t), u_2(t), u_3(t), u_4(t) \geq 0$$

The Pareto outcome for this solution concept is solved in a similar manner to that of the open-loop Nash equilibrium concept.

In the non-cooperative Nash equilibrium analysis, each player has adjoint variables with respect to each state variable, such that for player 1, $\lambda_1 \dots \lambda_4$ and for player 2, $\phi_1 \dots \phi_4$. For a Pareto-efficient outcome, the joint optimisation of the players' objective functionals results in one set of adjoint variables, such that the only adjoint variables for the Hamiltonian are $\lambda_1 \dots \lambda_4$.

The Hamiltonian for the game is specified as:

$$H = \varphi(1 - \theta)(Y + u_1^2 - I - u_2^2) + (1 - \varphi)(ru_3^2 + I - u_4^2) + \lambda_1(\beta S + (u_4 - u_3)rY - u_4I - u_1Y + u_2u_4\alpha_2D) + \lambda_2((\alpha + u_4\alpha_2)D - u_1Y) + \lambda_3(u_4I + \chi Y) + \lambda_4(ru_3Y + (1 - u_2)\alpha_2u_4D - \mu S) \quad (4.53)$$

Theorem 4.2.

The Pareto-efficient equilibrium solutions are:

$$u_1^* = \frac{Y}{2\varphi(1-\theta)} (\lambda_1 + \lambda_2) \quad (4.54)$$

This calculation means that with an increase in the weight assigned to the shareholder, the company will repay less debt in each financial period. This implies that the shareholder retains a high level of confidence from the higher weight assignment and does not enforce high debt repayment when their interests are fully protected in the company.

$$u_2^* = \frac{\alpha_2 u_4 D}{2\varphi(1-\theta)} (\lambda_1 - \lambda_4) \quad (4.55)$$

To attain a Pareto-efficient equilibrium, the shareholder employs greater monitoring and increases the same with respect to the value of debt.

$$u_3^* = \frac{Y}{2(1-\varphi)} (\lambda_1 - \lambda_4) \quad (4.56)$$

From Equation (4.56), with a lesser Pareto weight assignment $1 - \varphi$ to the manager, the manager requires a higher investment level to attain optimality. This provides an optimal mechanism as the manager is induced to select the optimal investment level in each period in order to attain optimality in the strategic relationship.

$$u_4^* = \frac{1}{2(1-\varphi)} (\lambda_1(\alpha_2 u_2 D + rY - I) + \lambda_2 \alpha_2 D + \lambda_3 I + \lambda_4(1 - u_2)\alpha_2 D) \quad (4.57)$$

The optimal quality of effort required from the manager is determined by the decision variables in the agency relationship. This calculation means that the quality of effort provided by the manager will increase with a decrease in the

weight assigned to the manager's objective functional. This induces the manager to improve their quality of effort in order to attain optimality.

Theorem 4.2 shows that the manager has no incentive to divert resources or opportunistically protect their interests as this would require higher investment and quality of effort to reach their optimal pecuniary incentives.

Proof of Theorem 4.2.

$$\frac{\partial H_1}{\partial u_1} = 2u_1(1 - \theta) - (\lambda_1 + \lambda_2)Y = 0$$

$$u_1^* = \frac{Y}{2\varphi(1-\theta)} (\lambda_1 + \lambda_2)$$

$$\frac{\partial H_1}{\partial u_2} = -2u_2(1 - \theta) + (\lambda_1 - \lambda_4)\alpha_2 u_4 D = 0$$

$$u_2^* = \frac{\alpha_2 u_4 D}{2\varphi(1-\theta)} (\lambda_1 - \lambda_4)$$

$$\frac{\partial H_2}{\partial u_3} = 2ru_3 + (\lambda_4 - \lambda_1)rY = 0$$

$$u_3^* = \frac{Y}{2(1-\varphi)} (\lambda_1 - \lambda_4)$$

$$\frac{\partial H_2}{\partial u_4} = -2u_4(1 - \varphi) + \lambda_1(\alpha_2 u_2 D + rY - I) + \lambda_2 \alpha_2 D + \lambda_3 I + \lambda_4(1 - u_2)\alpha_2 D = 0$$

$$u_4^* = \frac{1}{2(1-\varphi)} (\lambda_1(\alpha_2 u_2 D + rY - I) + \lambda_2 \alpha_2 D + \lambda_3 I + \lambda_4(1 - u_2)\alpha_2 D)$$

End of Proof.

The four systems of ordinary differential equations for the adjoint equations $\lambda_1 \dots \lambda_4$ that determine Equations (4.54) – (4.57) above are:

$$\begin{aligned} \dot{\lambda}_1 = \rho \lambda_1 - \frac{\partial H}{\partial Y} = \lambda_1(\rho + u_1 + (u_3 - u_4)r) - \varphi(1 - \theta) + \lambda_2 u_1 - \lambda_3 \chi - \\ \lambda_4 r u_3 \end{aligned} \quad (4.58)$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H}{\partial D} = \lambda_2(\rho - \alpha - u_4\alpha_2) - \lambda_1u_2u_4\alpha_2 - \lambda_4(1 - u_2)\alpha_2u_4) \quad (4.59)$$

$$\dot{\lambda}_3 = \rho\lambda_3 - \frac{\partial H}{\partial I} = \lambda_3(\rho - u_4) + \lambda_1u_4 + \varphi(1 - \theta) - (1 - \varphi) \quad (4.60)$$

$$\dot{\lambda}_4 = \rho\lambda_4 - \frac{\partial H}{\partial S} = \lambda_4(\rho + \mu) - \lambda_1\beta \quad (4.61)$$

Also, these co-states functions satisfy the terminal conditions:

$$\lambda_1(T) = 0 \quad \lambda_2(T) = 0 \quad \lambda_3(T) = 0 \quad \lambda_4(T) = 0$$

The third conditions, as before, are.

$$\dot{Y} = \beta S + (u_4 - u_3)rY - u_4I - u_1Y + u_2u_4\alpha_2D, Y(0) = Y_0; I(0) = I_0 \quad (4.62)$$

$$\dot{D} = (\alpha + u_4\alpha_2)D - u_1(t)Y, D(0) = D_0 \quad (4.63)$$

$$\dot{I} = u_4I + \chi Y, Y(0) = Y_0; I(0) = I_0 \quad (4.64)$$

$$\dot{S} = ru_3Y + (1 - u_2)\alpha_2u_4D - \mu S; S(0) = S_0 \quad (4.65)$$

4.5 Computational Experiments and Results

In this section, the numerical results of the model simulation are reported. The model was computed using a RK4 forward-backward sweep approach. Sensitivity analysis of some of the key parameters was also conducted.

4.5.1 Basis for Parameter Sensitivity

Sensitivity analysis examines how the modification in the results of a numerical model can be explained by alteration of its input factors, otherwise known as the 'what-if' analysis in complex simulation models (Pianosi et al. 2016). This helps to evaluate the validity of the simulation results with respect to uncertain input factors or model assumptions (Paton, Maier & Dandy 2013). The model in this study was simulated using the data collected from the case study company, following the assumptions discussed previously. To observe the impacts of

different parameters on the model outcome, four sensitivity analyses were carried out repeatedly for each solution concept, denoted as Encounter 1, Encounter 2, Encounter 3, and Encounter 4. These four encounters are referred to in the graphical results as E1, E2, E3, and E4. The sensitivity analysis discussed in this chapter is based on the following input factors: payout rate β , the ratio of new debt to existing company debt α , and the cost of debt α_2 . These variables were selected because they are uncertain factors and can be influenced by company or financial market behaviour.

Encounter 1: No parameter sensitivity.

Encounter 2: $\beta = 0.60$. Holding all other parameters constant, the company's productive assets payout rate or value derived from its productive assets is assumed to double from $\beta = 0.30$ to $\beta = 0.60$. This is to observe the outcome of the model when the productivity of the company's assets is enhanced.

Encounter 3: $\alpha = 0.3, \alpha_2 = 0.2$. Originally, the company's rate of change in the use of debt at the start of the game is observed to be $\alpha = 0.01$, while cost of debt is assumed to be 10% or 0.1. Holding all other parameters constant, the model is simulated to observe the impact of a higher rate of change of debt $\alpha = 0.3$ and a higher cost of debt $\alpha_2 = 0.2$ simultaneously.

Encounter 4: $\beta = 0.60, \alpha = 0.3, \alpha_2 = 0.2$. Finally, holding all other parameters constant, the impact of a higher asset payout rate, the higher rate of change of debt and a higher cost of debt are simultaneously observed.

The case study data used to simulate the model are presented in Table 4.3 below.

Table 4.3 Financial data from 2018 financial statements of Metcash Limited (Metcash Limited 2018)

S/N	Parameters	Definition and code	Data
1	r	Riskless rate $r > 0$, assumed to be constant - Current (as of May 2019) Australian Government bonds yield for 10-year residual maturity	0.02
2	b	Manager's reservation utility - Financial data (AUD \$b)	0.002
3	β	Payout rate of company's productive assets, assumed to be constant - Assumed	0.30
4	$Y(0)$	Initial value of cash flow - Financial data (AUD \$b)	0.29
5	$I(0)$	Cost of pecuniary incentive - Financial data (AUD \$b)	0.003
4	$S(0)$	Initial value of company's productive assets - Financial data (AUD \$b)	1.30
5	$D(0)$	Initial value of company debt - Financial data (AUD \$b), calculated as the current liabilities' interest-bearing borrowings plus total non-current liabilities	0.27
6	θ	Company's applicable effective tax rate, ranging between 0 and 1	0.28
7	α	An average of the rate of change in Metcash Limited's use of debt over a six-year financial period (2013 - 2018)	0.01
8	ρ	Discount rate - Assumed	0.001
9	α_2	Finance cost - Assumed	0.1
10	μ	Rate of asset depreciation - Assumed	0.00001
11	χ	Proportion of management fee is specified as 2%	0.02

4.5.2 Nash Equilibrium Results

4.5.2.1 Open-loop Nash equilibrium

The result of the open-loop Nash equilibrium concept for each Encounter - E is provided in Table 4.4 below. The parameter sensitive input factors are specified in addition to the initial values of the states and the final optimal outcomes of the states. The Nash equilibrium strategies and optimal states are then illustrated in Figures 4.4 and 4.5 respectively. Where unspecified, the figures illustrating the results of the financial management model do not have units on the y-axis.

Table 4.4 Open-loop Nash equilibrium outcomes for Encounters (1 - 3)

E	β	α	α_2	$Y(0)$	$D(0)$	$I(0)$	$S(0)$	$Y(T)$	$D(T)$	$I(T)$	$S(T)$
1	0.30	0.01	0.1	0.29	0.27	0.003	1.30	0.6389	0.2315	0.0123	1.3001
2	0.60	0.01	0.1	"	"	"	"	0.9969	0.1993	0.0159	1.3001
3	0.30	0.30	0.2	"	"	"	"	0.6389	0.3158	0.0123	1.3001
4	0.60	0.30	0.2	"	"	"	"	0.9969	0.2791	0.0159	1.3002

Open-loop Nash Strategies

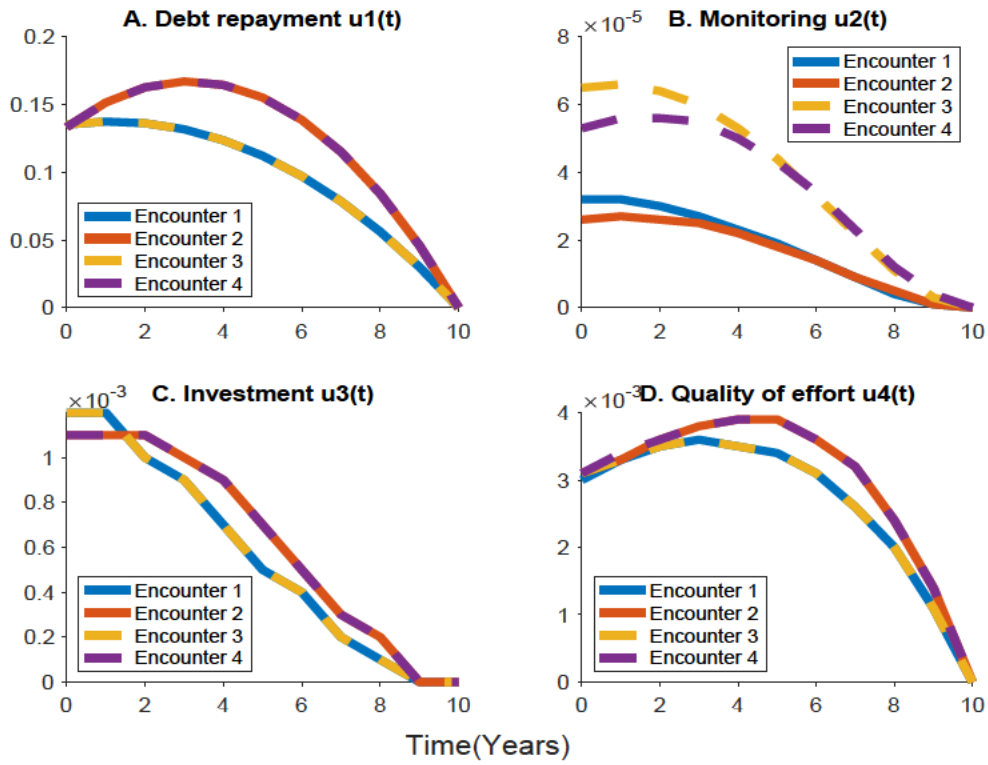


Figure 4.4 Open-loop Nash equilibrium strategies

Trajectories of state variables

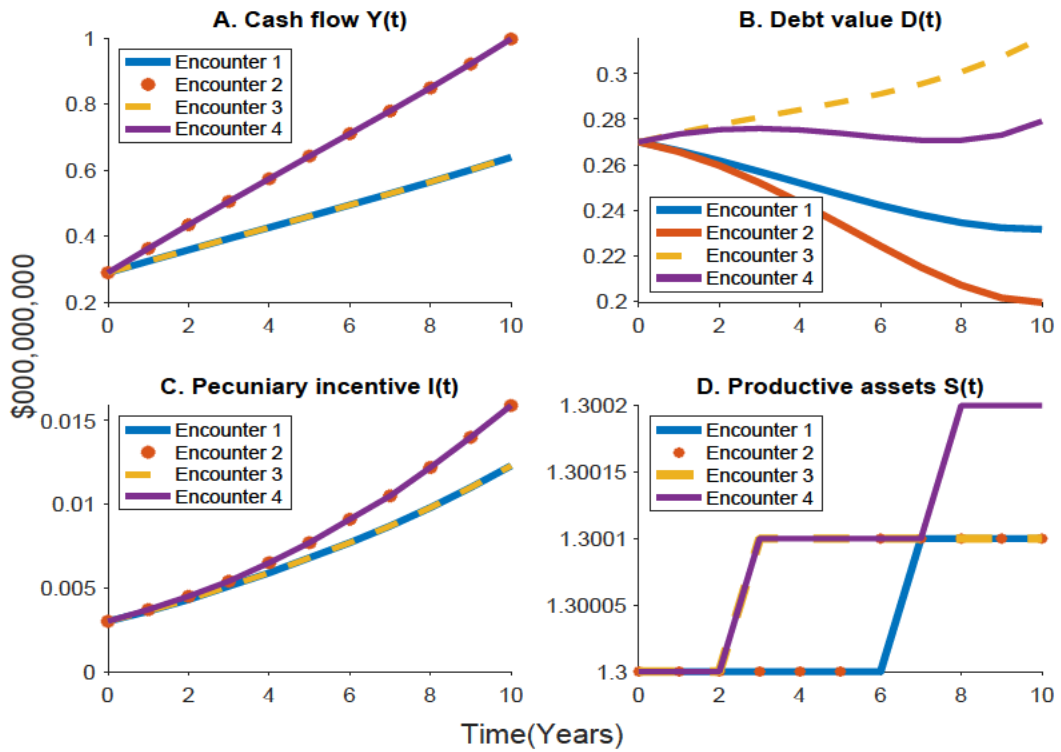


Figure 4.5 Equilibrium state trajectories of the open-loop Nash equilibrium game

Figure 4.4 represents the open-loop Nash equilibrium strategies required to derive the optimal states or outcomes presented in Figure 4.5. It is noted that the shareholder employs a greater debt repayment, rather than monitoring, to optimise their utility. Hence, it may be deduced that the shareholder finds the greater use of debt a more efficient mechanism than monitoring. It is also observed that the shareholder starts the game with an increasing use of debt repayment and lowers this from Period 4 as the contract stabilises and the shareholder is more familiar with the manager's actions. The sensitivity analysis indicates that the shareholder repays more debt in Encounters 2 and 4. This implies that with an increase in the company's assets payout rate, the shareholder adopts a higher debt repayment strategy in order to reduce the free cash flow available to the manager. This agrees with prior studies that have noted the importance of debt in reducing free cash flow, such as Jensen (1986) and Ortiz-Molina (2007). On the other hand, with a higher use of debt and associated cost of debt, the shareholder adopts greater monitoring. This implies that the level of monitoring selected by the shareholder is determined mainly by the company's value of debt and cost of debt simultaneously.

Figure 4.4 also shows that the manager relies more on their quality of effort than on investment as their Nash equilibrium strategy. The quality of effort from the manager is observed to be increasing for the first five years of the contract. Prior studies, such as Thomsen and Conyon (2012), Mu, Wang and Yang (2017), and Goetz et al. (2019), have relied heavily on the effort of the manager as the only strategy in an agency relationship. The results in this new study show the additional benefit of investment in the principal-agent relationship. Furthermore, it is also shown that with a higher asset payout rate, the manager adopts a higher level of investment and a greater quality of effort. This may be explained by the use of greater monitoring by the shareholder. This suggests that the manager will adopt higher Nash equilibrium strategies when the shareholder applies greater debt and, consequently, greater monitoring.

Figure 4.5 illustrates the optimal states outcomes. The company's cash flow increases in all encounters but is highest when the company's productive assets have a higher payout rate in Encounters 2 and 4, irrespective of the increase in

the use of debt. The company's debt value declines in most encounters except in Encounter 3, when the ratio of new debt to existing company debt is higher. It is also shown in Graph B in Figure 4.5 that a higher payout rate, as specified in Encounters 2 and 4, reduces the debt value of the company as the company is able to repay more debt. The pecuniary incentive of the manager increases over time but is highest when the productive assets payout rate is higher. This corresponds with the theory that a manager's remuneration is a function of the company's performance. In the model specification, the company's cash flow is described as a function of the company's investment. Thus, the manager has the incentive to optimise the company's investment level to improve its cash flow, which improves the manager's pecuniary incentive. Finally, the company's value of productive assets is seen to grow slowly due to the low riskless rate of return on the company's cash flow.

4.5.2.1.1 *Optimal debt-equity ratio (open-loop case)*

The optimal debt-equity $D^*(t)/E^*(t)$ ratios required in the open-loop case of the dynamic financial game are presented in Table 4.5 below over a ten-year period and in Figure 4.6, with no units on the y-axis.

Table 4.5 *Optimal debt-equity ratio for the open-loop case*

Period	Encounter 1 $D^*(t)/E^*(t)$	Encounter 2 $D^*(t)/E^*(t)$	Encounter 3 $D^*(t)/E^*(t)$	Encounter 4 $D^*(t)/E^*(t)$
0	0.2045	0.2045	0.2045	0.2045
1	0.1958	0.1900	0.2028	0.1968
2	0.1872	0.1759	0.2009	0.1887
3	0.1789	0.1623	0.1989	0.1804
4	0.1708	0.1492	0.1970	0.1721
5	0.1631	0.1368	0.1952	0.1640
6	0.1560	0.1254	0.1937	0.1564
7	0.1495	0.1152	0.1927	0.1497
8	0.1438	0.1065	0.1924	0.1441
9	0.1391	0.0997	0.1929	0.1401
10	0.1356	0.0950	0.1946	0.1383

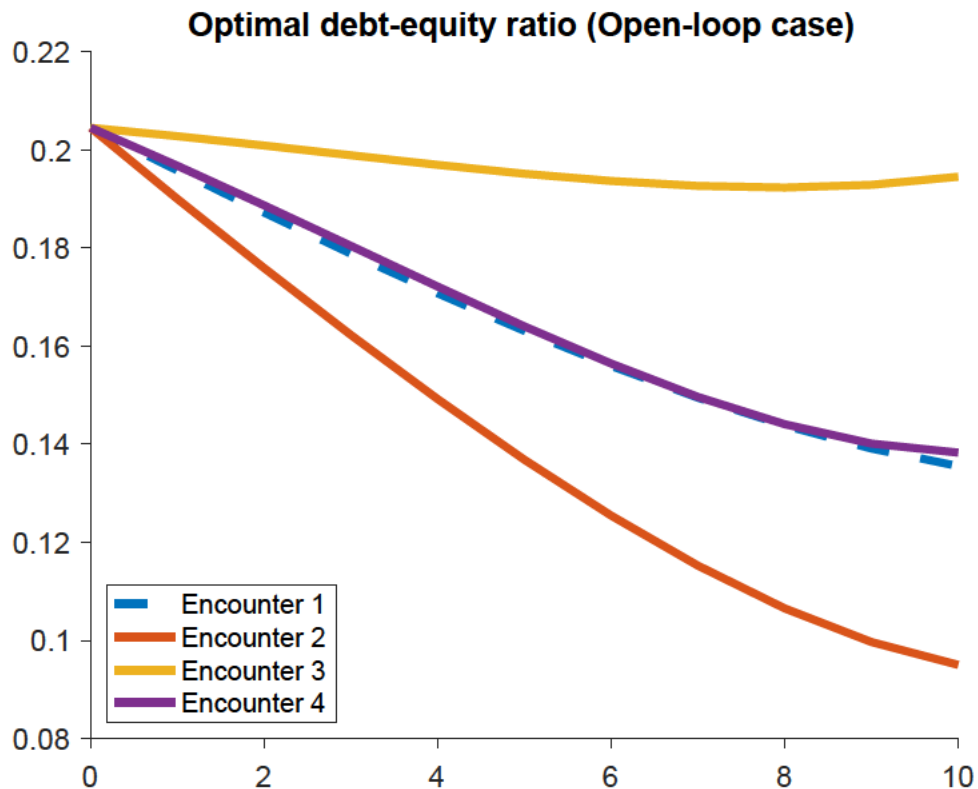


Figure 4.6 Optimal debt-equity ratio (Open-loop case)

Figure 4.6 illustrates that the optimal capital structure required for the shareholder-manager agency relationship is lower than the current capital structure of the company, irrespective of parameter sensitivity. This implies that the optimal mechanism required to minimise the agency relationship in the company requires a lower debt-equity ratio in order to optimise the company value. It is also noted that the optimal debt-equity ratio for the firm is highest when the company uses more debt with an associated cost of debt and lowest when the company's assets are more productive. A lower debt-equity ratio at a higher payout rate, as seen in Encounter 2, may be described as a vote of confidence in the manager, thus requiring a less stringent debt mechanism from the shareholder.

4.5.2.2 Feedback Nash equilibrium

The results of the Feedback Nash equilibrium concept for each Encounter are provided below in Table 4.6.

Table 4.6 Feedback Nash equilibrium outcomes for encounters (1 - 3)

E	β	α	α_2	$Y(0)$	$D(0)$	$I(0)$	$S(0)$	$Y(T)$	$D(T)$	$I(T)$	$S(T)$
1	0.30	0.01	0.1	0.29	0.27	0.003	1.30	0.6389	0.2315	0.0123	1.3001
2	0.60	0.01	0.1	"	"	"	"	0.9969	0.1993	0.0159	1.3001
3	0.30	0.30	0.2	"	"	"	"	0.6389	0.3158	0.0123	1.3001
4	0.60	0.30	0.2	"	"	"	"	0.9969	0.2791	0.0159	1.3002

The results of the optimal mechanism required in the feedback Nash equilibrium game are presented In Figure 4.7 below.

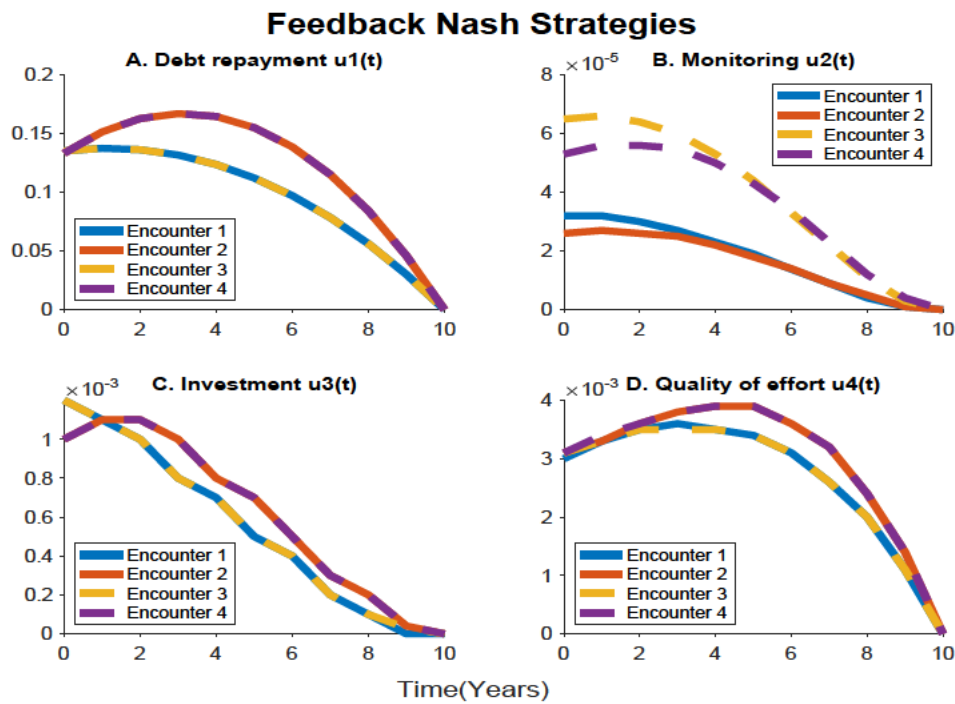


Figure 4.7 Feedback Nash equilibrium strategies

The outcomes of the feedback game for the company performance indicators are presented in Figure 4.8 below:

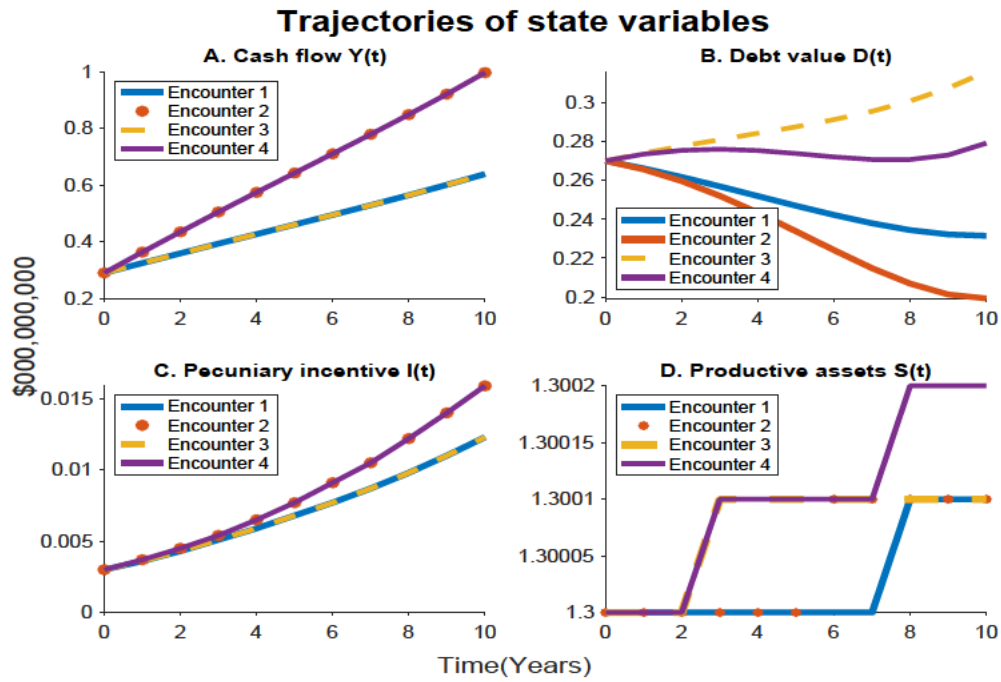


Figure 4.8 Equilibrium state trajectories of the feedback Nash equilibrium game

4.5.2.2.1 Optimal debt-equity ratio (feedback case)

The optimal debt-equity $D^*(t)/E^*(t)$ ratios required in the feedback case of the dynamic financial game are presented in Table 4.7 below over a ten-year period and in Figure 4.9, with no units on the y-axis.

Table 4.7 *Optimal debt-equity ratio for the feedback case*

Period	Encounter 1 $D^*(t)/E^*(t)$	Encounter 2 $D^*(t)/E^*(t)$	Encounter 3 $D^*(t)/E^*(t)$	Encounter 4 $D^*(t)/E^*(t)$
0	0.2045	0.2045	0.2045	0.2045
1	0.1958	0.1900	0.2028	0.1968
2	0.1872	0.1759	0.2009	0.1887
3	0.1789	0.1623	0.1989	0.1804
4	0.1708	0.1492	0.1970	0.1721
5	0.1631	0.1368	0.1952	0.1640
6	0.1560	0.1254	0.1937	0.1564
7	0.1495	0.1152	0.1927	0.1497
8	0.1438	0.1065	0.1924	0.1441
9	0.1391	0.0997	0.1929	0.1401
10	0.1356	0.0950	0.1946	0.1383

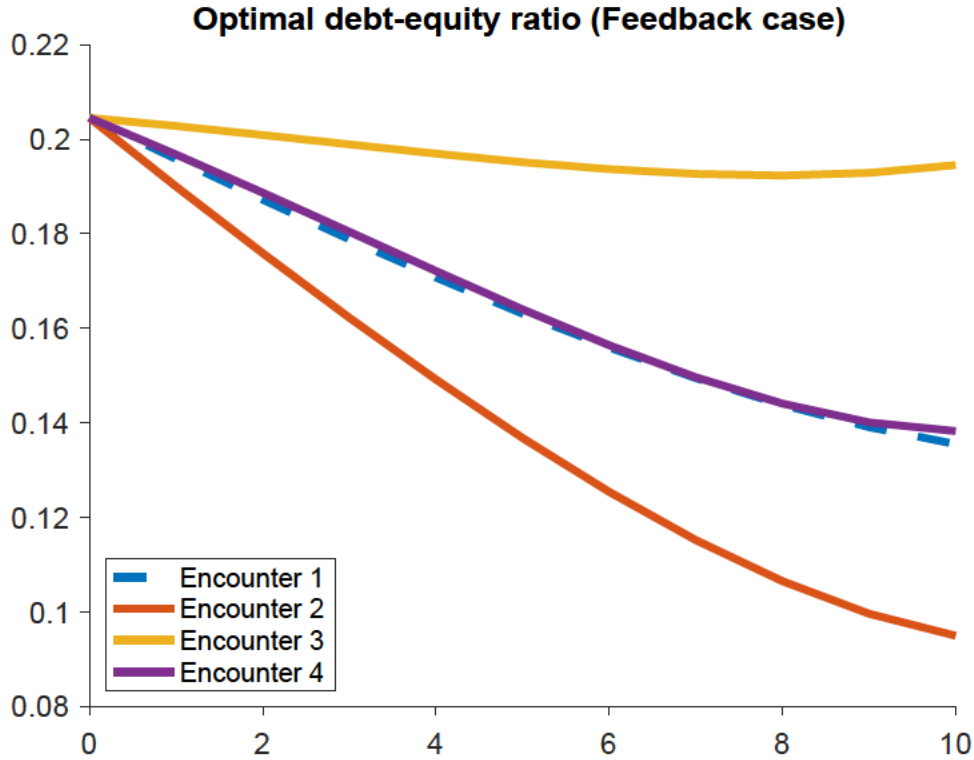


Figure 4.9 Optimal debt-equity ratio (feedback Nash equilibrium case)

The optimal results of the feedback Nash equilibrium solution are similar to those of the open-loop Nash equilibrium solution. However, it is observed that the manager's investment strategy starts lower in the feedback game, as they observe the current states of the game.

4.5.3 Pareto-efficient Results

The numerical results of the cooperative solution concept were also obtained. For each weight assignment $0 < \varphi < 1$, the Pareto-efficient solution was plotted on the Pareto frontier. To obtain the Pareto frontiers, the Pareto-efficient strategies and states were obtained for each $\varphi = [0.1, \dots, 0.9]$. Weights 0 and 1 were excluded because a player will not remain in a game if their interest is set to 0. The returned optimal values at each weight share were then imputed into the individual utility functions (objective functionals) of each player independently, thereby producing the manager and shareholder's payoff for each weight share. The values of the manager's payoff were then plotted against those of the shareholder, to observe the trade-off for both players. The results were plotted

as $(\varphi, 1 - \varphi)$, where φ is the shareholder's weight assignment and $1 - \varphi$ is the manager's weight assignment.

To calculate the payoffs J_1 and J_2 for the shareholder and the manager, the area under the objective functional curve was calculated using the standard integration by parts method.

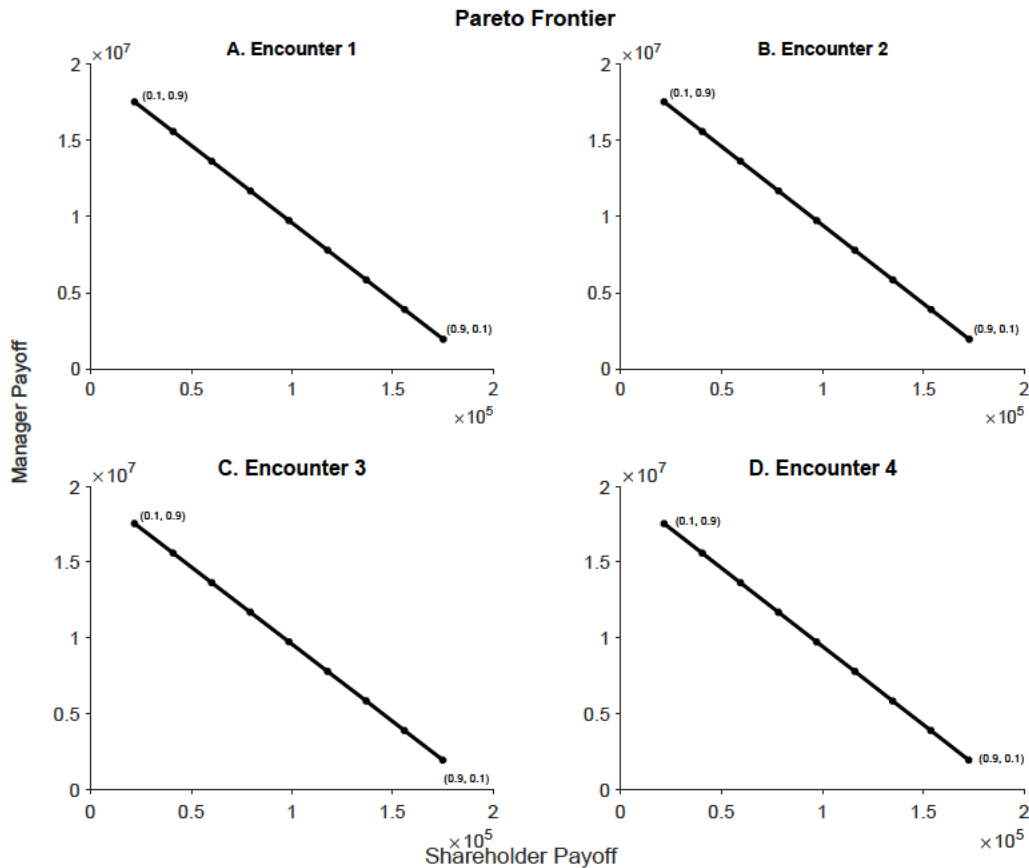


Figure 4.10 Pareto frontier

Figure 4.10 shows the payoff trade-off between the shareholder and the manager, explained as a linear relationship. With an increase in the shareholder's weight assignment, the manager's payoff decreases steadily. This implies that the more the shareholder protects their interests, the less the optimal payoff is for the manager. The natural choice of a final solution may put one player at a disadvantage in the agency relationship, hence the best compromise solution should be decided upon. This could be determined by the optimal states of the contract. It is noted that the manager's worst payoff is greater in value than the

shareholder's best payoff. This suggests that the assignment of the weight (0.9, 0.1) is a recommended best compromise solution.

Hence, the results of each player's best payoff are presented to observe the difference in the game outcome. The shareholder's optimal payoff (0.9, 0.1) is assumed to be the best compromise solution in the contract, while the manager's optimal payoff (0.1, 0.9) is discussed in turn as a suboptimal payoff or a reneging on the terms of cooperation.

4.5.3.1 *Optimal debt-equity ratio (Pareto case)*

The optimal debt-equity $D^*(t)/E^*(t)$ ratios required in the Pareto case of the dynamic game are presented in Tables 4.8 and 4.9 below over a ten-year period, and in Figure 4.11, with no units on the y-axis.

Table 4.8 *Optimal debt-equity ratio for the Pareto case (0.9, 0.1)*

Period	Encounter 1 $D^*(t)/E^*(t)$	Encounter 2 $D^*(t)/E^*(t)$	Encounter 3 $D^*(t)/E^*(t)$	Encounter 4 $D^*(t)/E^*(t)$
0	0.2045	0.2045	0.2045	0.2045
1	0.1958	0.1900	0.2028	0.1969
2	0.1873	0.1759	0.2009	0.1888
3	0.1789	0.1623	0.1990	0.1805
4	0.1708	0.1492	0.1971	0.1723
5	0.1631	0.1369	0.1952	0.1642
6	0.1560	0.1254	0.1937	0.1566
7	0.1495	0.1153	0.1927	0.1498
8	0.1438	0.1065	0.1924	0.1443
9	0.1392	0.0997	0.1930	0.1403
10	0.1356	0.0950	0.1946	0.1384

Table 4.9 Optimal debt-equity ratio for the Pareto case (0.1, 0.9)

Period	Encounter 1 $D^*(t)/E^*(t)$	Encounter 2 $D^*(t)/E^*(t)$	Encounter 3 $D^*(t)/E^*(t)$	Encounter 4 $D^*(t)/E^*(t)$
0	0.2045	0.2045	0.2045	0.2045
1	0.1955	0.1897	0.2025	0.1964
2	0.1865	0.1751	0.2002	0.1879
3	0.1779	0.1612	0.1990	0.1792
4	0.1696	0.1477	0.1979	0.1705
5	0.1618	0.1351	0.1956	0.1621
6	0.1545	0.1235	0.1936	0.1543
7	0.1480	0.1132	0.1936	0.1473
8	0.1423	0.1045	0.1920	0.1417
9	0.1376	0.0997	0.1910	0.1377
10	0.1340	0.0931	0.1926	0.1359

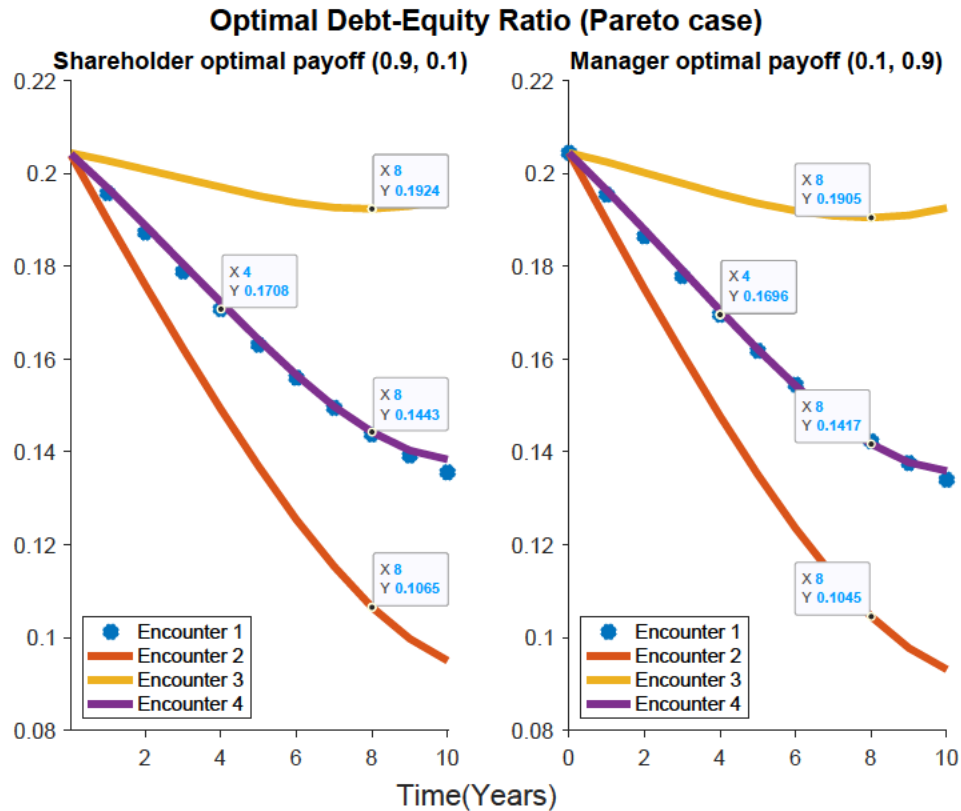


Figure 4.11 Optimal debt-equity ratio (Pareto case)

Figure 4.11 shows that the optimal capital structure required at the shareholder optimal payoff requires a higher use of debt to enforce the shareholder's interests. However, it is noted that the differences in the capital structure required in both cases are not significant. The optimal debt-equity ratio is lowest when the company's asset payout rate is highest without an increase in the use of debt.

4.5.4 Comparison of the Open-loop, Feedback, Pareto Solution Concepts for the Weight Share [0.9, 0.1] and [0.1, 0.9]

In this section, a comparison of the Nash equilibrium strategies, the Pareto-efficient strategies, and the results for the different parameter sensitivity analyses are discussed.

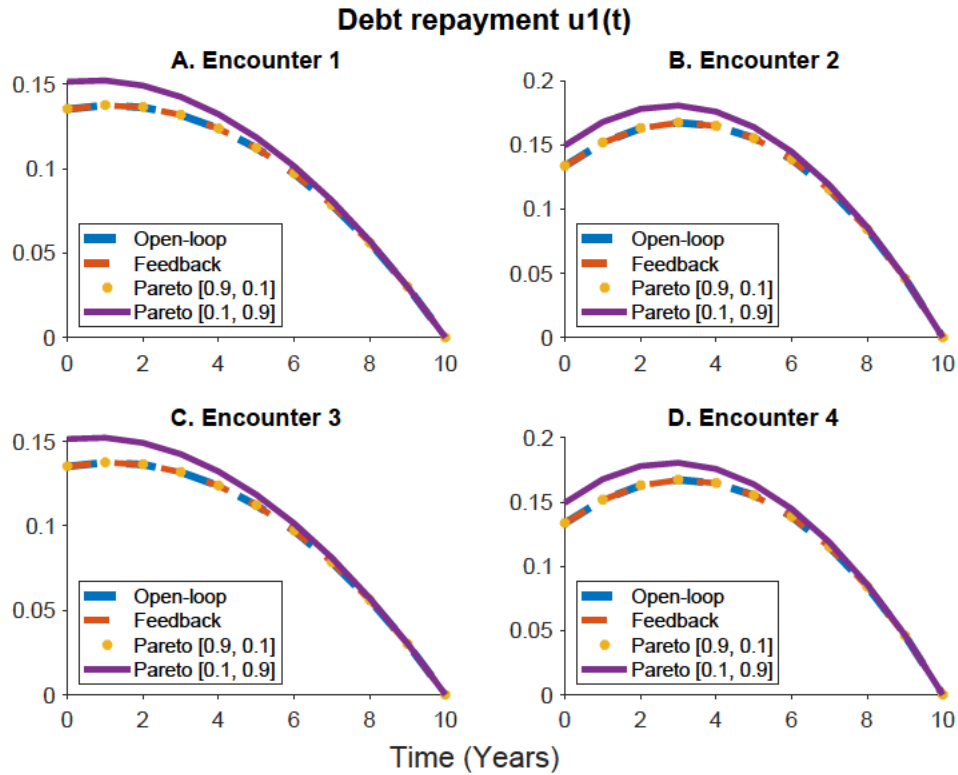


Figure 4.12 Debt repayment strategy (Open-loop, Feedback, Pareto [0.9, 0.1], [0.1, 0.9])

Figure 4.12 shows that a higher debt repayment strategy is required when the manager's interest is fully protected at the expense of the shareholder's interest. This implies that the shareholder employs higher debt repayment when the manager reneges on the terms of cooperation, assuming that the original terms of cooperation represent the best compromise solution selected at the Pareto-efficient weight share [0.9, 0.1], the shareholder's optimal payoff. This provides an incentive for the manager to stick to terms of cooperation, and thus serves as an optimal mechanism. Intuitively, higher debt repayment is observed in Encounters 2 and 4, when the company's cash flow is improved by a higher payout rate.

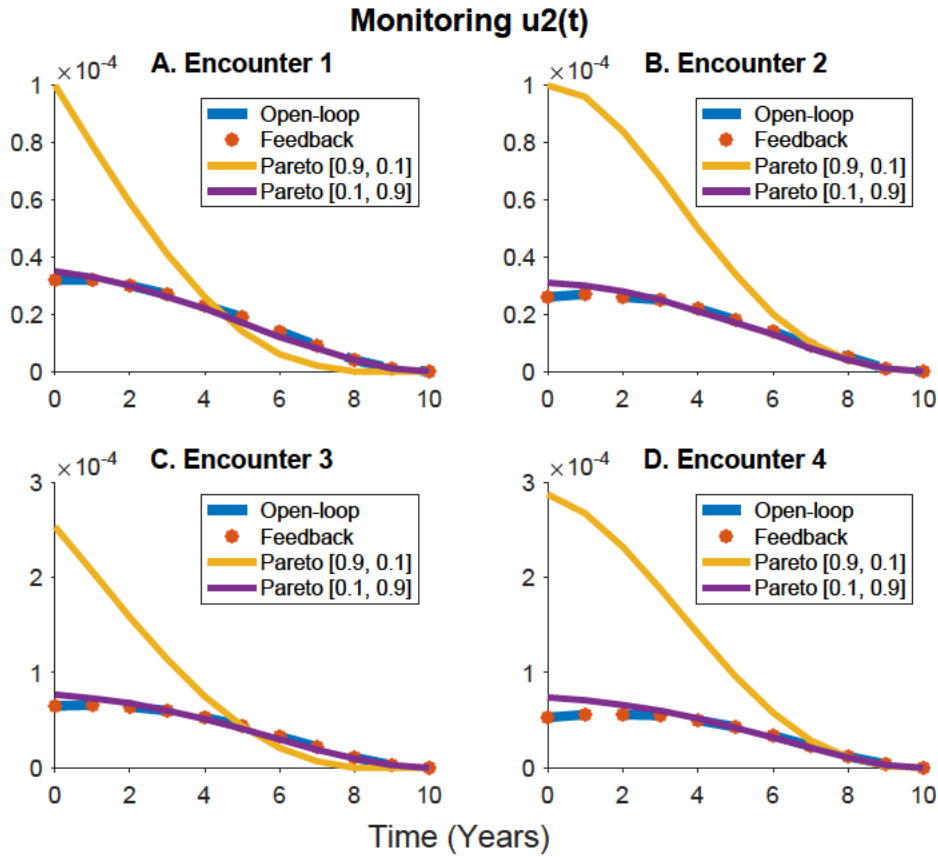


Figure 4.13 Monitoring strategy (Open-loop, Feedback, Pareto [0.9, 0.1], [0.1, 0.9])

In Figure 4.13, a significantly greater monitoring is applicable at the best compromise Pareto-efficient solution. It is interesting to note that in all the non-cooperative cases, the shareholder employs a similar level of monitoring, but requires greater monitoring to protect the optimal payoff during cooperation. Additionally, greater monitoring is required when the company's use of new debt is greater, and more so, when the payout rate of productive assets is higher, as shown in Encounters 3 and 4. This indicates that the optimal level of monitoring would increase with an increase in the company's use of new debt and with a higher productive asset rate. As the contract comes to an end, the monitoring required is observed to have no marginal difference when compared with the non-cooperative cases.

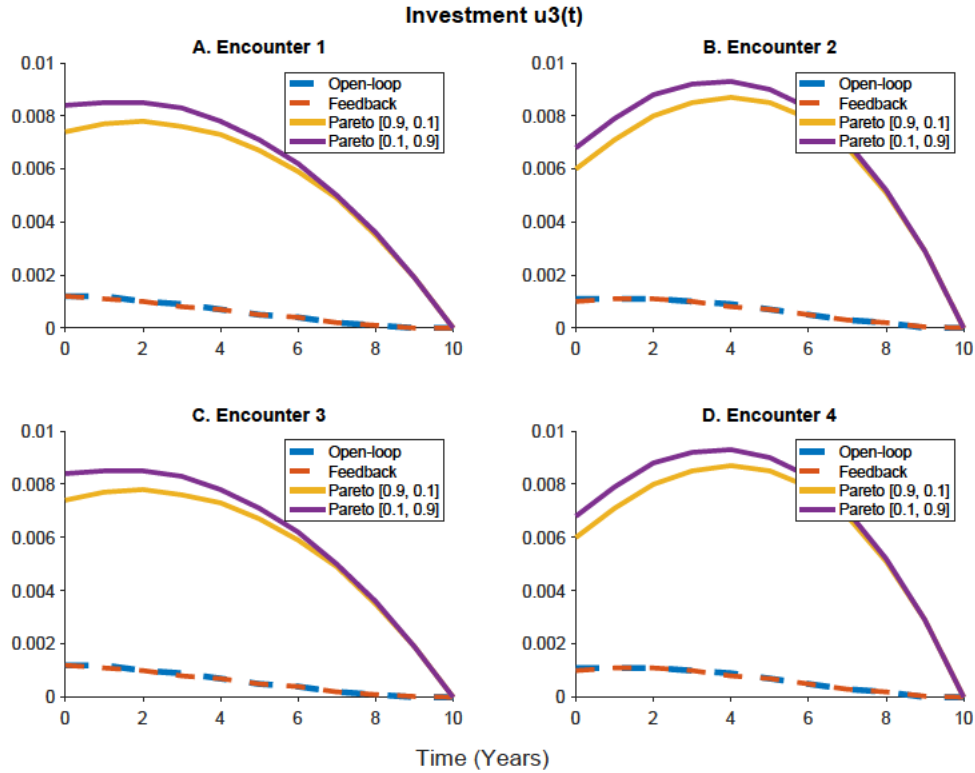


Figure 4.14 Investment strategy (Open-loop, Feedback, Pareto [0.9, 0.1], [0.1, 0.9])

Figure 4.14 illustrates the optimal investment strategy applicable to the manager. It is shown that the manager requires a higher investment level in the Pareto-efficient outcome when compared to the Nash equilibrium outcomes, particularly when they renege on the terms of cooperation. This implies that the manager must commit to a higher investment level if they select an opportunistic weight assignment. This can disincentivise the manager from reneging on cooperation so as to minimise the pressure to invest by sticking with the best compromise solution.

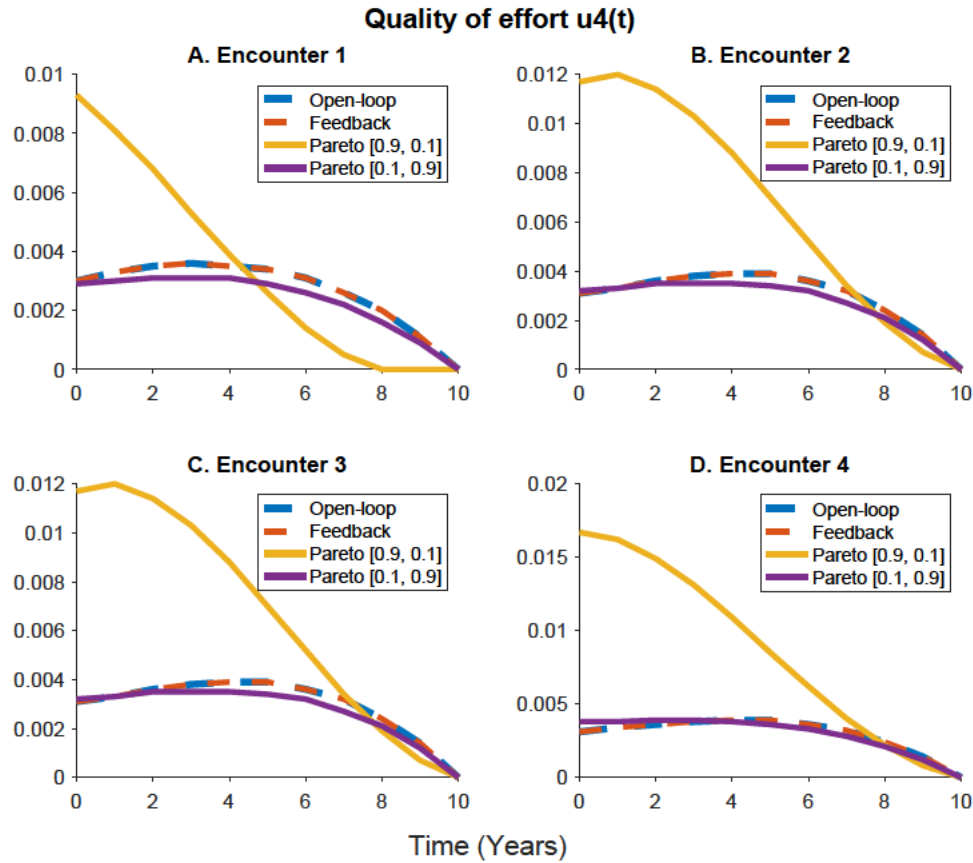


Figure 4.15 Effort strategy (Open-loop, Feedback, Pareto [0.9, 0.1], [0.1, 0.9])

Figure 4.15 compares the optimal quality of effort required of the manager at the different solution concepts. It shows that the manager deploys a higher quality of effort at the shareholder's optimal payoff. Thus, the model sufficiently induces the manager to increase their quality of effort to optimise the company value. The quality of effort required of the manager is also considerably parameter sensitive. The manager exerts the smallest effort at a lower asset payout rate when the company uses less new debt. On the other hand, the manager exerts the greatest effort when the company's productive asset payout rate is improved, as this increases their pecuniary incentives, and the company's use of new debt is higher due to its disciplining effects. These results offer interesting insights as they provide information about the optimal quality of effort preferred by the manager over time. This contrasts to research such as Bolton and Dewatripont (2005), in which the manager chooses to provide a generic high or low level of effort. These results also reveal the decreasing or increasing effects of the manager's actions

on the company's outcome over time. These highlight the benefits of a dynamic continuous-time game-theoretic tool in analysing strategic interactions.

4.5.4.1 Trajectories of state variables

The outcomes of the state variables, which indicate the impact of the model on company performance, are presented in Figure 4.16 below.

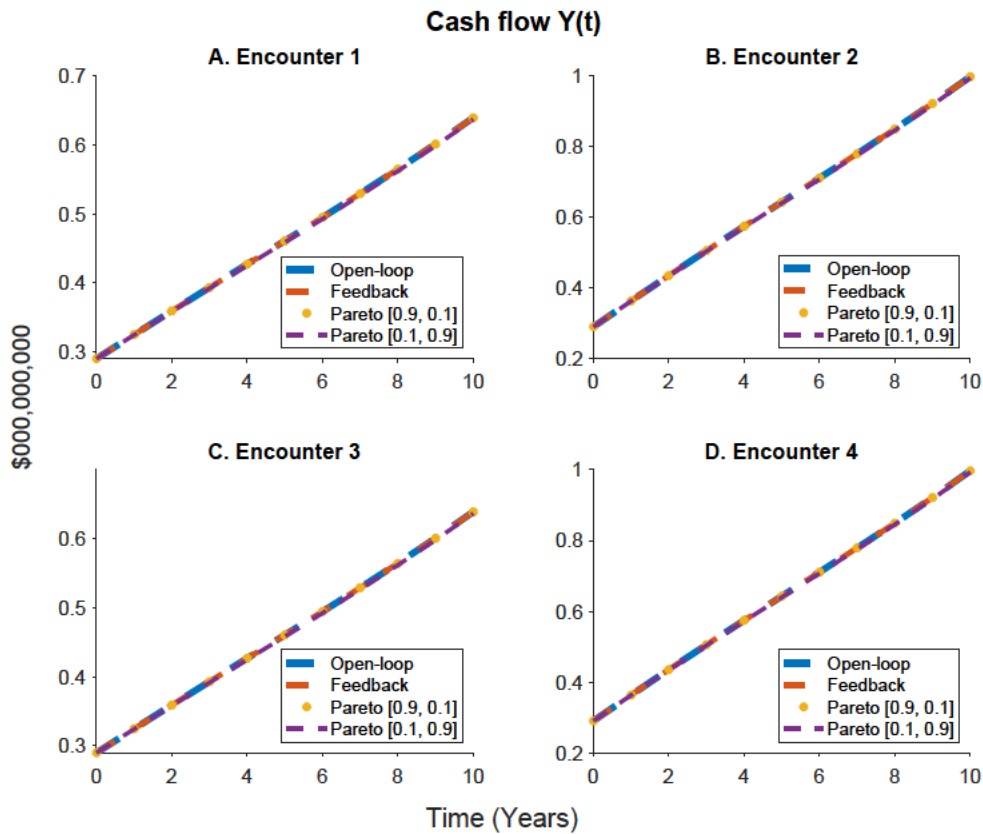


Figure 4.16 Cash flow over time

Figure 4.16 illustrates the effect of the shareholder and manager's optimal strategies on the cash flow of the company over time. Intuitively, a higher cash flow represents a larger firm value (Mu, Wang & Yang 2017). The result of the computational studies, as shown in Figure 4.16, indicate that the company's cash flow is less sensitive to the different solution concepts, but more sensitive to the value of the input factors. The company's cash flow substantially increases when the company's assets are more productive, irrespective of the value of the new debt of the company, as seen in Encounters 2 and 4.

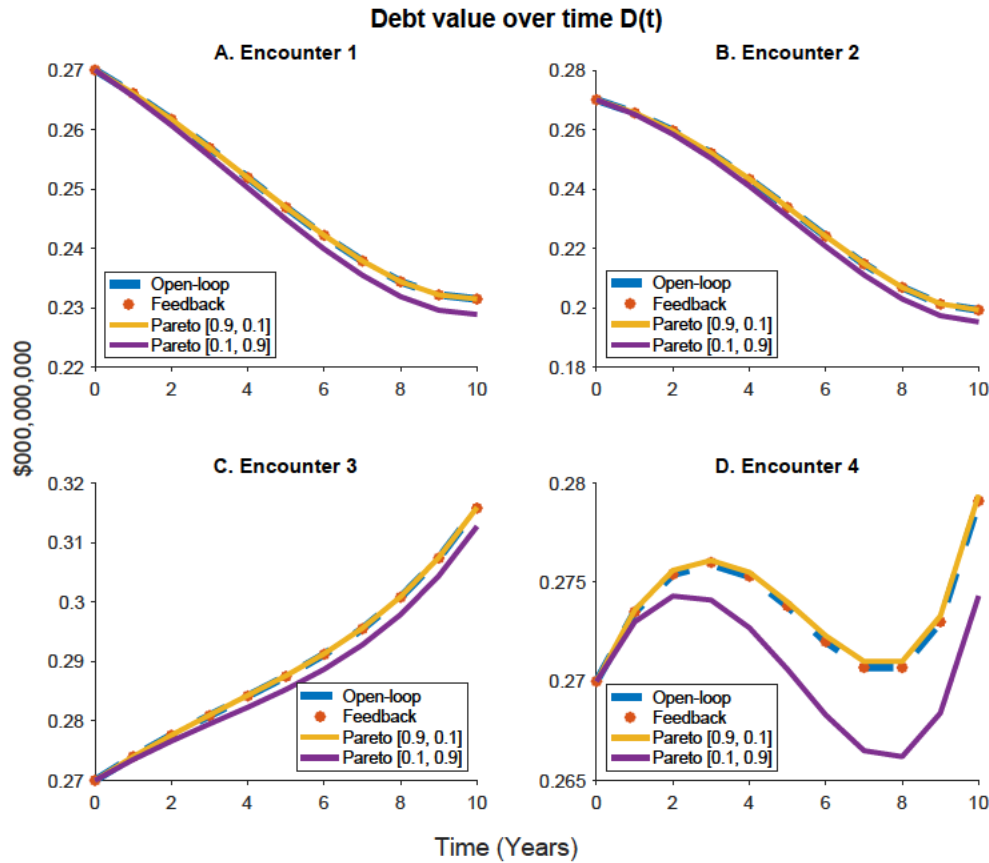


Figure 4.17 Debt value over time

As shown in Figure 4.17, with no parameter sensitivity, as specified in Encounter 1, there is a clear trend of decreasing debt value over time. This means that the optimal debt required to minimise the information asymmetry problem is less than the current use of debt in the company. However, debt value steadily rises in Encounter 3 because the use of new debt is greater. In Encounter 4, the optimal debt peaks until year 3 before a decline is observed because of the effect of the higher payout rate, which improves the firm's cash flow. The optimal debt required also climbs towards the end of the contract. Generally, lower debt can imply less need for debt as there is surplus cash flow. Furthermore, when the manager's interests are fully protected at the expense of the shareholder, the company's optimal required debt is less than in any other case, with a more significant difference observed in Encounter 4. This result agrees with those of prior studies, such as He (2011), which suggests information asymmetry causes firms to use a suboptimal level of debt. This result implies that the shareholder requires a higher

optimal debt level to enforce cooperation. Thus, the shareholder will require higher debt to protect their interests throughout the contract.

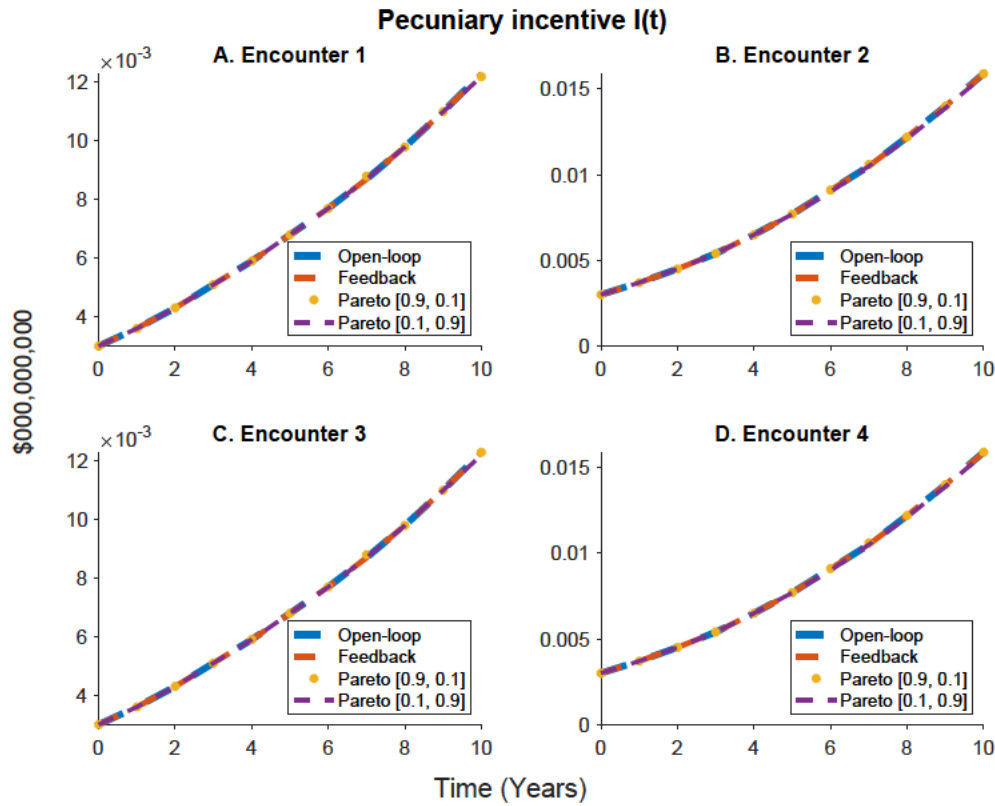


Figure 4.18 Pecuniary incentive over time

As shown in Figure 4.18, the optimal pecuniary incentive payable to the manager steadily rises throughout the contract because of the company's profitability, as observed from the results of the cash flow in Figure 4.16. The manager's reservation utility is a fixed percentage of the company's cash flow. Similar to the results shown in Figure 4.16, the result of the pecuniary incentive shown in Figure 4.18 is observed to be less sensitive to the solution concepts, but more sensitive to the value of the input factors. The manager earns more in Encounters 3 and 4 when the company's productive assets are enhanced, irrespective of the use of new debt. Although the manager earns the same pay irrespective of the solution concepts, however, the optimal strategies required by the manager to obtain the optimal pecuniary incentive are not the same.

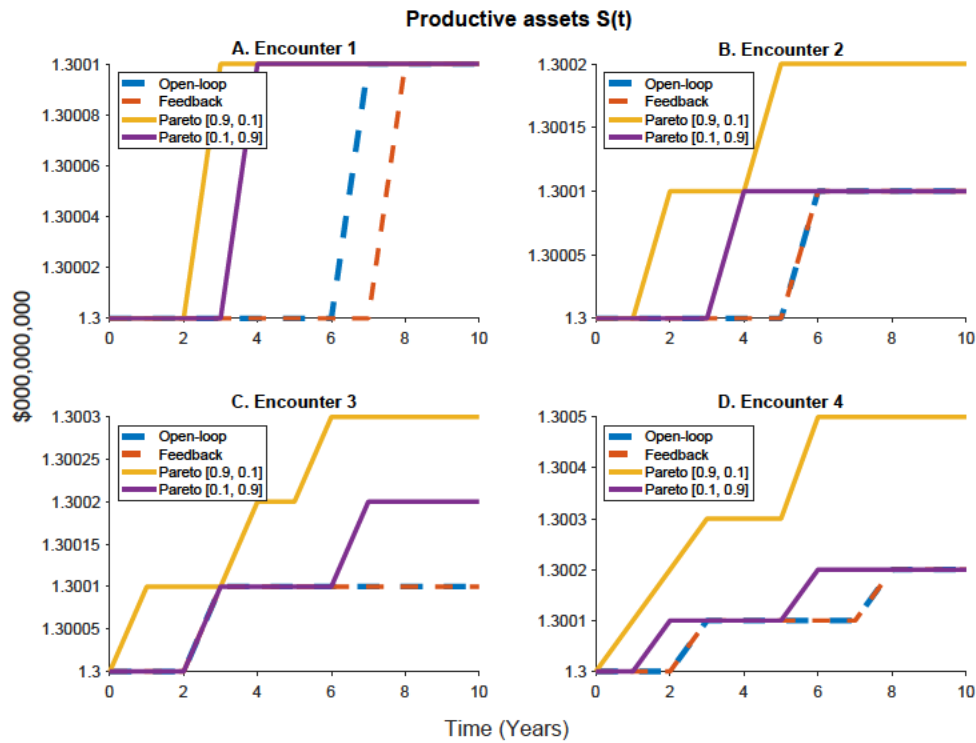


Figure 4.19 Productive assets over time

Figure 4.19 shows that the company's productive assets gradually increase to a maximum in all four encounters. This result can be explained by the low riskless rate on the company's investment. However, higher growth is observed for the Pareto-efficient outcome in all encounters. This suggests there is a positive relationship between the optimal mechanism and the company's growth of productive assets. With an increase in the company's asset payout rate, there is an observed increase in the value of productive assets and a further increase when the value of new debt is increased.

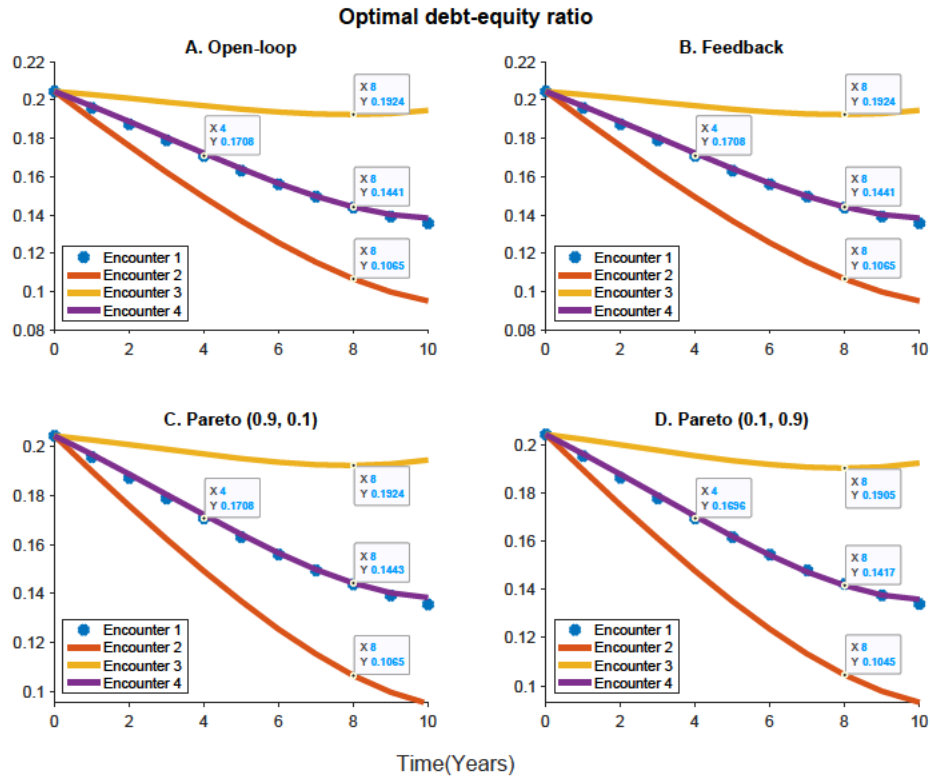


Figure 4.20 Optimal debt-equity ratio (Open-loop, Feedback and Pareto cases)

As illustrated in Figure 4.20, the optimal debt-equity ratio required at the best compromise Pareto-efficient outcome is minimally higher than at a suboptimal Pareto outcome. This implies that the shareholder optimally protects their interests using higher leverage. However, the required optimal capital structure to minimise the agency problem is less than the current debt value of the company.

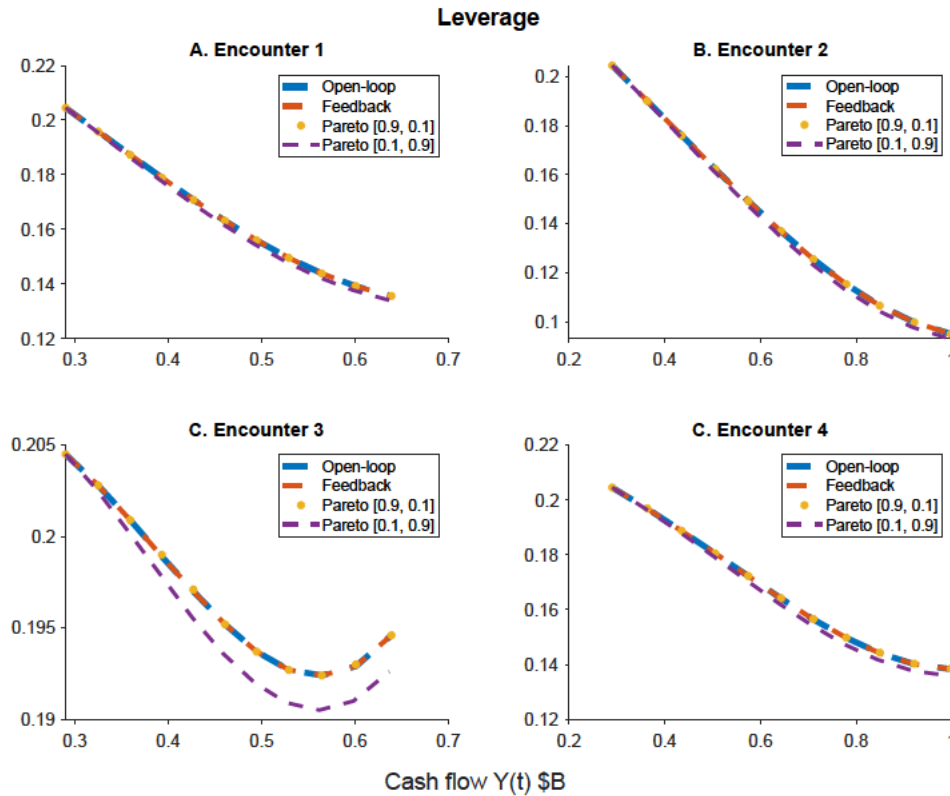


Figure 4.21 The dynamic effects of cash flow on leverage

Figure 4.21 shows the relationship between the company's optimal capital structure and cash flow. Intuitively, the optimal leverage declines as cash flow grows over time, this is consistent with the findings of DeMarzo and Sannikov (2006). In Encounter 3, when the company applies more debt without an increase in the asset payout rate, the manager prefers much lower leverage to optimise their welfare, as this can reduce financing pressure.

4.6 Implications for Corporate Governance, Optimal Capital Structure and Dynamic Game Theory

The financial management and optimal mechanism design model presented in this chapter provides useful suggestions and recommendations for management contract design, as follows:

1. The model provides Nash equilibrium strategies required for an optimal management contract and an optimal capital structure. These strategies require the shareholder to employ a higher debt repayment strategy, rather

than monitoring, to optimise their payoff in the game. This may infer that debt repayment is a more effective strategy than monitoring in a shareholder-manager agency relationship. Furthermore, with a higher company asset payout rate, the shareholder requires a higher debt repayment. This can reduce the free cash flow available to the manager due to the added value derived from improved company productive assets. Contrary to a common view in capital structure modelling, in which the manager fixes the firm's leverage throughout the contract (see, for example, Morellec (2004)), the results of this new study present a case in which the shareholder, via the board of directors, can apply different levels of debt repayment strategies depending on the company's outcomes in each period.

2. The optimal monitoring level will increase with an increase in the use of new debt and the associated cost of debt. Thus, the level of monitoring adopted by the shareholder is mainly determined by the value of new debt and the cost of debt simultaneously. This result differs from those of Heinrich (2002), which suggest optimal monitoring is a function of the institutional environment.
3. The result of the analysis shows that the manager relies more on their optimal quality of effort than investment as a strategy in the game. This finding supports previous research by Pitchford and Snyder (2004). It establishes the additional benefit of dynamic investment in optimal contracts to minimise the information asymmetry.
4. Generally, a higher asset payout rate improves the company's cash flow, reduces its debt level, increases the manager's pecuniary incentives, and improves the value of productive assets.
5. For the Pareto-efficient or centralised decision outcome, the Pareto front trade-off indicates a negative correlation between the shareholder's payoff and the manager's payoff.
6. The shareholder applies a higher debt repayment strategy when the manager reneges on the shareholder's optimal payoff, and a lesser debt repayment strategy when the manager agrees to the terms of cooperation. On the other hand, the optimal level of monitoring is higher at the shareholder's optimal payoff to protect the shareholder's interest.

7. The manager requires a higher optimal investment level to derive similar optimal pecuniary incentive if they renege on the best compromise Pareto-efficient solution. This may disincentivise the manager from reneging on the contract. By contrast, a higher optimal quality of effort is required at the best compromise Pareto-efficient solution, particularly at a higher asset payout rate and a higher rate of new debt. This implies that the manager is required to produce more effort when the company's assets are more productive, and the company uses more new debt. This reveals the manager's optimal actions with fluctuations in cash flow.
8. The manager will invest less at a lower payout rate. This reduces their pecuniary incentives and can induce them to invest more as their pecuniary incentives are dependent on cash flow. The manager will earn more pecuniary incentive when the asset payout rate is higher, irrespective of change in the use of new debt.
9. At the current value of new debt, the debt value declines, but follows a monotonic pattern at a higher asset payout rate and a higher value of new debt, as more cash flow can reduce the need for debt. The company will require more debt at the start and at the end of the contract to optimise the capital structure and minimise the agency relationship when the company's productive assets' payout rate and the use of new debt are higher.
10. Overall, the optimal dynamic contract minimising the agency problem reduces the optimal capital structure of the company. However, the shareholder will require more debt at their optimal payoff than at the manager's optimal payoff. This result supports the findings of Mu, Wang and Yang (2017), which showed that moral hazard could destabilise the capital structure of the company, forcing the company to use more debt than required. Hence, these findings further support the work of Vilasuso and Minkler (2001), which suggests that the optimal capital structure minimises the agency costs of using debt.

4.6.1 Renegotiation Proofness

It may be questioned why the manager would not play non-cooperatively throughout the contract or, in other words, why the manager would not renege on

the terms of a Pareto-efficient cooperation/contract, assuming this to be at weight share $[0.9, 0.1]$, if reneging offers less monitoring and a lower quality of effort. This can be seen in Figure 4.13 for a similar pecuniary incentive. The argument against this is that the shareholder will apply a higher debt repayment strategy, which may cause financing pressure on the manager and, potentially, lead to bankruptcy or financial distress. In addition, the manager would require a higher investment to achieve optimal pecuniary incentives. This result is consistent with He's (2011) definition of incentive-compatibility, as the manager finds it optimal to exert the recommended effort to avoid bankruptcy due to excessive debt repayment and a higher investment requirement. This induces the manager to select the Pareto-efficient contract, thus providing renegotiation proofness and an optimal mechanism design.

The financial management model derives the optimal capital structure and optimal mechanism to minimise the shareholder-manager agency problem by providing equilibrium and optimal strategies for an optimal management contract in a dynamic framework using differential game theory. In contrast to an earlier analysis by Heinrich (2002), the continuous-time modelling framework in this study provides a more tractable framework to analyse the optimal strategies required for the optimal contract.

4.7 Conclusion

This chapter set out to analyse the problem of optimal capital structure when there is information asymmetry between the shareholder and the manager, leading to conflicts of interest. The main goal of the chapter was to develop an optimal capital structure that minimises the agency problem using complementary corporate governance mechanisms in a continuous-time dynamic framework. This provides results for situations in which the shareholder does not rely excessively on optimal pay or pecuniary incentives as the only strategies in contracting but develops more robust results for complementary mechanism design in a dynamic continuous-time setting. A dynamic framework provides more comprehensive optimal strategies for the shareholder and the manager throughout the tenure of the contract, and a recommendation for a dynamic optimal capital structure.

The dynamic conflicts of interest between the shareholder and the manager were specified in a differential game framework. The shareholder seeks to maximise the value of the company, while the manager seeks to maximise the pecuniary incentives accruable to them from the contract. A non-cooperative model was first constructed using the open-loop and feedback Nash equilibrium, producing closely related results. Then, a cooperative model was developed as a Pareto-efficient outcome. A numerical solution was provided using financial information from a case study company, and sensitivity analyses were carried out, based on key parameters, to illustrate the impact of uncertain input factors on the model outcome. The results of the model indicated the following:

- a. A higher asset payout rate requires the shareholder to apply a strategy of more debt repayment to reduce the free cash flow available to the manager, forcing the manager to exert more effort than at a lower asset payout rate. A higher asset payout rate increases the company's cash flow, reduces its debt level, increases the manager's cash or stock incentives, and increases productive assets.
- b. If the manager reneges on the best compromise Pareto-efficient outcome, the shareholder will increase the debt repayment strategy. Similarly, the manager will require a higher investment level to secure similar pecuniary incentives if they renege on the terms of cooperation.
- c. It is observed that at a higher asset payout rate and higher new debt rate, the company requires more debt at the start and end of the management contract.
- d. The optimal mechanism design lowers the optimal debt-equity ratio of the company, but with higher leverage required at the best compromise Pareto-efficient solution.

These results and recommendations could inform decision-making on improving company value and minimising the agency problem in a dynamic setting. The model generated quantitative implications for how firm characteristics, such as effective tax rates, asset payout rates, and new debt ratios, affect a company's optimal capital structure.

This chapter examined the agency problem between the shareholder and the manager, the next chapter focuses on the agency problem between the debtholder and the manager.

Chapter 5 – Debtholder-Manager Conflicts of Interest and the Optimal Capital Structure

5.1 Introduction

The conflicts of interest between the debtholder and the manager, defined as the agency cost of the use of debt, once again highlight the problem of determining an optimal capital structure that minimises the agency cost. This chapter focuses on modelling the principal-agent relationship between the debtholder and the manager, with the inherent conflicts of interest, to highlight good corporate governance practices for creating an optimal capital structure for a company. The agency cost of debt can be mitigated by designing an optimal capital structure as a corporate governance mechanism that specifies the optimal issuance of debt, and the proportion of debt to equity that minimises the marginal agency cost of debt.

In Chapter 4, the optimal capital structure was deployed, in addition to other corporate governance mechanisms, to mitigate the agency problem between the shareholder and the manager. The issuance of debt has been established as a disciplining measure to keep the manager focused on the shareholder's profit maximisation. However, the issuance of debt introduces new agency problems, as the debtholder now becomes an additional claimholder in the company.

An agency problem associated with a company's issuance of debt is widely known as the agency cost of debt. This introduces the problem of determining the optimal issuance of debt that could mitigate the agency cost of debt. The use of debt creates the potential for manager opportunism. This might compromise the debtholder's interests through payment of excessive dividends to shareholders and/or by committing to additional debt (Scott 2015). Significantly, one major cause of the agency cost of debt in a company's capital structure is the asset substitution problem (Green & Talmor 1986). Due to the limited liability of shareholders, debt finance provides shareholders with an impetus to select riskier projects against the preferences of debtholders (Jensen & Meckling 1976). This is termed as the asset substitution problem, in which the shareholder's desire to maximise their value increases their risk appetite. The agency cost from asset substitution results in the reduction of a company's total firm value due to

consequences of risk-shifting by the company's manager on behalf of the shareholder (Vanden 2009). This implies that the shareholder replaces the 'first-best' exercise policy of maximising the firm value with the 'second-best' exercise policy of maximising the equity value (Wang, Xu & Yang 2018). Moreover, the payoff of a shareholder is convex in the profit stream of an indebted firm, whereas the payoff of a debtholder is concave (Heinrich 2002). The problem is also significant because a company's payout policy is influenced by the extent of the agency conflicts between its shareholders and debtholder (Lepetit et al. 2018). Although agency cost does not consistently increase with the use of debt (Mao 2003), higher tax rates exacerbate the risk-shifting incentives and debt-overhang problem, while moderate tax rates decrease the same (Wang, Xu & Yang 2018). The agency conflicts of interest are also worsened by director interlocks (Ramaswamy 2019).

Unlike Brander and Poitevin (1992), but like most other debtholder-manager agency studies (e.g., Beladi & Quijano 2013), in this study the manager is assumed to act on behalf of the shareholder due to the firm's indebtedness.

Therefore, hereafter, only the manager is referenced (not the shareholder). This means that the debtholder-shareholder incentive conflict is discussed here as the debtholder-manager incentive conflict.

A debt contract can be perfectly represented as an incentive contract. In the incentive contract, the principal induces the agent by taking disciplinary actions when the agent commits a suboptimal effort. Similarly, in the debt contract, the debtholder legally obliges the firm to enforce its interest payments (cost of debt), irrespective of the firm's financial position. The debtholder can impose a penalty on the firm and its manager when cheating arises (Shah & Abdul-Majid 2019).

In the principal-agent relationship between the debtholder and the manager, the debtholder's primary objective is to recover the debt face value provided to a company and the associated cost of debt. To ensure this is realisable, the debtholder prefers that the manager does not unnecessarily shift the company's risk or substitute assets in a way that may jeopardise the debt repayment. However, the debtholder is informationally disadvantaged with respect to the manager's decisions on asset substitution. This leads to the problem of moral

hazard and information asymmetry. The sequence of events starts as follows: the debtholder provides a debt contract to the manager, subject to certain debt covenants, to ensure the fulfillment of the debt contract. The manager, however, has opportunistic tendencies to seek to maximise the company's financial income from asset substitution and minimise the total weighted average cost of capital in order to achieve an efficient capital structure. This leads to each party preferring a different goal in the company. A manager encounters the following significant issues: What is the optimal level of coupon or cost of debt and cost of equity that minimises the marginal effect of the agency cost of debt on the firm? What is the optimal capital structure required to minimise the impact of the debtholder's disciplinary actions on the firm in the case of non-cooperation? What are the incentives provided by the debtholder to discourage risk-shifting? In contrast, the debtholder faces the problem of how to maximise their investments in the company and the design of sufficient and sustainable incentives to induce the manager to protect their interest in the firm.

Using differential game theory, the goal of the non-cooperative strategic analysis of the problem was to obtain an optimal capital structure and optimal cost of debt and equity that minimises the agency problems via Nash equilibria. This provides the company with information on the optimal issuance of debt and cost of financing that it must incur to minimise the agency cost of debt if it cannot agree on a common interest with the debtholder (i.e., take the minimal risk to avoid a debt default). Since the debtholder can pre-empt the possibility of an asset substitution or risk-shifting behaviour, they incorporate this information and hedge themselves against this risk by ex-ante inflating the company's cost of debt (Lopez-Gracia & Mestre-Barberá 2015). Additionally, a cooperative solution concept was considered to minimise the agency problem using certain terms of cooperation.

The corporate governance instrument/mechanism used to resolve the conflicts of interest between the debtholder and the firm, as discussed in this chapter, is the instrument of optimal capital structure by selecting an optimal debt-equity ratio.

As discussed in this chapter, the conflicts of interest between the debtholder and the manager lead to a dynamic game throughout the debt maturity period. The main goal was to design an optimal capital structure for the company via an

optimal debt-equity ratio as a mechanism to mitigate the conflicts of interest between the debtholder and manager, as well as jointly optimise the interests of the players to resolve the agency problem. In addition, the four sub-goals were to:

- Formalise corporate governance and the debtholder-manager agency relationship as a dynamic game problem.
- Analyse the relationship as a non-cooperative game using the Nash open-loop and feedback equilibrium outcomes.
- Specify the differential game model (joint optimisation/Pareto outcome) as a dynamic contract model of corporate governance for resolving the existing agency problems when deciding on an optimal capital structure.
- Specify an optimal capital structure for the company that minimises the agency cost of debts for the company via differential game theory.
- Compare the effects of different balancing cost factors on elements of the players' payoffs.

5.1.1 Structure of the Chapter

This chapter is organised as follows: Sections 5.2 provides the rationale for conducting this part of the study. Section 5.3 presents the model setup with information asymmetry for the debtholder-manager agency relationship. Section 5.4 analyses the model as a non-cooperative dynamic game model using Nash equilibrium concepts and further obtains a Pareto-efficient outcome as a cooperative outcome. Section 5.5 provides the computational results; Section 5.6 presents the implications and insights from the study. Finally, Section 5.7 summarises and concludes the chapter.

5.2 Background Literature: Motivation for this Study

Previous studies have evaluated the significance of the risk-shifting problem via theoretical frameworks (Ericsson 2000; Leland 1998), simulation methods (Parrino & Weisbach 1999), managerial surveys (Graham & Harvey 2001), and empirical research (Eisdorfer 2008). Although equity-based compensations can align the manager and shareholder's interests in an organisation, they can increase the agency cost of debt because the debtholder anticipates more risk-

shifting due to the manager's increased equity assets in the company (Beladi & Quijano 2013). Beladi and Quijano established this in their robust study of the loan contracts of non-financial US corporations between 1990 and 2010. However, limiting equity-based compensation can reduce the manager's incentive to select the optimal effort that maximises the shareholder's value (Edmans & Liu 2011). Duran and Lozano-Vivas (2014) found evidence of risk-shifting in banks in the 2008 financial crisis. Their research also adds to the number of empirical analyses of risk-shifting, albeit for a financial firm.

A case study for the asset substitution problem was described by Copeland, Weston and Shastri (2014, p. 457) as follows:

consider a firm which plans to make an investment with end-of-period payoffs of \$200 and \$180 with equal probability. The project is financed with a debt issue with a face value of \$170. Holders of this debt can anticipate \$170 due to the expected payoffs of the project and proposed debt payment irrespective of the state of the world. However, after debt issue, assume that the firm substitutes the proposed project with another project with end-of-period payoffs of \$230 and \$150 with equal probability. The debt payment becomes risky as the bonds either receive \$170 or \$150 with equal probability. This impacts on the debtholder's wealth as a result of a loss in the bond value. The debtholder incurs a loss of \$30 with a 50% probability. Contrarily, if the shareholders are the only other significant claimholders in the firm, they can anticipate an additional \$30 with a 50% probability, and this would increase their stock value at par with the amount that decreases the bond value.

The case study described above contextualises the asset substitution problem and the impact on the debtholder. Financing policies are significant in reducing the agency cost of managerial volition (Stulz 1990). Some theoretical contributions to the literature on the elimination of the cost of asset substitution include the work of Green (1984). He suggests replacing straight debt financing with the use of convertible debt financing, while the convex and concave domains of the debt contract are stabilised to present the security locally in the form of equity. This he proposed as a solution in a single period. Hennessy and Tserlukevich (2008), however, proved this to be unrealistic in a dynamic context because equity remains risk-loving as a firm tends to bankruptcy. Although Huerga and Rodríguez-Monroy (2019) found evidence that mandatory convertible bonds can enhance shareholder value by improving financial sustainability, without increasing the default risk to the debtholder, they

concluded that it is more effective to align the interest of the manager with the debtholder. Other studies that have considered the use of capital structure design as an internal solution to resolve the asset substitution problem include Vanden (2009). Vanden suggests the use of structured financing and imitates the work of Childs, Mauer and Ott (2005), which identifies structured financing as a company having adequate financial flexibility to continuously manage its degree of short-term debts. By seeking an internal solution, this means minimising the resultant loss on total firm value.

Burkart and Ellingsen (2004), and Chod (2017) proposed trade credit as an agency-based measure to mitigate asset substitution. However, their results are limited because they only favour the possibility of lending goods rather than lending cash, which is not always a realistic alternative for companies.

Short-term debt has been recommended as one panacea to the moral hazard problem of asset substitution because this is less reactive to changes in the company's assets (Barnea, Haugen & Senbet 1980). Furthermore, it bridges the information gap between the debtholder and manager, since it spurs a frequent reporting by the manager on the company's performance and operating risk (Jun & Jen 2003).

Another solution discussed in the existing literature, such as Myers (1977), is that the productive life of a company's assets should be evened with the debt maturity offered by the creditor. This implies that companies with a high magnitude of fixed assets will select long-term debts (Stohs & Mauer 1996). This current study improves on the existing body of literature by developing a model that is flexible for analysis in both a long-term and short-term (debt maturity) period.

Lopez-Gracia and Mestre-Barberá (2015) found evidence that some Spanish small-medium enterprises (SMEs) defer to long-term debt to moderate the conflicts of interest between the manager and the debtholder as this can serve as a measure to restrain managers from improper use of the company's resources. However, their results limit the outcomes to SMEs only. Diamond (1989) suggested that reputation considerations can minimise the risk-shifting incentive over a long-time horizon.

Other solutions proposed to mitigate the asset substitution problem include debt covenants (Smith & Warner 1979), and managerial compensation contracts (Brander & Poitevin 1992). Debt covenants may limit the firm's level of investment as a covenant cannot fully distinguish between a non-rewarding and a rewarding investment, so they may unduly impede a good investment (Edmans & Liu 2011). Antill and Grenadier (2019) analysed the debtholder-manager relationship but with a focus on a manager who deliberately selects a preferred time to default. In this study, the manager finds the contract and relationship of benefit to the firm. Tran (2019) furthered the discussion on the use of debt covenants, in addition to reputation-building, as mechanisms to minimise the agency problem.

Edmans and Liu (2011) also recommended the use of debt compensation in the forms of pensions and deferred compensation in addition to equity-based compensation as an optimal solution to discourage risk-shifting or asset substitution by managers. The debt-equity ratio of the manager's compensation structure, they argued, could be 20-80, a ratio that should be lower than the company's debt-equity ratio. Moreover, only the shareholders via the compensation committee should implement this. The incentives for shareholders to enforce this include the knowledge that this can reduce the high interest rate and cost of debt covenant imposed by the debtholder. However, with a board of directors hijacked by the manager, or the assumption that shareholders have the same risk-shifting interest with the manager, this may not be implementable. Nonetheless, there has been a growth in the use of debt-based compensation, as the investment banking group, UBS, and Credit Suisse now employ the use of contingent convertible (CoCo) bonds. Arguably, the debt-based pay may not be applicable or appropriate for every company.

More recently, the dual ownership theory has also been argued as a solution to the risk-shifting problem. This postulates that if a debtholder simultaneously holds both equity and debt in a company, this can minimise the incentive conflict by increasing the debtholder's monitoring scope and internalising the conflict (Sudheer, Wang & Zou 2019). The limitations of this proposition, however, are: debtholders will not always seek an equity interest in a company; not all debt providers have the legal rights to buy equity interests; and not all firms will be

willing to sell equity interests to their providers in order to avoid excess monitoring.

Eisdorfer (2010) found proof that risk-shifting can be more beneficial to shareholders than originally recognised. If firm owners can modify the asymmetry of the firm's future value, then they could benefit from 'two birds in hand', that is, deriving intrinsic value from risk-shifting, while minimising the firm's total risk, equity risk and bankruptcy risk. The limitations of Eisdorfer's (2010) study, however, are: its applicability to a two-date period (static setting); it constrains the proposition to only non-symmetry investment pay-off distribution, which is usually prevalent in financial markets investments; and, finally, it imposes higher risk on the debtholder, thereby retaining the well-known consequences for the debtholder.

The conflicts of interest between the debtholder and the manager, and the asset substitution problem has been well established, with several theoretical contributions made in an attempt to eliminate the problem, as reviewed above. However, there is limited research available that integrates the conflicts of interest emanating from the use of debt and the optimisation of the firm's capital structure, particularly via the design of a mathematical model as a corporate governance model. Many studies, such as Allen and Morris (2014), Bhattacharya (1979), and Kunz (2018), have developed game-theoretic analyses of agency issues in management accounting and corporate finance, with some discussions on the optimal capital structure. Some studies have developed a model based on the risk-shifting or asset substitution problem, including Moreno-Bromberg and Rochet (2018), and Vanden (2009). Moreno-Bromberg and Rochet's model captures the effect of the risk-shifting problem on the firm's cash flow (the state variable) via a higher payout rate. The limitation of this model, however, is that it only strives to establish the impact of the risk-shifting problem on the firm value but does not attempt to resolve the problem. On the other hand, Vanden developed a model to resolve the asset substitution problem using structured financing by correlating the relevant payout features with the issuing firm's debts. The limitation of this model is that while it seeks an internal solution for the company to minimise the impact observed by the debtholder, it eliminates the tax effect in the model design.

Eichberger and Harper (1997) also developed a standard debt contract that incorporated incentive compatible constraints. These constraints suggest that the loan repayment must be made to the debtholder, irrespective of the company's outcomes in each period. The difference between their model and the current study is that their model assumes the manager to be an entrepreneur with zero funds to repay their loans. This is an unrealistic assumption in the real world as a company is assumed to be a going concern. Moreover, their model focused on an incentive compatible contract that achieved the wealth maximisation goal of the debtholder with little consideration of the asset substitution problem and the consequences for the firm.

This study builds upon the existing research, filling the gaps and developing a corporate governance mechanism that minimises the conflicts of interest between the debtholder and the manager while, at the same time, optimising the capital structure of the firm.

The foremost agency problem discussed in this chapter is the moral hazard problem, that is, the actions of each player are unobservable to the other player. The debtholder is unable to observe the manager's unutilised effort and whether the manager takes appropriate actions to repay the debt finance. A lack of observable actions makes it difficult for the debtholder to assess the manager's productivity level. These asymmetries of information describe the relationship between the two agents (the debtholder and the manager) as a game of incomplete information (Amaro de Matos 2001).

Differential game theory was adopted because of its suitability in analysing non-cooperative games, as specified above. In a differential game, the objective of one decision-maker (debtholder or manager) impacts on the objective of the other, hence the problem becomes a game (de Zeeuw 2014). In engineering studies, such as those focused on military pursuit game modelling, zero-sum games are considered more applicable. By contrast, in the economics literature, players often have common and opposite utility functions, hence non-zero-sum differential games are more applicable (Van der Ploeg & de Zeeuw 1991).

This study adds to the body of research on agency relationships in corporate finance. While the agency relationship between the shareholder and manager

has enjoyed much attention, the agency relationship between the debtholder and manager has been largely neglected, despite the significance of the associated agency cost of debts.

5.3 Model Setup

The goal of this section is to specify a suitable modelling framework for the applicable decision-making undertakings fundamental to the conflicts of interest between the manager and the debtholder, using data and variables from financial accounting. Using a differential game framework, the different objectives of the debtholder and manager were established as an objective functional for each player, thus supplying two different objective functionals. Each objective functional includes control variables/strategies, state variables and other constant parameters. The objective functionals reveal the contradictory interests of both players and thus carry significant information.

Once the objective functionals were specified, they were then governed by certain state equations that affect the relationship established in the model. These state equations, specified as differential equations, incorporate a major part of the dynamism of the model as a continuous-time model. They also reveal the information about the evolution of the state variables. Once the model was established, it was then solved initially as a non-cooperative dynamic game, as it was presumed that the manager and the debtholder do not completely agree on the same interest or wealth maximisation strategies, but yet must develop strategies that can optimise each player's objective. Next the Pareto outcome was considered as a further step to achieve cooperation, by providing incentives to align the manager's interests to the debtholder. Therefore, in the non-cooperative game analysis, the primary mechanism design is the optimal capital structure that provides the optimal use of debt to equity and the optimal cost of finance required by the company to minimise the company's marginal agency cost of debt. In the cooperative analysis of the model via the Pareto-efficient outcome, the agency cost of debt is minimised by the design of mechanisms, such as the use of punitive measures to disincentivise the manager from asset substitution.

The results obtained were solved partly analytically, as well as numerically to apply the model to a case study company. The numerical code was simulated in the MATLAB2020a(64-bits) programming environment. The numerical algorithm was devised to generate an approximation for a pair of Nash equilibrium piecewise continuous strategies to yield the optimal state value and optimal debt-equity ratio for the non-cooperative game analysis. Similarly, this produced optimal results for the Pareto case. The fourth order RK4 numerical method was used to solve the boundary value problem using the forward backward sweep approach. The error in the optimal strategies needed to be negligibly small by setting delta to 0.001. The results obtained were computed graphically, discussed and sensitivity analysis was carried out to provide the implications of the model.

In the design of the game model between the debtholder and the manager, a key assumption is that the company only issues limited-liability securities (stocks and loans). These loans may include bilateral loans, finance lease obligations, and US private placements (USSP).

The players in the continuous-time game between the manager and debtholder are listed in Table 5.1, as follows.

Table 5.1 List of players in the game

In the game model, the players $i = 1, 2$ are outlined as:	
1.	Manager (Agent)
2.	Debtholder (Principal)
	In the finite time horizon game, $N=2$ players act over a bounded interval of time $[t_0, T]$ and within the systems described by the vector of state variables, liquid reserve and debt value $M(t), D(t)$. The values of these variables are given at each instant t by a point in $M(t), D(t)$. The evolution of these state variables follows certain vector differential equations, where the vectors $l(t), c(t)$ of control variables are at the players' disposition. Each player selects a control path for his vector of control variables, a piecewise continuous function $l(t), c(t)$ on $[t_0, T]$.

5.3.1 State Variables and Systems of the Game (State Equations)

The state variables are a set of variables that describe the mathematical state of the dynamical system of the model. These are financial accounting variables involved in the strategic interactions between the debtholder and the manager and thus form elements of the debt contract. They provide information about the outcome of the game's strategic interaction. The systems of the game are state equations that govern the behaviour of the specified state variables. The state variables and their systems are now described below.

I. Company's liquid reserve

The company's liquid reserve is very significant because it covers the company's ongoing operating expenses, such as its cost of debt or current finance cost. The liquid reserve $M(t)$, otherwise tagged as current asset, evolves by adding the operating income $\beta S(t)$, the financial income $rM(t)$ (liquid reserve is assumed to be remunerated at rate r) minus cost of debt $c(t)$ and the cost of equity $l(t)$. This evolution of $M(t)$ can be referred to as the company's net earnings, stated as:

$$\dot{M}(t) = \beta S(t) + rM(t) - c(t) - l(t) \quad (5.1)$$

II. Value of debt

A simple tax setting is considered. The firm's income is taxed at the effective tax rate θ , when $\theta > 0$, the use of debt shields some of the firm's income from tax charges. $c(t)$ denotes the cost of debt associated with the use of debt $D(t)$ at any time t . It is assumed that the company's value of debt changes throughout the lifecycle of the firm depending on its need for new financing in the next period. Thus, the capital structure is dynamic, a distinction from most capital structure models. However, based on the agency relationship between the debtholder and the manager, the debtholder may provide more or less debt facility to the firm depending on the manager's discretion to act opportunistically or not in a previous period. Hence, more debt facility may serve as an incentive. Thus, $D(t)$ reflects the company's total long-term indebtedness at any given point in time, therefore it is crucial to the capital structure discussion. The company's value of debt is defined as its cost of debt plus its need for new debt, where α represents the ratio of the new value of debt to the existing market value of debt.

$$\dot{D}(t) = \alpha D(t) + c(t) \quad (5.2)$$

III. Productive assets

The company's productive asset impacts the value of the company and its equity in any period. The value of the company's productive assets $S(t)$ is assumed to grow or decline exponentially depending on the difference between the riskless rate (r) and the payout rate (β):

$$S(t) = S_0 e^{(r-\beta)t}, \quad (5.3)$$

where S_0 is the initial productive asset.

IV. Value of equity

Building on the work of Moreno-Bromberg and Rochet (2018), the fundamentals of a non-financial corporation are observed via its balance sheet, which is a summary of a company's assets, liabilities and capital at a specific point in time. The company's total assets are expected to equal its liabilities at every point in time. In a company's statement of financial position, total equity $E(t)$ is defined as:

$$E(t) = M(t) + S(t) - D(t) \quad (5.4)$$

One of the frequently argued weaknesses of the effectiveness of the weighted average cost of capital is that it does not always reflect the true overall cost of capital. This is due largely to the difference in book value and market value (Beranek 1977; Reilly & Wecker 1973). This weakness is managed in this model by using the accounting equation for $E(t)$, rather than specifying the book value from the financial statements.

Equations (5.1 - 5.4) above are the sets of equations describing the paths of the control variables and the evolution of the state variables, $D(t), M(t)$. In the two-person differential game, the players influence the evolution of these state variables.

5.3.2 Manager's Interest, Strategy and Objective Functional

As established, the manager has an opposing interest to the debtholder. The manager's objective is to minimise the company's cost of finance and maximise the value from its asset substitution. The weighted average cost of capital is a

compelling financial theory that is applied extensively by both investors and company management. It is referred to as the cost of financing a company's activities, otherwise known as the cost of capital. This is the minimum return a company must realise on its capital asset base as anticipated by its providers of capital (Reilly & Wecker 1973). Therefore, the primary financial goal of a company is to find the optimal capital structure that yields the lowest weighted average cost of capital and maximises the value of the company (Zelgalve & Bērzkalne 2011).

Acknowledging that the weighted average cost of capital is a cost function the manager seeks to minimise; this is set as the objective functional of the manager while maximising the financial income of the company $rM(t)$ and the rate of return on the company's liquid reserve from asset substitution. To achieve an optimal capital structure, it is assumed that the manager prefers the responsibility of cost of equity $l(t)$ (otherwise known as dividends) to the responsibility of cost of debt (interest payment) $c(t)$. Cost of debt increases the performance pressure on managers and requires more measurable effort (Harris & Raviv 1988). While the cost of equity may be avoided in an extreme case of decline in earnings due to the limited liability of shareholders, or as a result of jointly agreed-upon reinvestment decisions, the cost of debt must be met. In addition, the manager prefers to dilute the company's shares when they fear overreliance on debt. Therefore, the manager chooses the cost of equity $l(t)$ as their control variable/strategy.

As described above, the objective functional of the manager over a finite time horizon is:

$$J_1 = \min \int_0^T e^{-\rho t} (\omega_1 \frac{E(t)}{V(t)} l(t)^2 + \omega_2 \frac{D(t)}{V(t)} c(t)^2 (1 - \theta) - \omega_3 rM(t)) dt \quad (5.5)$$

The ratio of the company's capital $V(t)$ financed by equity $E(t)$ can be represented as $\frac{E(t)}{V(t)} = \mu(t)$, such that the remaining ratio financed by debt $D(t)$ is

$$\frac{D(t)}{V(t)} = 1 - \mu(t).$$

The objective functional of the manager over a finite time horizon is therefore stated as:

$$J_1 = \min \int_0^T e^{-\rho t} (\omega_1 \mu(t) l(t)^2 + \omega_2 (1 - \mu(t)) c(t)^2 (1 - \theta) - \omega_3 rM(t)) dt \quad (5.6)$$

Where $\omega_1, \omega_2, \omega_3 > 0$ are balancing cost factors, these costs incorporate other transaction and operational costs associated with implementing financial decisions in a firm. A constant discount rate $\rho \geq 0$ is assumed. This can be tagged as the current-value formulation of the problem. The debtholder's objective functional is introduced next.

5.3.3 Debtholder's Interest, Strategy and Objective Functional

The risk-averse debtholder provides the company with debt finance based on the company's market value, credit rating, existing relationship with the debtholder, and other available information in the financial market. These statistics are used by the debtholder to categorise the borrower as a safe borrower. The debtholder, who is assumed to be secure and senior, ultimately seeks to maximise the principal value of debt issued to the company at $t = 0$. This increases over time, depending on the company's debt acquisition ratio α , while minimising the monitoring costs of obtaining their interest payments. When the firm's debt is not fully secured, the secured debtholder is preferred over the ordinary debtholder as they can withhold the assets used as collateral (Tirole 2006). The debtholder evidently selects the cost of debts $c(t)$ - (their interest payments) as their control variable/strategy to maximise their interest. The principal value of debt and the cost of debt accrued are the fixed claim available to the debtholder (Sudheer, Wang & Zou 2019). The debtholder's payoff functional is specified as:

$$J_2 = \max \int_0^T e^{-\rho t} (\gamma_2 D(t) - \gamma_1 c(t)^2) dt \quad (5.7)$$

where $\gamma_1, \gamma_2 > 0$ are balancing cost factor.

The contradicting objective functionals of the two players have been specified, next, the state variables and equations are discussed.

5.3.4 Discussion of the Model

The manager's information advantage about the value, productivity and decision-making of the company leads to the information asymmetry and agency problem of asset substitution. This causes the manager to uphold the contradictory interest of asset substitution, in contrast to the wealth maximisation goal of the debtholder. The goal of this study was to establish the conflicts of interest

between the debtholder and the manager as a differential game problem and obtain the optimal capital structure to minimise the agency cost of debt as a corporate governance mechanism. To achieve this goal, the interest of each player was established as an objective functional in a continuous-time horizon, as seen in Equations (5.6) and (5.7). Equations (5.6) and (5.7) above establish a difference in the interests of the manager and the debtholder. In these equations, each player seeks to optimise their interest over a finite time period. The designed objective functionals specify the agency problem. The goal of the manager is to minimise the company's cost of financing and maximise its financial income from asset substitution, while the debtholder's aim is to recoup the debt contracted while minimising the monitoring costs of obtaining their interest repayments. The debt face value and cost of debt are the maximum benefits a debtholder can make from a firm. There is no extra value accrued to the debtholder from the increase in the company value. However, a decline in company value may signal the likelihood of a debt default. In each period t , each player pursues their own objective.

The objectives specified in Equations (5.6) and (5.7) are governed by elements of the capital structure. Following the theory of differential game and optimal control, each player is able to select their optimal or Nash strategy via an equilibrium combination of $c(t)$ and $l(t)$. The optimal strategies and state variables result in the optimal capital structure $D(t)/E(t)$ that minimises the conflicts of interest between the debtholder and the manager.

As a dynamic game model, the results of the model present dynamic issues of the over-time strategic interactions of the two players, such as the trigger strategies proposed by the debtholder to ensure that the optimal contract is not renegotiated. Moreover, these relevant dynamic issues relate to the dynamic aspects of the agency problem over time.

5.3.4.1 *Balancing cost factors*

In specifying the objective functionals, it is presumed that there are certain costs associated with optimising elements of these functionals. Therefore, $\omega_1, \omega_2, \omega_3$ are set as the balancing cost factors of the manager's objective functional. Similarly, γ_1, γ_2 are set as the balancing cost factors for the debtholder's objective

functional. These balancing cost factors are also known as weight on the cost (Agusto & Leite 2019) of achieving the objective functionals via the optimal strategies. $\omega_1, \omega_2, \omega_3$ are specified as inherent transaction and operational costs incurred by the manager in order to meet their finance costs and maximise their financial income. Similarly, debtholders face an opportunity cost per unit of capital provided to a company (Eichberger & Harper 1997). Thus, the debtholder incurs an opportunity cost γ_2 to provide $D(t)$. Additionally, the debtholder incurs monitoring costs γ_1 to recover the cost of debt $c(t)$ from the manager, as debtholders cannot monitor the state of the debt contract game without costs. This includes legal and auditing fees (Eichberger & Harper 1997). It is noted that the values of the weight assigned to the balancing cost factors are merely theoretical for illustrative purposes.

In summary, the model is set out as:

$$J_1 = \min \int_0^T e^{-\rho t} (\omega_1 \mu(t) l(t)^2 + \omega_2 c(t)^2 (1 - \theta)(1 - \mu(t)) - \omega_3 r M(t)) dt$$

$$J_2 = \max \int_0^T e^{-\rho t} (\gamma_2 D(t) - \gamma_1 c(t)^2)$$

Subject to:

$$\dot{M}(t) = \beta S(t) + rM(t) - c(t) - l(t), M(0) = M_0; S(0) = S_0$$

$$\dot{D}(t) = \alpha D(t) + c(t), D(0) = D_0$$

$$S(t) = S_0 e^{(r-\beta)t}, S(0) = S_0$$

$$E(t) = M(t) + S(t) - D(t)$$

$$\frac{\dot{D}(t)}{\dot{E}(t)} = \frac{\alpha D(t) + c(t)}{\beta S(t) + rM(t) - c(t) - l(t) + S_0 e^{(r-\beta)t} - \alpha D(t) - c(t)}$$

Following this definition for $E(t)$, $\mu(t) = 1 - \frac{D(t)}{M(t)+S(t)}$

At the start of the game, $t = 0$, the manager (agent) requests a debt asset to maximise the company's earnings, while the debtholder provides one. Over the course of the game, the debtholder (principal) becomes interested in the profitability of the company and its liquidity position and fears that the manager may make decisions that affect debt repayment. Observing the available information about the company's liquid reserve, value of debt, productive assets,

and value of equity informs the debtholder about the ability of the company to pay its debts or default. As the game evolves, the manager increases the value of debt at the minimum rate of α . As the company's value of debt increases, the agency cost of debt may increase. The optimal capital structure provides the optimal issuance of debt that minimises the marginal agency cost of debt to the company. The game is assumed to last until the company's debt maturity, thereby making the model flexible for short-term maturity and long-term debt maturity. A summary of the game is presented in Table 5.3, as follows.

Table 5.2 A summary of the game

	Description and features of the game
Game	Two-person non-zero-sum game
Players	Player 1: Manager (Agent) Player 2: Debtholder (Principal)
Period of the game	The game is played over time, and a single period in the over-time horizon is reckoned as a year, that is $t = 1$. This is because momentous decisions such as issuance of new debts or review of debt contract rarely happen in shorter periods (e.g., monthly). The final time T is taken as $T = 10$.
Strategies	Player 1: Cost of equity Player 2: Cost of debt
Payoffs	Player 1: $J_1 = \min \int_0^T e^{-\rho t} (\omega_1 \mu(t) l(t)^2 + \omega_2 c(t)^2 (1 - \theta)(1 - \mu(t)) - \omega_3 r M(t))$
	Player 2: $J_2 = \max \int_0^T e^{-\rho t} (\gamma_2 D(t) - \gamma_1 c(t)^2)$

Moves	In the finite horizon differential game, the debtholder P_2 makes the first move by offering a debt contract to the manager. The manager initially accepts the terms and conditions of the contract but has incentives to do otherwise, most importantly taking actions that can cause it to default on the debt. As a result, the debtholder takes a keen interest in the activities of the firm thereby incurring additional monitoring costs and proposing to renegotiate the contracts when the information obtained via monitoring or financial statements signals a default. This initiates the game and the conflicts of interest therein. The game is said to be played simultaneously.
Information available in the game	The game is said to be one with perfect information because the actions of the manager are somewhat observed through the state variables $(M(t), D(t))$ of the game. This becomes available after the beginning of play, by reviewing the financial information of the company. However, the game is said to be with incomplete information because the preferences of one player are not totally known to the other player.

Once the differential game was established as a mathematical model, it was then analysed and solved via adequate equilibrium concepts, first as a non-cooperative game using the Nash open-loop and Nash feedback; and second, as a cooperative game using the Pareto solution concept to obtain the equilibrium strategies required by each player to optimise their utility functions.

Table 5.3 Parameters used in the model

S/N	Parameters	Definition
1	r	Riskless rate $r > 0$, assumed to be constant
2	β	Payout rate of a company's productive assets, assumed to be constant
3	$c(t)$	Cost of debt (interest rate)
4	$l(t)$	Cost of equity (dividend)
5	$E(t)$	Value of company equity
6	$M(t)$	Liquid reserve
7	$S(t)$	Value of company's productive assets
8	$D(t)$	Value of company debt
9	$V(t)$	Company's total capital: $V(t) = E(t) + D(t)$
10	θ	Effective tax rate
11	α	Ratio of new debt to existing value of company debt
12	ρ	Discount rate

5.4 Model Solutions

5.4.1 Non-cooperative Game Analysis - Nash Equilibrium

5.4.1.1 *Optimal decisions with the open-loop Nash equilibrium solution*

In a non-cooperative game, the debtholder does not support the manager's intention to maximise the company's value from asset substitution. The open-loop Nash equilibrium solution concept assumes that the only available information for action at time t is that of the initial state $M(0)$ and $D(0)$. The players in the game seek to optimise their interests at time 0 and do not alter their strategies afterwards based on current information in the game. In this model, this means that the manager and debtholder do not review their strategies based on considerations of the company's earnings, assets and debt level over time, but rather focus on their initial policies on the cost of debt, cost of equity and anticipated earnings. The open-loop information system is illustrated in Figure 5.1 below.

Open-loop System of the Game

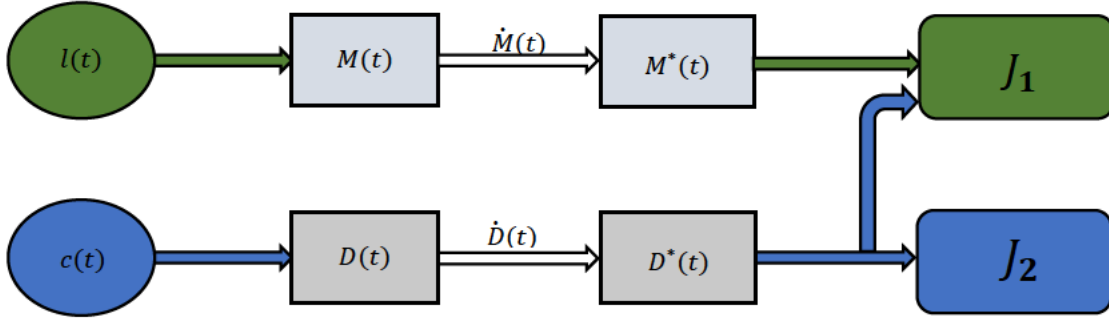


Figure 5.1 Open-loop system of the game

As shown in Figure 5.1, both players' strategies $l(t)$ and $c(t)$ influence the states of the game $M(t), D(t)$. The systems of the game, $\dot{M}(t), \dot{D}(t)$ react to the information from the strategies and states of the game. They produce equilibrium state trajectories at the Nash pair of strategies and a player cannot improve their outcome (J_1, J_2) if they move from this strategy while the other player sticks to theirs.

The Hamiltonians are derived by adjoining the state equations to the objective functional for each player with adjoint or co-state functions, $\lambda_j, j = 1, 2$ for player 1 (the manager) and $\phi_j, j = 1, 2$ for player 2 (the debtholder, where j represents the number of state variables). Therefore, Hamiltonians for players 1 and 2 are defined as:

$$\mathcal{H}_1(M, D, l, c, \lambda_j, t) = f_i(M, D, l, c, t) + \lambda_j g(M, D, l, c, t)$$

$$\mathcal{H}_2(M, D, l, c, \phi_j, t) = f_i(D, c, t) + \phi_j g(M, D, l, c, t)$$

For the purpose of brevity, the subscript t is taken off the vectors:

$$H_1 = \frac{\omega_1 l^2 (M+S-D)}{M+S} + \frac{\omega_2 c^2 D(1-\theta)}{M+S} - r\omega_3 M + \lambda_1(\beta S + rM - c - l) + \lambda_2(\alpha D + c) \quad (5.8)$$

$$H_2 = -\gamma_2 D + \gamma_1 c^2 + \phi_1(\beta S + rM - c - l) + \phi_2(\alpha D + c) \quad (5.9)$$

where J_2 is multiplied by minus to change the maximisation problem to a minimisation problem.

Theorem 5.1

The open-loop Nash equilibrium solutions are:

$$l^* = \frac{\lambda_1(M+S)}{2\omega_1(M+S-D)} \quad (5.10)$$

The optimal cost of equity for the firm from Equation (5.10) is the ratio of the value of an added dollar of debt or earnings multiplied by the firm's total assets, to the firm's equity multiplied by two times the balancing cost factor of the use of equity at any time t . This calculation means that the higher the weight on the cost of implementing equity, the lower the ratio of the company's total assets to its equity.

$$c^* = \frac{1}{2\gamma_1}(\phi_1 - \phi_2) \quad (5.11)$$

This calculation means that the company's optimal cost of debt is a function of the company's liquid reserve and current debt value. This means that the higher the difference in the marginal rate of change of the liquid reserve and debt value, the higher the company's cost of debt. From Equation (5.11), the optimal cost of debt for the firm is the ratio of the value of an added dollar of debt to the cost of financing the interest charges times two. In contrast to the results of Elton and Gruber (1974), in which the cost of equity funds equals the cost of debt funds, in this new study, when the conflicts of interest between the debtholder and the manager are introduced into the optimal capital structure model, the cost of equity is found to be less than the cost of debt.

Equations (5.10) and (5.11) are the characterisations of the Nash strategies.

Proof of Theorem 5.1

The necessary conditions establish the equilibrium time path for the variables. The **first** condition requires that each control variable selected at any time t must have an optimising effect on the Hamiltonian. This implies:

$$\frac{\partial H_i(t)}{\partial l(t) \text{ or } \partial c(t)} = 0 \quad \text{for all } i = 1, 2 \text{ and all } t$$

The equilibrium conditions are:

$$\frac{\partial H_1}{\partial l} = \frac{2\omega_1 l(M+S-D)}{(M+S)} - \lambda_1 = 0$$

$$l^* = \frac{\lambda_1(M + S)}{2\omega_1(M + S - D)}$$

$$\frac{\partial H_2}{\partial c} = 2\gamma_1 c - \phi_1 + \phi_2 = 0$$

$$c^* = \frac{1}{2\gamma_1}(\phi_1 - \phi_2)$$

End of Proof.

To check for concavity conditions:

$$\frac{\partial^2 H_1}{\partial l^2} = \frac{2\omega_1(M + S - D)}{(M + S)} \geq 0,$$

since ω_1 is a non-negative balancing cost factor and financially, $M + S - D$ which means the value of equity cannot be less than zero, $\frac{\partial H_1}{\partial l^2} \geq 0$, the problem is indeed a minimisation problem.

Similarly,

$$\frac{\partial^2 H_2}{\partial c^2} = 2\gamma_1 \geq 0,$$

since γ_1 is a non-negative balancing cost factor, therefore, it is indeed a minimisation problem.

The **second** necessary condition requires the rate of change with respect to time in each co-state variable to be equivalent to the negative of the partial derivative of the Hamiltonian with respect to the associated state variable. The adjoint variable can be defined as a marginal change associated with an increment in each state variable for each player. This means that, if one dollar of debt or liquid reserve is added to the company's finance portfolio, for the manager, this corresponds to the effect on the company's cost of capital and, for the debtholder, the impact on a debt default tendency.

From the Hamiltonian functions (5.7) and (5.8), the adjoint equations are:

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H_1}{\partial M} = (\rho - r)\lambda_1 + \omega_3 r - \frac{D(\omega_1 l^2 - \omega_2 c^2(1-\theta))}{(M+S)^2} \quad (5.12)$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H_1}{\partial D} = (\rho - \alpha)\lambda_2 + \frac{\omega_1 l^2 - \omega_2 c^2(1-\theta)}{(M+S)} \quad (5.13)$$

$$\dot{\phi}_1 = \rho\phi_1 - \frac{\partial H_2}{\partial M} = \phi_1(\rho - r) \quad (5.14)$$

$$\dot{\phi}_2 = \rho\phi_2 - \frac{\partial H_2}{\partial D} = \phi_2(\rho - \alpha) + \gamma_2 \quad (5.15)$$

Also, these co-states functions satisfy the terminal conditions:

$$\lambda_1(T) = 0 \quad \lambda_2(T) = 0 \quad \phi_1(T) = 0 \quad \phi_2(T) = 0$$

The terminal conditions for the adjoint variables denote the future value (shadow price) of a marginal change in the state variables at the final time (Goetz et al. 2019).

The **third** condition is basically that the state equations are achieved:

$$\dot{M} = \beta S + rM - c - l, \quad M(0) = M_0 \quad (5.16)$$

$$\dot{D} = \alpha D + c, \quad D(0) = D_0 \quad (5.17)$$

Equations (5.10 - 5.17) above are identified as the optimality system. The optimality system, which generates the equilibrium outcomes, is a forward-backward system of differential equations stated as follows:

$$\dot{M} = \beta S + rM - c - l, \quad M(0) = M_0 \quad (5.18)$$

$$\dot{D} = \alpha D + c, \quad D(0) = D_0 \quad (5.19)$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H_1}{\partial M} = (\rho - r)\lambda_1 + \omega_3 r - \frac{D(\omega_1 l^2 - \omega_2 c^2(1-\theta))}{(M+S)^2} \quad (5.20)$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H_1}{\partial D} = (\rho - \alpha)\lambda_2 + \frac{\omega_1 l^2 - \omega_2 c^2(1-\theta)}{(M+S)} \quad (5.21)$$

$$\dot{\phi}_1 = \rho\phi_1 - \frac{\partial H_2}{\partial M} = \phi_1(\rho - r) \quad (5.22)$$

$$\dot{\phi}_2 = \rho\phi_2 - \frac{\partial H_2}{\partial D} = \phi_2(\rho - \alpha) + \gamma_2 \quad (5.23)$$

$$\lambda_1(T) = 0 \quad \lambda_2(T) = 0 \quad \phi_1(T) = 0 \quad \phi_2(T) = 0$$

with Nash equilibrium strategies:

$$l^* = \frac{\lambda_1(M+S)}{2\omega_1(M+S-D)} \quad (5.24)$$

$$c^* = \frac{1}{2\gamma_1}(\phi_1 - \phi_2) \quad (5.25)$$

Next, some of the optimal state, control and adjoint variables were obtained analytically.

The adjoint Equations (5.22) and (5.23) are independent of other unknown variables and hence can be solved analytically. First, Equation (5.22) is:

$$\dot{\phi}_1 = (\rho - r)\phi_1$$

and the solution is:

$$\phi_1 = k_1 e^{(\rho-r)t}$$

where k_1 is the constant of integration, and solving for the constant of integration using the terminal (transversality) condition gives:

$$\phi_1 = 0 \tag{5.26}$$

From Equation (5.26), $\phi_1 = 0$ because the value of an added dollar of debt does not directly impact the system of $M(t)$ in the model.

For Equation (5.23):

$$\dot{\phi}_2 = \frac{d\phi_2}{dt} = (\rho - \alpha)\phi_2 + \gamma_2$$

Using the integrating factor method of integration for $\rho \neq \alpha$, where $e^{-(\rho-\alpha)t}$ is the integrating factor, results in:

$$\phi_2 = \frac{-\gamma_2}{(\rho - \alpha)} + k_2 e^{(\rho-\alpha)t}$$

From the transversality condition $\phi_2(T) = 0$, the constant of integration k_2 is determined. Hence,

$$k_2 = \frac{-\gamma_2}{(\rho - \alpha)} e^{-10(\rho-\alpha)}$$

Therefore:

$$\phi_2 = \begin{cases} \gamma_2(t - T), & \rho = \alpha \\ \frac{\gamma_2(e^{(\rho-\alpha)(t-T)} - 1)}{(\rho - \alpha)} & \rho \neq \alpha \end{cases} \tag{5.27}$$

From Equation (5.24), the Nash strategy $c(t)$ associated with the debtholder is:

$$c(t) = \begin{cases} \frac{\gamma_2(T-t)}{2\gamma_1}, & \rho = \alpha \\ \frac{\gamma_2(1-e^{(\rho-\alpha)(t-T)})}{2\gamma_1(\rho-\alpha)} & \rho \neq \alpha \end{cases} \quad (5.28)$$

The optimal outcome for the state variable $D(t)$ from Equation (5.16) is solved as follows:

$$\dot{D} = \frac{dD}{dt} = \alpha D + c$$

$$\frac{dD}{dt} = \alpha D + \frac{\gamma_2[1-e^{(\rho-\alpha)(t-T)}]}{2\gamma_1(\rho-\alpha)}$$

Using the integrating factor method of integration, where $e^{-\alpha t}$ is the integrating factor:

$$D(t) = k_3 e^{\alpha t} - \frac{\gamma_2}{2\gamma_1(\rho-\alpha)} \left[\frac{1}{\alpha} + \frac{e^{e^{(\rho-\alpha)(t-T)}}}{(\rho-2\alpha)} \right] \quad (5.29)$$

The initial value of $D(t)$ is obtained as $D(0) = 0.27$, hence:

$$k_3 = 0.27 + \frac{\gamma_2}{2\gamma_1(\rho-\alpha)} \left[\frac{(\rho-2\alpha) + \alpha e^{-T(\rho-\alpha)}}{\alpha(\rho-2\alpha)} \right] \quad (5.30)$$

Assume $a = \frac{\gamma_2}{2\gamma_1(\rho-\alpha)}$ and $b = \frac{\gamma_2 e^{-T(\rho-\alpha)}}{2\gamma_1(\rho-\alpha)}$

Therefore:

$$D^*(t) = 0.27 e^{\alpha t} + \frac{a}{\alpha} (e^{\alpha t} - 1) + \frac{b}{(\rho-2\alpha)} (e^{\alpha t} - e^{(\rho-\alpha)t}) \quad (5.31)$$

The results from the open-loop can be substantiated by considering the feedback Nash equilibrium solution concept, as discussed next.

5.4.1.2 *Optimal decisions with the feedback Nash equilibrium concept*

Unlike the open-loop Nash case, which only relies on the initial state information, the feedback Nash equilibrium uses information about the current state of the game in addition to the initial state or remains memoryless. The feedback system of the game is illustrated in Figure 5.2 below.

Feedback System of the Game

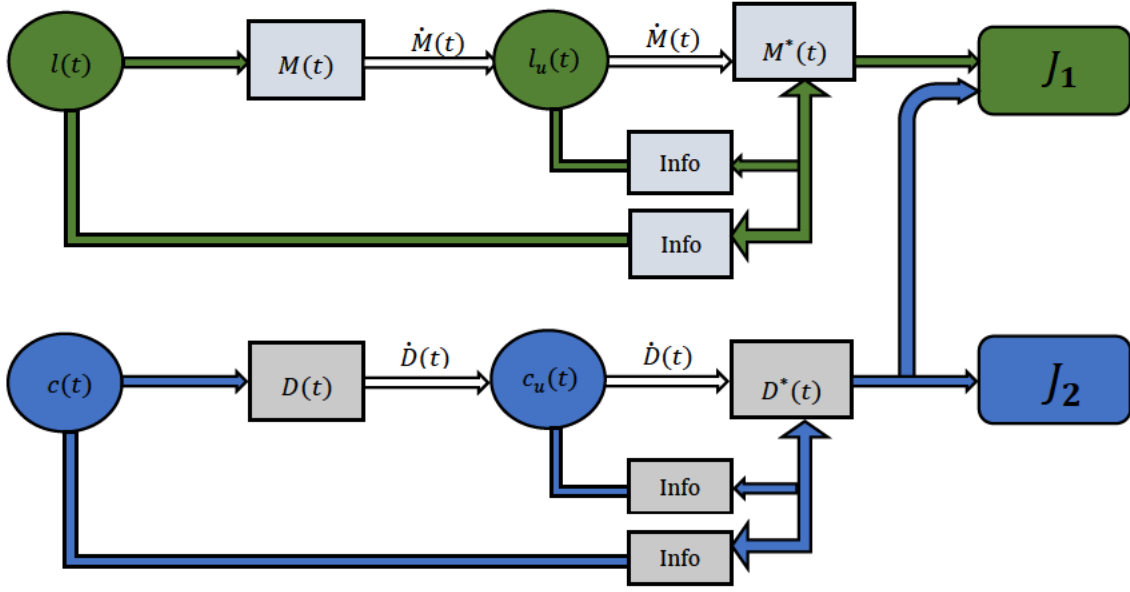


Figure 5.2 Feedback system of the game

In the open-loop outcome of the game, players act upon the initial strategies of other players in the game $l(t)$ and $c(t)$ alone, noted as pre-commitment. In contrast, in the feedback system of the game, the states of the game $M(t), D(t)$ are functions of the initial strategies. However, players get the opportunity to consider the current states of the game at any time t , align their strategies with this information, and therefore update their strategies. Hence, the equilibrium state trajectories and final utility of the players are functions of information (Info) from the initial strategies $l(t), c(t)$ and updated strategies $l_u(t), c_u(t)$. This feedback information system is reflected in the solution method as a cross-derivative of the Nash strategy of one player in the Hamiltonian of the other.

The set of necessary conditions to be satisfied in the feedback Nash equilibrium case is similar to those of the open-loop Nash Equilibrium case.

The Hamiltonians are presented as before:

$$H_1 = \frac{\omega_1 l^2 (M+S-D)}{M+S} + \frac{\omega_2 c^2 D (1-\theta)}{M+S} - r\omega_3 M + \lambda_1 (\beta S + rM - c - l) + \lambda_2 (\alpha D + c)$$

$$H_2 = -\gamma_2 D + \gamma_1 c^2 + \phi_1 (\beta S + rM - c - l) + \phi_2 (\alpha D + c)$$

The derivations of the equilibrium conditions are similar to Theorem 5.1, as specified for the open-loop case:

$$\frac{\partial H_1}{\partial l} = \frac{2\omega_1 l(M + S - D)}{(M + S)} - \lambda_1 = 0$$

$$l^* = \frac{\lambda_1(M + S)}{2\omega_1(M + S - D)}$$

$$\frac{\partial H_2}{\partial c} = 2\gamma_1 c - \phi_1 + \phi_2 = 0$$

$$c^* = \frac{1}{2\gamma_1}(\phi_1 - \phi_2)$$

However, a significant difference exists in the adjoint equations for the open-loop and feedback cases. The adjoint equations for each player in the feedback case incorporate the response of the other player to changes in the state variables, thereby impacting the decision-making of that player. This is expressed as a cross-derivative, thereby updating the Nash pair of strategies for both players as necessary. This explains how each player feeds existing information in the game back into their decision-making process.

From the Hamiltonian function (5.7) and (5.8), the adjoint equations are:

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H_1}{\partial M} - \frac{\partial H_1}{\partial c} \frac{\partial c^*}{\partial M}$$

$$\begin{aligned} \dot{\lambda}_1 = (\rho - r)\lambda_1 + \omega_3 r - \frac{D(\omega_1 l^2 - \omega_2 c^2(1-\theta))}{(M+S)^2} + \left[\frac{2\omega_2 c(1-\theta)D}{(M+S)^2} \right] \left[\frac{2\omega_2 c(1-\theta)D}{(M+S)} - \lambda_1 + \right. \\ \left. \lambda_2 \right] \end{aligned} \quad (5.32)$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H_1}{\partial D} - \frac{\partial H_1}{\partial c} \frac{\partial c^*}{\partial D}$$

$$\begin{aligned} \dot{\lambda}_2 = (\rho - \alpha)\lambda_2 + \frac{\omega_1 l^2 - \omega_2 c^2(1-\theta)}{(M+S)} - \left[\frac{2\omega_2 c(1-\theta)}{(M+S)} \right] \left[\frac{2\omega_2 c(1-\theta)D}{(M+S)} - \lambda_1 + \lambda_2 \right] \end{aligned} \quad (5.33)$$

$$\dot{\phi}_1 = \rho\phi_1 - \frac{\partial H_2}{\partial M} - \frac{\partial H_2}{\partial l} \frac{\partial l^*}{\partial M}$$

$$\dot{\phi}_1 = \phi_1(\rho - r)$$

$$\dot{\phi}_2 = \rho\phi_2 - \frac{\partial H_2}{\partial D} - \frac{\partial H_2}{\partial l} \frac{\partial l^*}{\partial D}$$

$$\dot{\phi}_2 = \phi_2(\rho - \alpha) + \gamma_2$$

The third condition remains the same and the state equations are achieved:

$$\dot{M} = \beta S + rM - c - l$$

$$\dot{D} = \alpha D + c$$

Also, the co-states functions satisfy the terminal conditions:

$$\lambda_1(T) = 0 \quad \lambda_2(T) = 0 \quad \phi_1(t) = 0 \quad \phi_2(T) = 0$$

In the absence of cooperation in the feedback Nash equilibrium case, the manager obtains information about the response of the debtholder to the company's current liquid reserve, value of debt and change in its cost of equity strategy. The manager then acts upon this information in selecting their Nash equilibrium strategy. The debtholder is expected to do vice-versa. However, from the solution of $\dot{\phi}_1$ and $\dot{\phi}_2$, the debtholder does not modify their strategies to the firm's change in cost of equity or liquid reserve information, since the cross-derivative information of the debtholder's response to changes in $M(t)$, $D(t)$ yields zero. Therefore, the debtholder does not incorporate any new information in their selection of an equilibrium strategy. In practical terms, in a non-cooperative feedback Nash equilibrium case, the debtholder sets their cost of debt regardless of the manager's response to changes in the company's liquid reserve and value of debt, while the manager adjusts the cost of equity incurred by the company and considers the debtholder's actions.

If a value function V is defined by:

$$V(M_0, D_0, t_0) := \max_{l, c} \int_{t_0}^{t_1} f(t, M(t), D(t), l(t), c(t)) dt$$

$$\text{Subject to } g(t, M, D, l, c), \quad M_0(t) = M_0; D_0(t) = D_0.$$

The value function yields the value of the integral when estimated at the optimal control and state. The relationship between the adjoint variable and the derivative of the value function with respect to the state variable can be expressed as:

$$\frac{\partial V_1}{\partial M}(M_0, t_0) = \lim_{\epsilon \rightarrow 0} \frac{V(M_0 + \epsilon, t_0) - V(M_0, t_0)}{\epsilon} = \lambda(t_0)$$

$$\frac{\partial V_2}{\partial D}(D_0, t_0) = \lim_{\epsilon \rightarrow 0} \frac{V(D_0 + \epsilon, t_0) - V(D_0, t_0)}{\epsilon} = \phi(t_0)$$

$$\text{Such that } \frac{\partial V_1}{\partial M}(M^*(t), t) = \lambda(t) \quad \text{for all } t_0 \leq t \leq t_1.$$

$$\frac{\partial V_2}{\partial D}(D^*(t), t) = \phi(t) \quad \text{for all } t_0 \leq t \leq t_1.$$

The strategies l^*, c^* are feedback Nash equilibria for the differential game, with a closed-loop information structure, if the value functions V_1, V_2 exist that satisfy:

$$V_1(M, D, t) \geq J_1[l(M, M_0, D, D_0, t), c^*(M, M_0, D, D_0, t)]$$

$$V_2(M, D, t) \geq J_2[l^*(M, M_0, D, D_0, t), c(M, M_0, D, D_0, t)]$$

The so-called value functions (V_1, V_2) must be optimal irrespective of the initial value of the steady state.

The feedback Nash equilibrium alongside the open-loop Nash equilibrium analyse the optimal capital structure as a mechanism to minimise the conflict of interest between the debtholder and the manager in a non-cooperative case. This provides insights into the analysis of the optimal capital structure when agency problems and conflicts of interest are introduced.

The non-cooperative game analysis elucidates the incongruity of interests between the debtholder and the manager and suggest that the manager does not align their interests with the debtholder's. Hence, the non-cooperative game analysis does not fully resolve the agency problem but provides optimal strategies to minimise the marginal agency cost of debt. A more efficient corporate governance mechanism, alongside the choice of an optimal capital structure, is one that makes it possible to align the interests of the manager and the debtholder by disincentivising the manager from substituting the company's assets, an action that places the debtholder's value maximisation at risk. To achieve this in the mathematical analysis, the Pareto outcome was considered an appropriate approach to jointly optimise the conflicting interests of the two players.

5.4.2 Cooperative Game Analysis

5.4.2.1 Optimal decisions with a Pareto-efficient outcome

To elicit corporate governance in the selection of an optimal capital structure and optimise the interests of the manager and the debtholder, cooperation may be sought between the players. The Pareto solution concept provides a joint optimisation of the debtholder and manager's utility functions over the time interval. It is therefore presumed that the equilibrium of a cooperative game will be Pareto optimal. According to the Coase theorem, if a rational debtholder observes the significant effect of the manager's actions on the company's default tendencies, they will readily negotiate on a Pareto optimal outcome. Equally, if a rational manager observes the severe effect of the debtholder's disciplinary actions on the company, they will readily negotiate on a Pareto outcome.

Pareto System of the Game

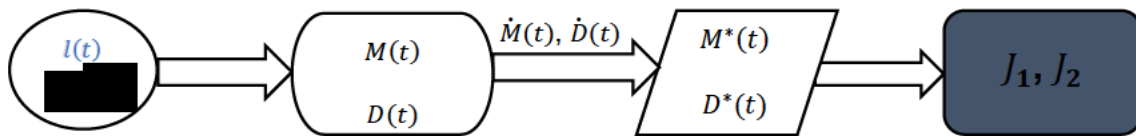


Figure 5.3 Pareto system of the game

Figure 5.3 illustrates the Pareto system of the game. The game becomes a seemingly optimal control system, in which both players agree to optimise their objectives with respect to a weight assignment as a corporate governance mechanism. Both players' strategies $l(t)$ and $c(t)$ jointly influence the states of the game $M(t), D(t)$. These are processed by the systems of the game, $\dot{M}(t), \dot{D}(t)$ and result in optimal state trajectories. The results from the optimal states and strategies are then independently imputed in each player's utility function to derive a Pareto frontier to compare the outcomes for both players.

In the solution concept, the interests of both players are prioritised with respect to the assigned φ , such that:

$$\varphi J_1 + (1 - \varphi) J_2$$

The results from different weight assignments are plotted on a Pareto front to observe the Pareto optimal payoffs for both players. Recall:

$$J_1 = \min \int_0^T e^{-\rho t} (\omega_1 \mu(t) l(t)^2 + \omega_2 c(t)^2 (1 - \theta)(1 - \mu(t)) - \omega_3 r M(t)) dt$$

$$J_2 = \max \int_0^T e^{-\rho t} (\gamma_2 D(t) - \gamma_1 c(t)^2) dt$$

To make the two objective functionals similar, J_2 is converted into a minimisation problem as:

$$J_2 = \min \int_0^T e^{-\rho t} (\gamma_1 c(t)^2 - \gamma_2 D(t)) dt$$

The joint objective functional of the game becomes:

$$\min \int_0^T \left(\varphi e^{-\rho t} (\omega_1 \mu(t) l(t)^2 + \omega_2 c(t)^2 (1 - \theta)(1 - \mu(t)) - \omega_3 r M(t)) \right) + e^{-\rho t} (1 - \varphi) (\gamma_1 c(t)^2 - \gamma_2 D(t)) dt \quad (5.34)$$

Subject to:

$$\begin{aligned} \dot{M}(t) &= \beta S(t) + r M(t) - c(t) - l(t) \\ \dot{D}(t) &= \alpha D(t) + c(t) \\ S(t) &= S_0 e^{(r-\beta)t} \\ E(t) &= M(t) + S(t) - D(t) \end{aligned}$$

Following this definition for $E(t)$, $\mu(t) = 1 - \frac{D(t)}{M(t)+S(t)}$

The Pareto outcome for this solution concept is solved similarly to the open-loop Nash equilibrium concept.

The Hamiltonian for the game is specified as:

$$H = \varphi \left(\frac{\omega_1 l^2 (M+S-D)}{M+S} + \frac{\omega_2 c^2 D (1-\theta)}{M+S} - r \omega_3 M \right) + (1 - \varphi) (\gamma_1 c^2 - \gamma_2 D) + \lambda_1 (\beta S + r M - c - l) + \lambda_2 (\alpha D + c) \quad (5.35)$$

Theorem 5.2

The Pareto-efficient equilibrium solutions are:

$$l^* = \frac{\lambda_1 (M+S)}{2\omega_1 \varphi (M+S-D)} \quad (5.36)$$

Achieving cooperation between the players presents a mechanism that resolves the agency problem. The optimal cost of equity for the firm from Equation (5.36)

is the ratio of the value of an added dollar of debt or earnings multiplied by the firm's total assets, to the firm's equity multiplied by two times the balancing cost factor of the use of equity, times the assigned φ at any time t . This calculation implies that the greater the weight on the cost of implementing equity and the manager's weight assignment φ , the lower the ratio of the company's total assets to its equity. This implies that the cost of implementing equity can lower the company's asset-equity ratio, that is, if it is more difficult to obtain equity, then a lower proportion of the company's assets is funded by equity. Similarly, the greater the weight φ assigned to the manager's objective function, the lower the optimal cost of equity required to attain optimality, implying that the firm may require higher debt if the manager behaves opportunistically.

$$c^* = \frac{(\lambda_1 - \lambda_2)(M+S)}{2\omega_2 D\varphi(1-\theta) + 2\gamma_1(M+S)(1-\varphi)} \quad (5.37)$$

The Pareto-efficient cost of debt is mainly determined by the ratio of the company's total assets to the tax-deductible value of the use of debt finance and the debtholder's assigned weight. From Equation (5.37), this implies that the greater the weight φ assigned to the manager's objective function, the higher the optimal cost of debt required by the debtholder. The contraposition is that the greater the weight assigned to the debtholder ($1 - \varphi$), the lower the optimal cost of debt. This motivates the manager to protect the interests of the shareholder.

Proof of Theorem 5.2

$$\frac{\partial H}{\partial l} = \frac{2\omega_1 \varphi l(M+S-D)}{(M+S)} - \lambda_1 = 0$$

$$l^* = \frac{\lambda_1(M+S)}{2\omega_1 \varphi(M+S-D)}$$

$$\frac{\partial H}{\partial c} = \frac{2c\omega_2 \varphi(1-\theta)D}{(M+S)} + 2\gamma_1 c(1-\varphi) - \lambda_1 + \lambda_2 = 0$$

$$c^* = \frac{(\lambda_1 - \lambda_2)(M+S)}{2\omega_2 D\varphi(1-\theta) + 2\gamma_1(M+S)(1-\varphi)}$$

End of Proof.

The two systems of ordinary differential equations for the adjoint equations λ_1 and λ_2 that determine Equations (5.36) – (5.37) above are:

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial M} = (\rho - r)\lambda_1 + \varphi\omega_3r - \frac{\varphi D(\omega_1 l^2 - \omega_2 c^2(1-\theta))}{(M+S)^2} \quad (5.38)$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H}{\partial D} = (\rho - \alpha)\lambda_2 + \gamma_2(1 - \varphi) + \frac{\varphi(\omega_1 l^2 - \omega_2 c^2(1-\theta))}{(M+S)} \quad (5.39)$$

The third condition, as before, is:

$$\dot{M} = \beta S + rM - c - l$$

$$\dot{D} = \alpha D + c$$

An equilibrium solution found for each $0 < \varphi < 1$ yields a point on the Pareto frontier. To obtain the Pareto frontiers, the equilibrium strategies and states were obtained for each $\varphi = [0.1, \dots 0.9]$. Weights 0 and 1 were excluded because a player will not remain in a game if their interest is set to 0. The returned equilibrium values at each weight share were then imputed into the individual utility functions (objective functionals) of each player independently, thereby producing the manager and debtholder's objective/utility for each weight share. The values for the manager's payoff were then plotted against those of the debtholder, to observe the outcome for both players.

5.5 Computational Experiments and Results

Nikooeinejad, Delavarkhalafi and Heydari (2016) thoroughly discussed the difficulty of solving two-points boundary value problems analytically and the need for numerical solutions for dynamic games. Due to the non-linearity of the developed model, the remaining solutions were obtained via numerical algorithm.

5.5.1 Varying the Balancing Cost Factors

To obtain interesting and useful results for the model, the weight or value assigned to the balancing cost factors can be varied to understand the impact of certain costs in optimising the players' objectives. This is the basis for the parameter sensitivity carried out in each solution concept. Three sensitivity analyses were carried out for the game analysis in each solution concept, denoted as Encounter 1, Encounter 2, Encounter 3. The three encounters are described in the graphical results as E1, E2, and E3.

Encounter 1 – $[\omega_1, \omega_2, \omega_3] = [2 \ 2 \ 5]$ and $[\gamma_1, \gamma_2] = [5, 2]$. This implies that the cost ω_3 of maximising the company's financial income $rM(t)$ is higher than the cost of minimising its finance cost. In the same encounter, it is hypothetically stated that the debtholder incurs a higher monitoring cost γ_1 to ensure the cost of debt $c(t)$ can be repaid than the opportunity cost γ_2 of providing debt.

Encounter 2 – $[\omega_1, \omega_2, \omega_3] = [2 \ 2 \ 5]$ and $[\gamma_1, \gamma_2] = [5, 10]$. The costs associated in Encounter 2 are similar to those of Encounter 1, however, with a significant increase in the opportunity cost γ_2 incurred by the debtholder to provide the debt face value $D(t)$.

Encounter 3 – $[\omega_1, \omega_2, \omega_3] = [50 \ 2 \ 2]$ and $[\gamma_1, \gamma_2] = [5, 2]$. In Encounter 3, there is a significant weight on the operational cost ω_1 of minimising the company's weighted average cost of equity than on other variables. This provides different outcomes for the optimal state trajectories of the game and the optimal capital structure of the firm, thus minimising the agency problem and providing useful insights.

The three different solution concepts were applied to the case study company, Metcash Limited. The financial variables from the company's 2018 financial statements, used to obtain numerical results for the model application, are presented below.

Table 5.4 Financial data from 2018 financial statements of Metcash Limited (Metcash Limited 2018)

S/N	Parameters	Definition and Code	Data
1	r	Riskless rate $r > 0$, assumed to be constant - Current (as of May 2019) Australian Government bonds yield for 10-year residual maturity	0.02
2	β	Payout rate of company's productive assets, assumed to be constant - Assumed	0.30
3	$M(0)$	Liquid reserve - Financial data (AUD \$b)	2.40
4	$S(0)$	Initial value of company's productive assets - Financial data (AUD \$b)	1.30
5	$D(0)$	Initial value of company debt - Financial data (AUD \$b), calculated as the current liabilities interest-bearing borrowings plus total non-current liabilities	0.27
6	θ	Company's applicable effective tax rate, ranging between 0 and 1	0.28
7	α	An average of the rate of change in Metcash Limited's use of debt over a six-year financial period (2013 - 2018)	0.01
8	ρ	Discount rate - Assumed	0.001
9	ω_1	Assigned balancing cost factor (Encounter 1 - 3)	[2 2 50]
10	ω_2	Assigned balancing cost factor (Encounter 1 - 3)	[2 2 2]
11	ω_3	Assigned balancing cost factor (Encounter 1 - 3)	[5 5 2]
12	γ_1	Assigned balancing cost factor (Encounter 1 - 3)	[5 5 5]
13	γ_2	Assigned balancing cost factor (Encounter 1 - 3)	[2 10 2]

5.5.2 Nash Equilibrium Results

5.5.2.1 Open-loop Nash equilibrium

The results of the open-loop Nash equilibrium concept are provided in Table 5.5. and Figures 5.4 and 5.5. Where unspecified, the figures illustrating the results of the game do not have units on the y-axis.

Table 5.5 Open-loop Nash equilibrium outcomes for Encounters (1 - 3)

Encounter	ρ	ω	γ	$M(0)$	$D(0)$	$M(T)$	$D(T)$
1	0.001	[2 2 5]	[5 2]	2.4000	0.2700	2.4939	0.3736
2	"	"	[5 10]	"	"	2.0922	0.7775
3	"	[50 2 2]	[5 2]	"	"	2.4798	0.3736

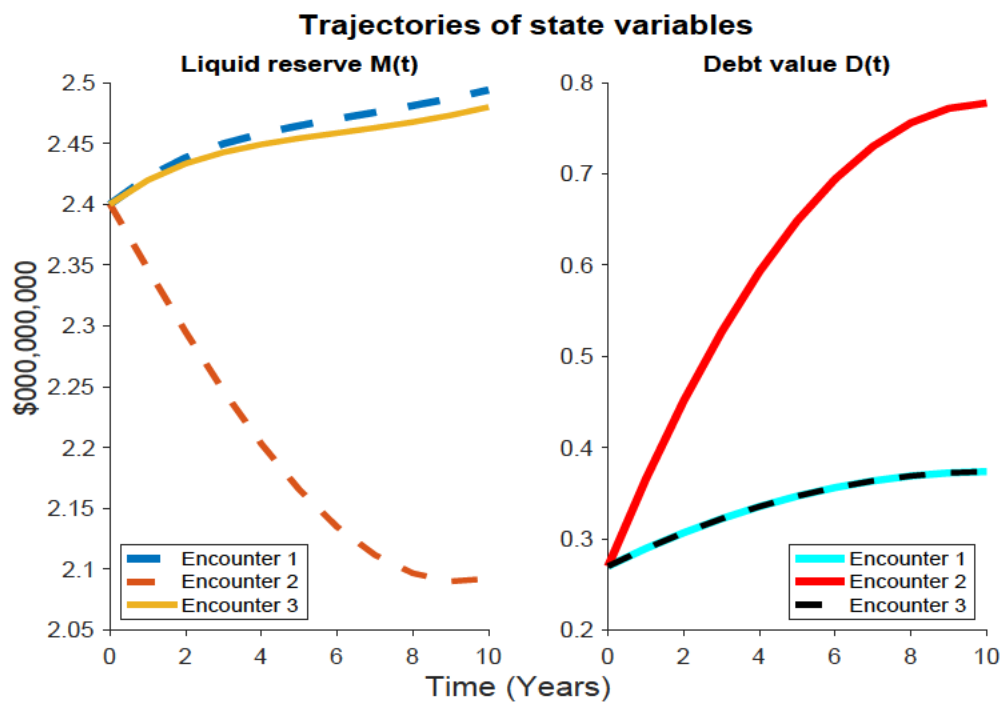


Figure 5.4 *Equilibrium state trajectories of the open-loop Nash equilibrium game*

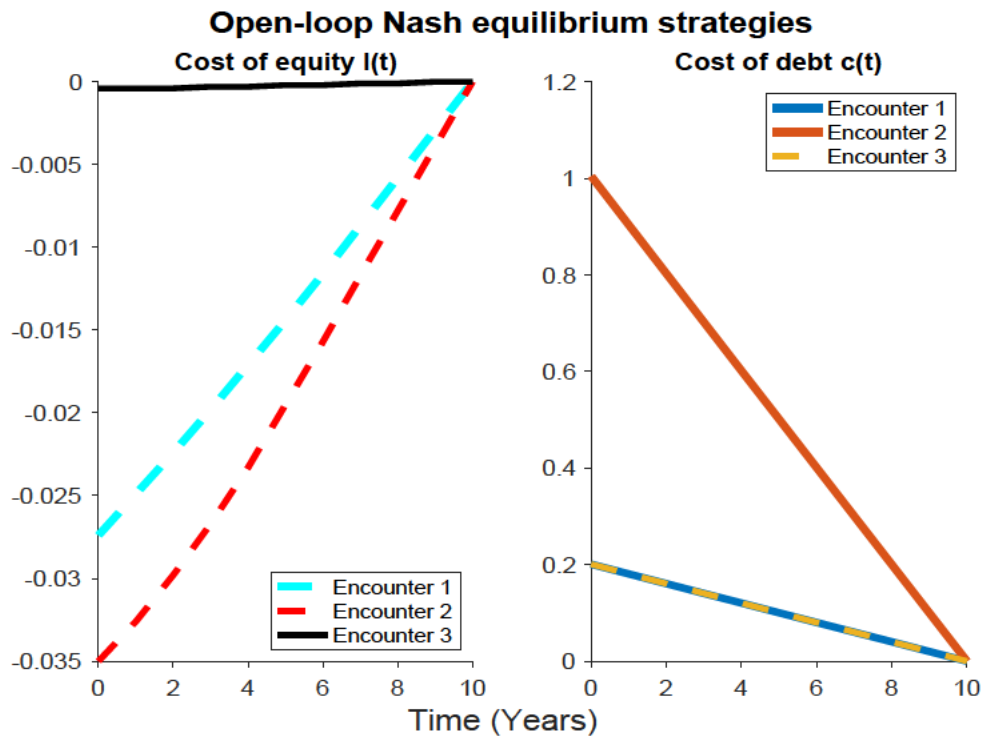


Figure 5.5 *Open-loop Nash equilibrium strategies*

The optimal debt-equity $D^*(t)/E^*(t)$ ratios obtained from the optimal states that minimise the associated cost of capital and optimise the capital structure are expressed in Table 5.6 and Figure 5.6 below over a ten-year period, with no units on the y-axis.

Table 5.6 *Optimal debt-equity ratio for the open-loop case*

Period	Encounter 1 $D^*(t)/E^*(t)$	Encounter 2 $D^*(t)/E^*(t)$	Encounter 3 $D^*(t)/E^*(t)$
0	0.0787	0.0787	0.0787
1	0.0929	0.1234	0.0930
2	0.1067	0.1745	0.1069
3	0.1198	0.2311	0.1201
4	0.1317	0.2915	0.1322
5	0.1422	0.3530	0.1428
6	0.1512	0.4127	0.1519
7	0.1584	0.4666	0.1593
8	0.1639	0.5110	0.1649
9	0.1678	0.5423	0.1688
10	0.1699	0.5578	0.1710

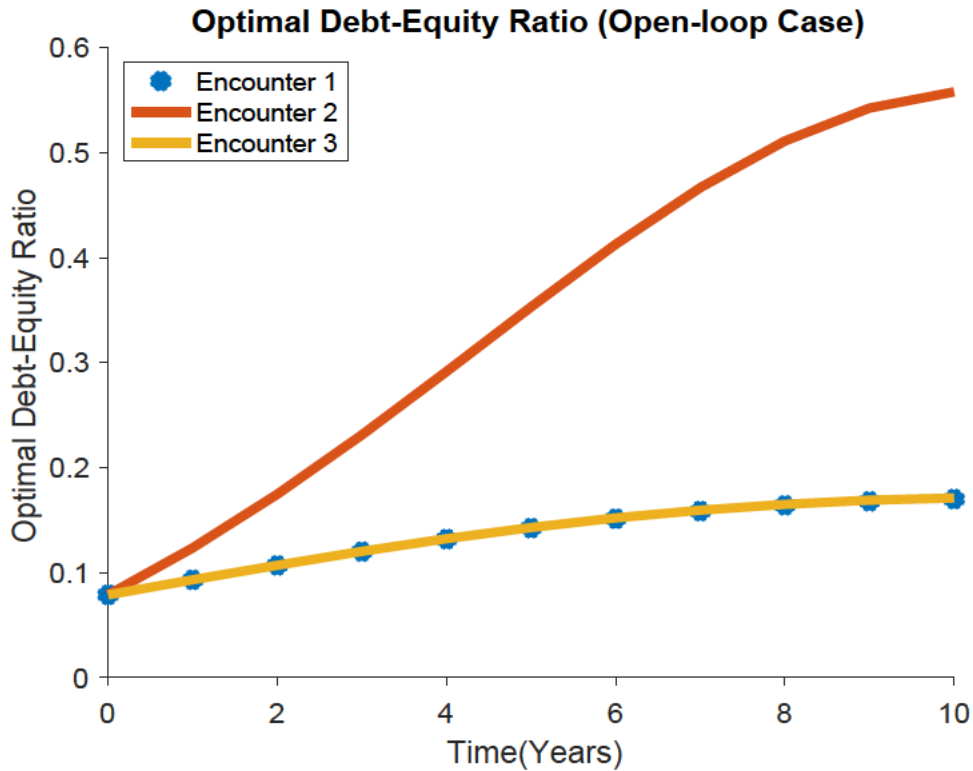


Figure 5.6 Optimal debt-equity ratio (Open-loop case)

Table 5.6 shows the optimal state values at the final time. Figure 5.6 presents the open-loop Nash equilibrium strategies at different encounters. The results suggest a higher cost of debt than a higher cost of equity. The optimal debt-equity ratio, as shown in Table 5.6, rises from 7.8% in the initial period to approximately 17% in Encounters 1 and 3, and up to 55% in Encounter 2, suggesting the higher use of debt, when possible, in the firm's capital structure. This finding partly supports those of Elton and Gruber (1974), which indicated that the maximum allowable debt may be used subject to the institutional constraint on the use of debt. The results also suggest that payouts (i.e., the cost of equity) should be returned to the firm's fund pool rather than as a cash outflow. This can be done by repurchasing shares rather than paying out dividends. Moreover, share repurchases may be encouraged by low capital gain rates (Allen & Morris 2014).

In the open-loop non-cooperative Nash equilibrium solution concept, due to the debtholder's interest in maximising the company's value of debt and cost of debt, which results in the conflict of interest in the game, the significance of the debtholder's interest impacts the equilibrium state trajectory of $D(t)$. This causes the variable to rise over time. This is particularly observable in both Encounters

1 and 3, as seen in Figure 5.4. In Encounter 1, the cost of financing the firm is presumed to be cheaper than improving the financial income of the firm for the manager. The cost of recovering debt is more expensive than the cost of recovering the interest charges. As a result, the company liquid reserve rises steadily. A similar outcome is observed in Encounter 3 when the cost of implementing equity is more expensive than debt.

While some debt investors are keen on interest charges, others are interested in principal protection. In Encounter 2, the conditions are similar to those in Encounter 1, except that the opportunity cost of providing debt is higher than the cost of recovering interest charges. The outcome of this is an increase in the value of debt and a higher cost of debt for the company. The implication here is that when a debtholder incurs more costs in the form of monitoring to ensure the firm's ability to pay back its debt finance, this has an impact on the total cost to the firm.

Throughout the trajectory, cost of debt did not equal the cost of equity except at the final time of the game. The outcome of the game, which is the equilibrium set of actions for both players, is that in the presence of a conflict of interest between the manager and the debtholder, the company's cost of debt is always higher than the cost of equity. This is observed in Figure 5.5, following the corresponding state trajectory in Figure 5.4, particularly when the debtholder's cost of principal protection is higher than recouping the interest payments. In the game, the debtholder maintains a high cost of debt, while the manager keeps a lower cost of equity.

With a pre-commitment to the game, these outcomes, which are individually-rational outcomes, provide each player with at least their security payoff level. A shift from these equilibrium outcomes by a selfish utility-maximising player will result in a lower payoff if the other player sticks to their strategy (Mehlmann 1988).

ϕ_1 as obtained in the analytical results in Equation (5.26) returned a zero value, implying that ϕ_1 was non-binding on player two. This is so because a liquid reserve does not have a direct effect on player two's objective functional, therefore yielding a zero as it's change in marginal variation. In contrast, λ_1 shows that a dollar of liquid reserve added to the company's portfolio results in a lower

cost of equity and debt over time and yields a positive marginal variation in the company's financial income. As the company's liquid reserve increases in Encounters 1 and 3, a lower cost of equity and debt is observed. This implies that the manager relies on less external funds, thereby reducing its cost of financing. This achieves the objective of the manager. ϕ_2 , additional use of debt increases the value of debt, and maximises the debtholder's objective, hence achieving a non-cooperative equilibrium for the players.

Overall, in the open-loop case, the optimal capital structure requires an increasing use of debt to equity in the dynamic game. The optimal capital structure for the ten-year debt maturity period, as presented in Table 5.8, does not exacerbate the agency cost of debt and keeps the manager in view of the debtholder's interest. This provides a corporate governance instrument that manages the conflict of interest between the debtholder and the manager.

5.5.2.2 **Feedback Nash equilibrium**

The results for the case study company using the feedback Nash Equilibrium are presented below. These provide the optimal outcomes for the company's financial performance indicators – the optimal cost of equity and debt required to obtain the optimal state outcomes and the optimal capital structure specification to resolve the agency problem between the debtholder and the manager.

Table 5.7 Feedback Nash equilibrium outcomes for Encounters (1 - 3)

Encounter	$M(0)$	$D(0)$	$M(T)$	$D(T)$	ρ	ω	γ
1	2.4000	0.2700	2.4940	0.3736	0.001	[2 2 5]	[5 2]
2	"	"	2.0987	0.7775	"	"	[5 10]
3	"	"	2.4798	0.3736	"	[50 2 2]	[5 2]

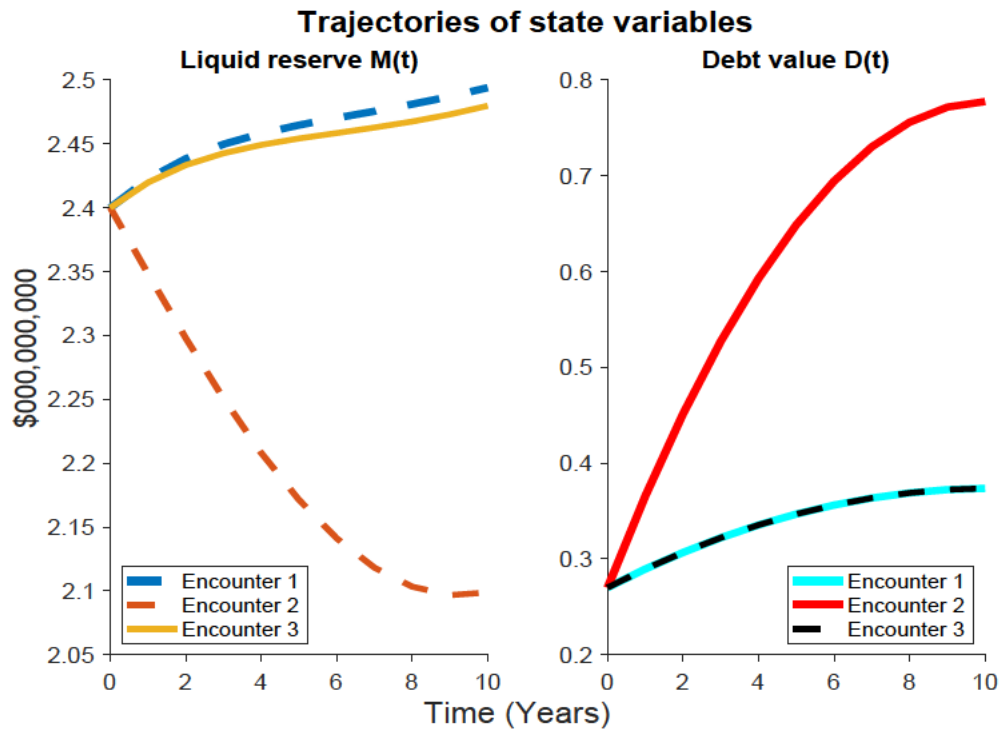


Figure 5.7 Equilibrium trajectories of the feedback Nash equilibrium game

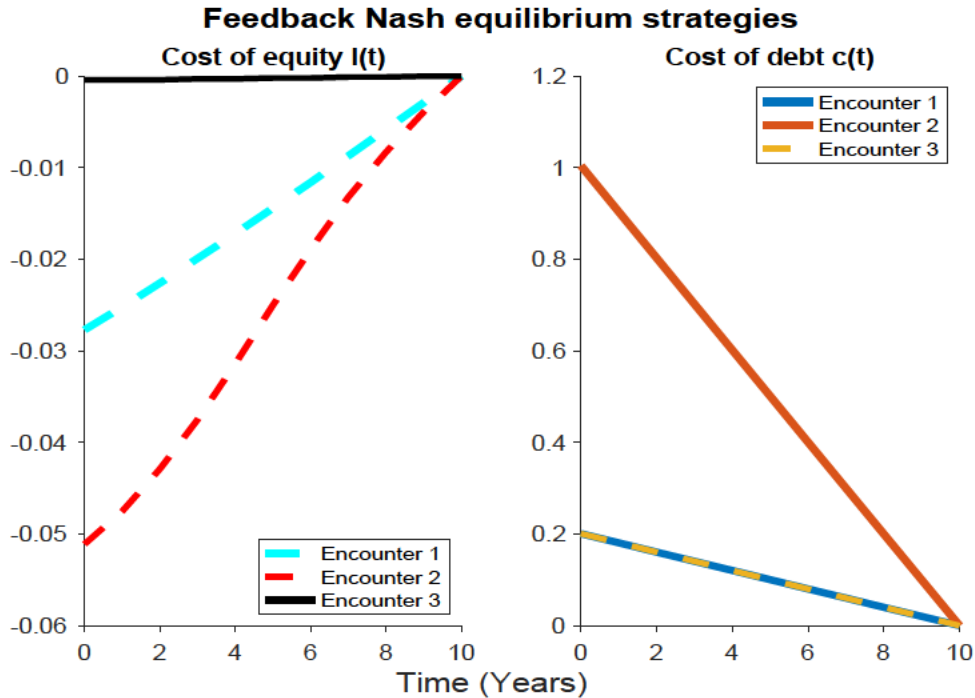


Figure 5.8 Feedback Nash equilibrium strategies

The optimal debt-equity ratio for the feedback case is presented tabularly and graphically below.

Table 5.8 *Optimal debt-equity ratio of the feedback case*

Period	Encounter 1	Encounter 2	Encounter 3
0	0.0787	0.0787	0.0787
1	0.0929	0.1233	0.0930
2	0.1067	0.1743	0.1069
3	0.1198	0.2308	0.1201
4	0.1317	0.2907	0.1322
5	0.1422	0.3520	0.1428
6	0.1512	0.4112	0.1519
7	0.1584	0.4648	0.1593
8	0.1639	0.5089	0.1649
9	0.1678	0.5398	0.1688
10	0.1699	0.5552	0.1710

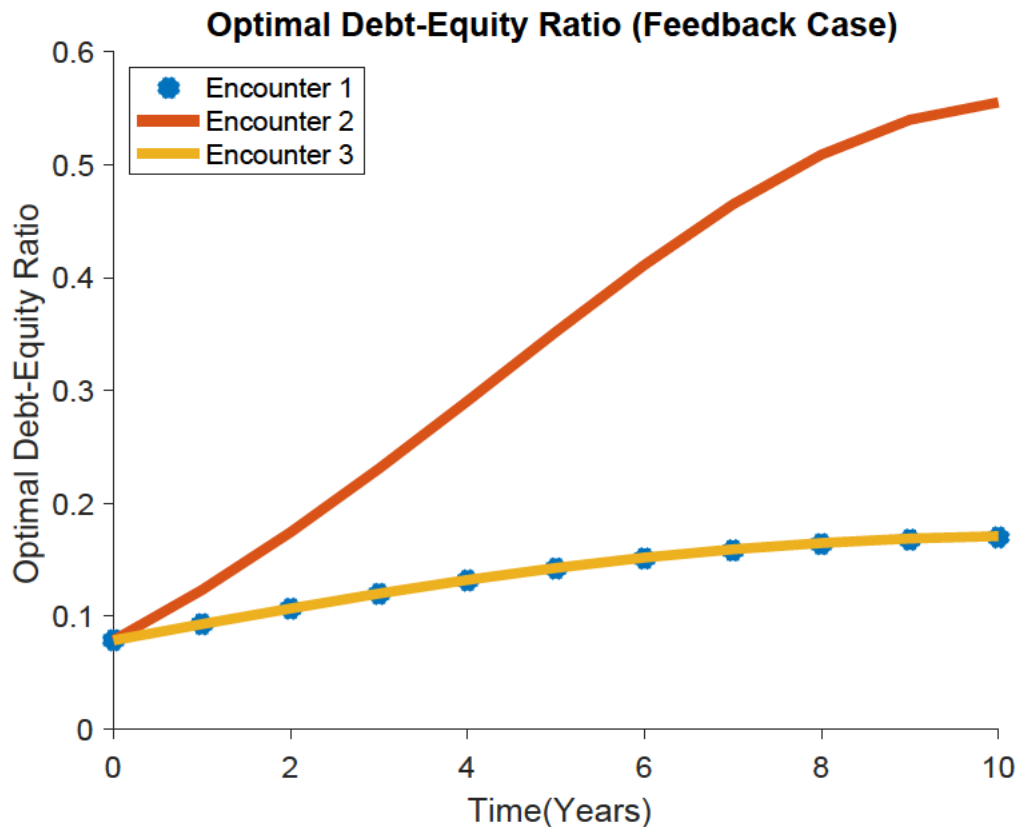


Figure 5.9 Optimal debt-equity ratio (feedback case)

With minimal reaction from the debtholder, the results for the feedback case are similar to those of the open-loop case, except that the manager adjusts his equilibrium strategy to a higher cost of equity. However, this shows only a marginal difference between the open-loop and feedback outcomes. The Nash strategies proposed by the optimal cost of debt and equity shown in Figure 5.8 cause an improvement in the firm's liquid reserve over the period in Encounters 1 and 3 when compared with the open-loop case.

Similar to those of the open-loop case, the feedback results suggest the prohibition of capital distributions (dividend payments) until after the debt matures. This supports the proposition of Copeland, Weston and Shastri (2014). Although the payout policy conveys information to the capital market about the health and ability of a company to produce cash flows (signalling motives), a firm is limited by the availability of its free cash flows. The optimal debt-equity ratio also rises in the same pattern as in the open-loop case. Hence the optimal capital structure that serves as a corporate governance mechanism to mitigate the conflict of interest between the debtholder and the manager is one with a greater

allowance for debt than equity. The optimal capital structure enhances the firm's ability to finance potential investments as the firm's financial income and operating income increases.

In Encounter 1, the manager's Nash strategy in the game is to adopt a higher cost of equity in the feedback case as they observe the action of the debtholder. In the fifth year in the over-time period, the value of the cost of equity in the feedback Nash equilibrium case is observed to be the same as that of the open-loop Nash equilibrium, which may be interpreted as predictability of the debtholder's response over the time horizon. That is, the manager is able to predict the debtholder's actions in the game from the fifth year. This highlights the learning advantage of the feedback Nash equilibrium solution technique.

In Encounter 2 in the feedback Nash equilibrium, the cost of equity is observed to begin much higher at ' -0.0511 ' than in the Encounter 1 feedback Nash equilibrium case, as well as the Encounter 2 open-loop Nash equilibrium case ' -0.0350 '. This implies that the manager adopts a higher cost of equity as a strategy when the company's value of debt has a higher balancing cost factor for the feedback analysis. For this case, the value of debt is seen to rise higher, while the company's liquid reserve declines heavily. This is similar to the open-loop Nash equilibrium outcome. This follows a logical intuition that the higher the cost of debt, the lower the firm's liquid reserve, as the company's current asset covers more finance costs. However, a tax advantage may be able to level this out.

One major highlight from the feedback case for the developed model is that while the manager is able to observe the debtholder's response to changes in state variables and incorporate this into their Nash strategy, the debtholder does not derive similar information about the manager's response to changes in state variables over time. This highlights the manager's information advantage.

5.5.2 Pareto-efficient results

The utilities of the players are used to develop the Pareto frontier. To calculate the utility for player 1 J_1 , due to the complexity of the first two elements of the integral function, the area under the curve was obtained approximately using the numerical integration method of the trapezoidal rule. The last element of the integral was calculated using the standard integration by parts method. J_2 for

player 2 was also obtained using the integration by parts methods. J_1 and J_2 were then plotted as follows.

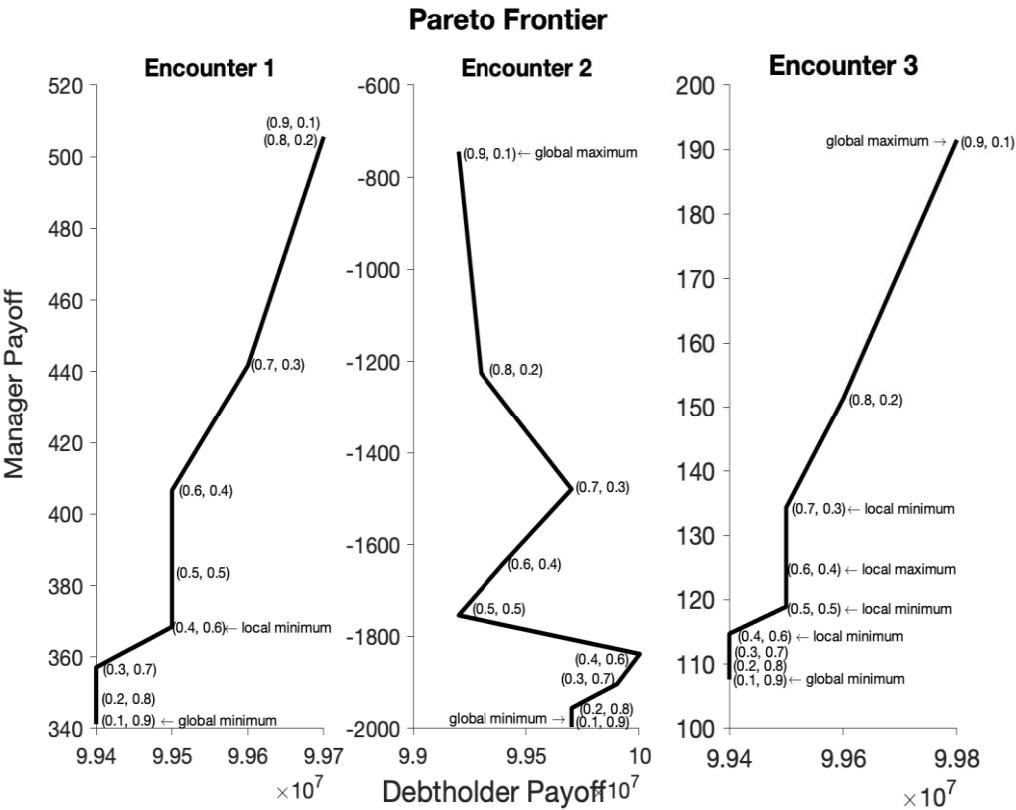


Figure 5.10 Pareto frontier

Table 5.9 Optimal debt-equity ratio Pareto case (0.1,0.9)

Period	Encounter 1 $D^*(t)/E^*(t)$	Encounter 2 $D^*(t)/E^*(t)$	Encounter 3 $D^*(t)/E^*(t)$
0	0.0787	0.0787	0.0787
1	0.0891	0.1041	0.0894
2	0.0992	0.1332	0.1000
3	0.1090	0.1659	0.1102
4	0.1182	0.2025	0.1200
5	0.1269	0.2430	0.1293
6	0.1352	0.2879	0.1381
7	0.1429	0.3379	0.1466
8	0.1503	0.3934	0.1546
9	0.1574	0.4558	0.1623
10	0.1643	0.5264	0.1700

Table 5.10 Optimal debt-equity ratio Pareto case (0.7,0.3)

Period	Encounter 1 $D^*(t)/E^*(t)$	Encounter 2 $D^*(t)/E^*(t)$	Encounter 3 $D^*(t)/E^*(t)$
0	0.0787	0.0787	0.0787
1	0.0895	0.1084	0.0900
2	0.1009	0.1433	0.1011
3	0.1117	0.1837	0.1120
4	0.1221	0.2300	0.1226
5	0.1320	0.2832	0.1327
6	0.1416	0.3441	0.1425
7	0.1508	0.4144	0.1519
8	0.1596	0.4962	0.1608
9	0.1682	0.5927	0.1700
10	0.1766	0.7080	0.1781

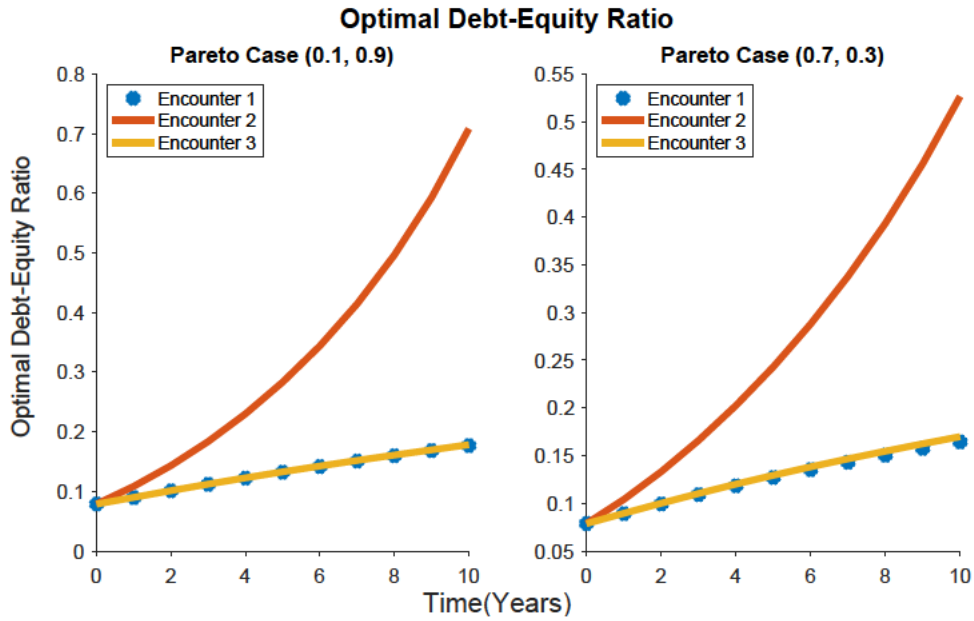


Figure 5.11 Optimal debt-equity ratio (Pareto case)

5.5.2.1 Comparing Pareto frontiers for Encounters 1, 2 and 3

Table 5.11 Comparing Pareto frontiers for the three encounters

Encounter	Balancing cost factors		Minimum cost	
	Obj. 1	Obj. 2	Manager	Debtholder
1	[2 2 5]	[5 2]	\$340	9.94×10^7
2	[2 2 5]	[5 10]	\$-2000	9.97×10^7
3	[50 2 2]	[5 2]	\$110	9.94×10^7

In Encounter 1 of the Pareto case, as shown in Figure 5.10, the balancing cost factors for the elements of both player's objective functionals were assigned as [2 2 5] for player 1, and [5 2] for player 2. Here, the manager's cost of maximising the firm's financial income $rM(t)$ is greater than minimising the weighted average cost of capital. The debtholder's cost of recouping their interest payment is higher than recovering the debt principal value. Recalling that both players' objectives have been converted to a minimisation problem, the global minimum for both

players are at the proportion [0.1, 0.9]. This weight assignment yields the minimum cost of \$340 for the manager, and $\$9.94 \times 10^7$ for the debtholder.

Encounter 2 gives a strikingly different Pareto to that of Encounter 1. In Encounter 2, the balancing cost factors for the weight on the cost of achieving both player's objective functionals were assigned as [2 2 5] for player 1, and [5 10] for player 2. The main difference between Encounter 1 and Encounter 2 is that the debtholder incurs a greater opportunity cost for providing the debt face value. Comparable to Encounter 1, the global minimum in the minimisation problem is at the ratio [0.1, 0.9], with the minimum costs of \$-2000 for the manager and $\$9.97 \times 10^7$ for the debtholder. This produces a lower cost for the manager, and a higher cost for the debtholder when compared with Encounter 1.

In Encounter 3, the balancing cost factors for the elements of both player's objective functionals were assigned as [50 2 2] for player 1, and [5 2] for player 2. This is similar to Encounter 1, however, with a significant increase in the balancing cost factor of the company's weighted cost of equity. This yields a global minimum of [0.1, 0.9], at a minimum of \$110 for the manager and $\$9.94 \times 10^7$ for the debtholder.

As shown in Table 5.11, the preferable encounter for the manager would be Encounter 2, since they enjoy the lowest cost here. However, the debtholder may refuse to select this encounter because they absorb the highest cost. This further proves that in a Pareto-efficient outcome it is impossible to improve the welfare of one player without having an impact on the other player. Following Encounter 2, the next best outcomes for both players are in Encounter 3. Therefore, with appropriate negotiations, both players may prefer Encounter 2 with the best compromise Pareto solution at a weight share of [0.1, 0.9], where 0.1 is the manager's weight share. It is important to note that, although the three encounters share global minima at the same weight shares, they all have a different combination of minimum costs.

5.5.3 Comparison of the Open-loop, Feedback, Pareto Solution Concepts for the Weight Share [0.1, 0.9] and [0.7, 0.3]

Comparing the state trajectories of the three solution concepts at certain weight shares for the Pareto case for all encounters offers some good insights. The

weight share $[0.1, 0.9]$ was selected because it offers the global minimum cost for the debtholder in the Pareto outcomes, and may be a preferred choice for the players. It is therefore specified as the best compromise Pareto solution. This is further compared to a suboptimal weight share $[0.7, 0.3]$ to observe the impact on the company's outcomes, that is if the manager moves away from the optimal weight share or reneges on the terms of cooperation. These are also compared with the Nash equilibrium outcomes. The results for the liquid reserve for this comparison are presented below.

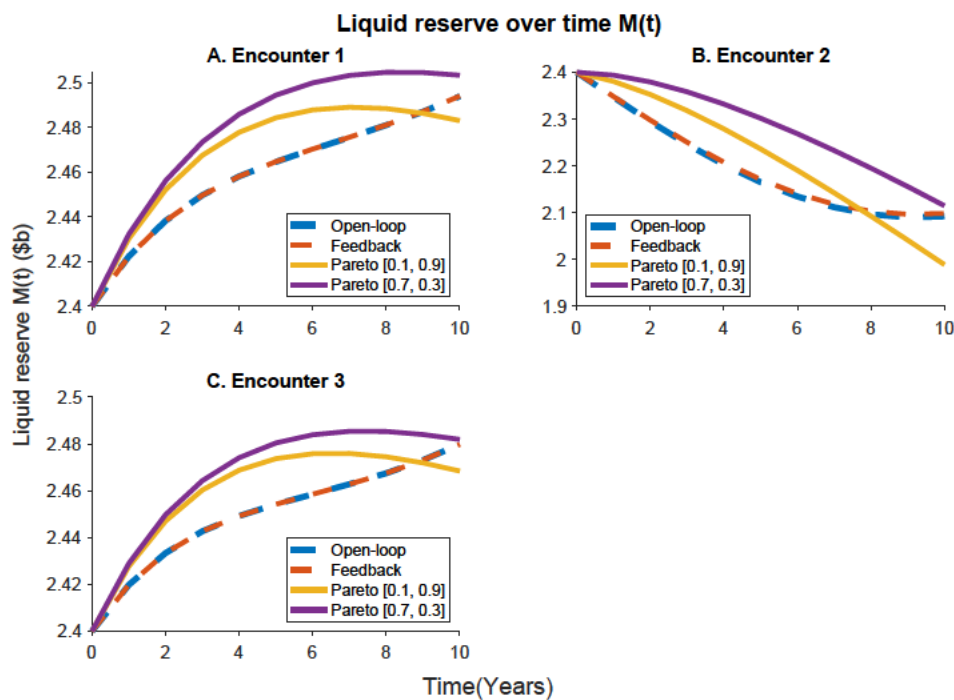


Figure 5.12 Firm liquid reserve over time (Open-loop, Feedback, Pareto $[0.1, 0.9]$, $[0.3, 0.7]$)

In Encounter 1, the company's liquid reserve for the open-loop and feedback cases follow a close and similar pattern, this proves the information uniqueness of the solution outcome. However, the Pareto case offers a higher liquid reserve for the company, particularly between the fourth and tenth year of the contract. This indicates that if the manager and debtholder are able to sustain cooperation in a $[0.1, 0.9]$ weight share, the company can enjoy an increasing value of liquid reserve, thereby implying a significant gain for cooperation. However, weight share $[0.7, 0.3]$ offers a higher marginal benefit in the Pareto case than with

weight share $[0.1, 0.9]$. This induces the manager to prefer a Pareto outcome if they are able to select weight share $[0.7, 0.3]$, rather than $[0.1, 0.9]$.

In Encounter 2, the open-loop, feedback and Pareto cases return a declining value for liquid reserve for the weight share $[0.1, 0.9]$ due to the additional monitoring cost incurred by the debtholder. This result is similar to those obtained by Mu, Wang and Yang (2017) in the shareholder-manager moral hazard problem with respect to the optimal capital structure. Moral hazard exacerbates the value derivable to the company. The feedback case also offers slightly higher value when compared with the open-loop case. The implication here is that the company's liquid reserve does slightly better when the players are able to update their strategies using the information available in the game than in a pre-commitment game. Although the three cases return declining values, the Pareto case offers higher values throughout the period. Similar to Encounter 1, the open-loop, feedback and Pareto outcomes meet around the ninth year of the time period.

Encounter 3 offers a similar trajectory to Encounter 1. However, Encounter 1 offers a slightly higher marginal benefit in the Pareto outcome. This implies that, irrespective of the cost of maximising the company's financial income or minimising the weighted average cost of equity, the company's liquid reserve produces a comparable outcome.

A similar comparison was made for the value of debt over time for the three solution concepts, as shown in Figure 5.13.

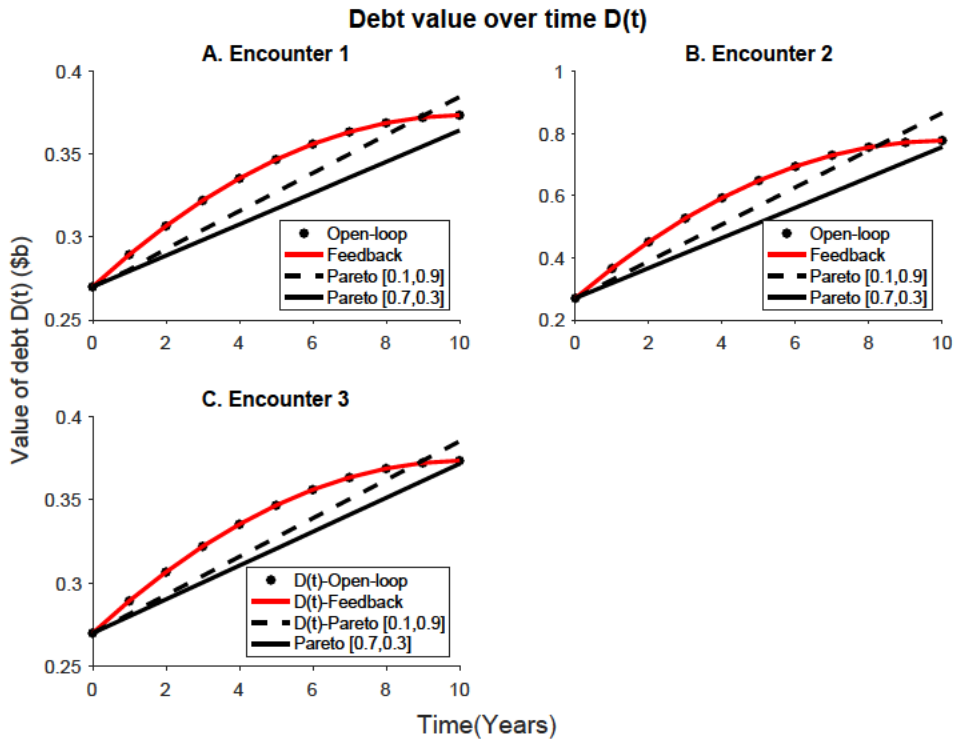


Figure 5.13 Value of debt over time (Open-loop, feedback, Pareto [0.1, 0.9], [0.3, 0.7])

As the figure shows, in Encounter 1, the company's value of debt for the open-loop and feedback cases follow the same trajectory in the time path. This increases steadily for the two cases but follows a straight-line pattern for the Pareto case without much impact in value. The Pareto case suggests that the company's value of debt can be predicted throughout the contract if cooperation is possible, thereby reassuring the debtholder of the debt level of the company.

In Encounter 2, $D(t)$ increases sharply to a high of 0.8, when compared to 0.38 in Encounter 1 or Encounter 3. Depending on the company's management team, the company may prefer to keep a lower debt level than suggested by Encounter 2, thereby taking strategies to discourage the debtholder preference for Encounter 2. However, since it offers the minimum cost in the Pareto case, and if the company management had a debt preference, they might choose Encounter 2. A risk-neutral manager might avoid the choice of Encounter 2.

In Encounter 3, the balancing cost factors of the debtholder are similar to those of Encounter 1. As a result, this yields no obvious difference in the trajectory of the company's debt value in the time period. This suggests the cost of maximising the company's financial income or minimising the cost of equity does not

significantly affect the company's value of debt. In summary, the Pareto-efficient results produce a linear and predictable growth in the company's debt when compared with the non-cooperative cases. In the three cases, the company's value of debt for the Pareto case aligns to those of the open-loop, and feedback cases over the time interval.

Following the linear growth in the company's debt value at the Pareto-efficient outcome, the company's optimal capital structure from the model allows for a greater use of debt, capped at 75%, as shown in Figure 5.14 below.

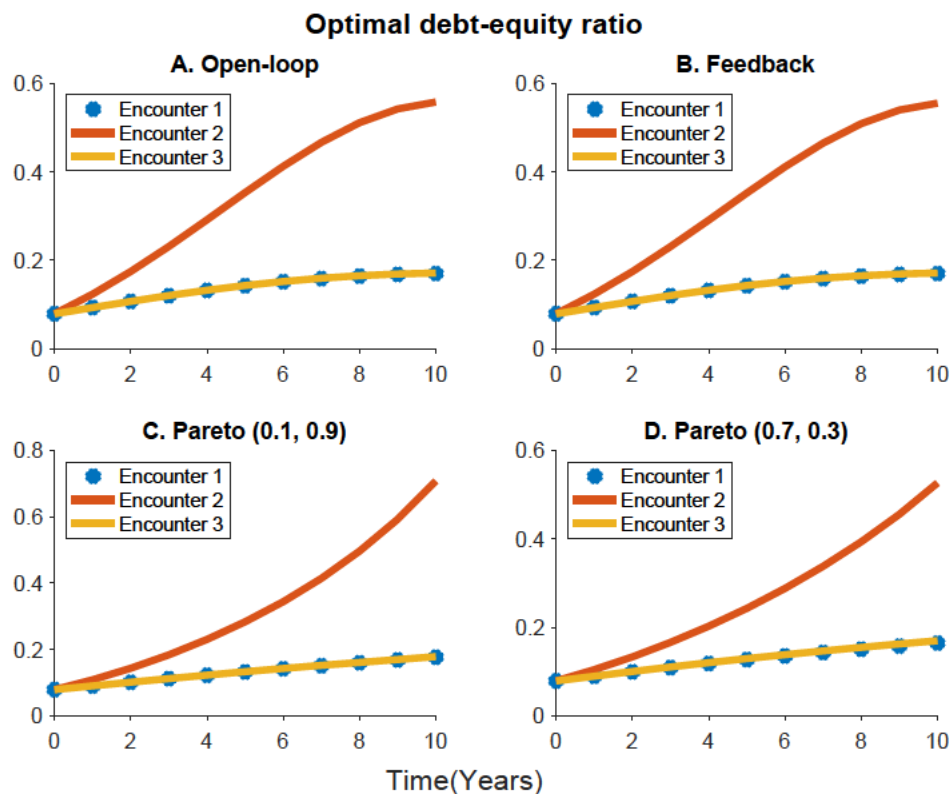


Figure 5.14 Optimal debt-equity ratio (Open-loop, feedback and Pareto cases)

Generally, gains from cooperation include a lower cost of debt, providing more debt facility with lower interest rates and a longer payback period. This shows that in the cooperative game analysed above, the Pareto optimal solution is to assign a lower φ to the manager and, presumably, a higher weight to the debtholder's objective function. If a higher debt-equity ratio provides more financing opportunities for the firm, then the gains of Pareto are also higher than the open-loop and feedback cases at the Pareto optimal weight assignment [0.1, 0.9]. The graph of the optimal capital structure in the cooperative game is slightly

convex or concave up, while the graph of the optimal capital structure for the non-cooperative game is concave up. This implies that a marginally lower capital structure is required when the players reach cooperation.

The major requirements for a *core* were also achieved. These are: the efficiency and individual rationality, the value of the coalition is fully utilised by the sum of payments to the players, and every player gets at least a payoff they could achieve without the coalition (Trost & Heim 2018). This is shown by a higher liquid reserve for the firm in the Pareto case when compared to the non-cooperative game. The value of debt for the firm also increases linearly in the period. This shows stability in the firm's use of debt when cooperation is achieved between the players. The Pareto optimal result provides a corporate governance mechanism to minimise the agency problem between the debtholder and the manager. It also provides an efficient financial management system for the firm.

5.6 Implications for Corporate Governance and Optimal Capital Structure

The results of the financial management model for corporate governance, optimal capital structure and dynamic differential game theory have several implications. These are discussed as follows:

1. The Nash strategies proposed by the optimal cost of debt and equity, as shown in Figures 5.5 and 5.8, caused an improvement in the firm's liquid reserve over time in Encounters 1 and 3 for the feedback and open-loop non-cooperative cases, as shown in Figures 5.4 and 5.7. The company's liquid reserve levelled out towards a maximum at the near end of the contract when the debtholder does not incur an excessive monitoring cost on the firm as specified in Encounters 1 and 3. This shows the decreasing marginal effect of the use of debt towards the end of the contract. This provides salient recommendations for a manager in estimating the weighted average cost of capital required to optimise the capital structure while cooperation is yet to be attained.
2. Similarly, in the absence of cooperation, during the debt maturity period, it is recommended that the company's payouts, that is the cost of equity, should be returned to the company's fund pool rather than as a cash

outflow (e.g., via share repurchase). Thus, a capital distribution may be avoided. This result supports that of Lepetit et al. (2018), which established that a company's payout policy is significantly dependent on the degree of agency conflicts between shareholders and debtholders. Further, the optimal payout policy (cost of equity) can serve as a complementary mechanism for the firm in cushioning the effects of the debtholder's stringent actions on the firm, particularly when cooperation is yet to be attained. This also provides an efficient capital structure that minimises the marginal agency cost of debt throughout the debt maturity.

3. Additionally, the cost of debt, which is the debtholder's response to the agency issue, declines over time because a long-term relationship can minimise information asymmetries between the debtholder and the firm. This reduces agency problems and, consequently, the agency cost of debt (Fukuda & Hirota 1996).
4. When compared with interest bearing borrowings and other long-term debt, the initial values of the financial data reveal a low debt-equity ratio was maintained at the start of the dynamic game. The optimal capital structure obtained in this study permits a greater use of debt to equity than is currently being used, up to the maximum recommended by the optimal debt-equity ratio obtained, as shown in Figure 5.14. This result is consistent with the findings of He (2011), Mu, Wang and Yang (2017), and Qu et al. (2018), which suggest a need for higher leverage when a moral hazard is present, even between the shareholder and manager. It contrasts, however, with the findings of Leland (1994), which indicated no moral hazard problem. This does not exacerbate the agency cost of debt, and serves as a disciplining measure (Tirole 2006) to keep the manager at least focused on the wealth maximisation goal of the debtholder. However, when the two players reach cooperation, a lower use of debt is required for an optimal capital structure. This implies that cooperation reduces the weight of the optimal leverage required by the company in the debtholder-manager agency relationship. The optimal capital structure and optimal cost of financing obtained are provided as corporate governance mechanisms. These minimise the marginal agency cost of debt associated with the issuance of debt.

5. Cooperation as a mechanism via the Pareto case minimises the conflicts of interest between the two players. It does so by disincentivising the manager from substituting the company's asset, which could jeopardise the debtholder's value maximisation. From the results of the study, the incentives proposed by the debtholder include the provision of a lower and more consistent cost of debt, as well as more debt facility for the company. These are described as a fair distribution of the gains from cooperation (Trost & Heim 2018). The company's change in the value of debt was linear in the Pareto case but non-linear in the non-cooperative case, as shown in Figure 5.13. This suggests a more reliable relationship between the players over time. The Pareto optimal solution in the cooperative analysis is to assign a lower weight φ to the manager's objective functional, and a higher weight to the debtholder's objective functional. Intuitively, this enhances the interest of the debtholder in the debt contract or strategic game relationship.
6. During cooperation, an opportunistic manager has an incentive, albeit minimal, to shift from the optimal pair of weight [0.1, 0.9] to an opportunistic weight assignment [0.7, 0.3], as this provides the firm with a minimally higher liquid reserve, as shown in Figure 5.12. This proves the theory of Pareto optimality, which states in part that it is impossible to allocate resources in a way that makes one player better off without making the other player worse off. If a selfish manager reneges on the Pareto optimal strategy, the debtholder responds by increasing the firm's cost of debt and reducing its available debt facility. This is observed by the lower market value of debt finance available to the firm, as shown in Figure 5.13 when the optimal pair of weight [0.1, 0.9] is compared to the suboptimal weight assignment [0.7, 0.3]. These are trigger strategies that enforce cooperation and ensure renegotiation proofness. Therefore, the debtholder provides a greater debt facility as an incentive to encourage the manager to stick to cooperation. Although a greater debt facility may increase the manager's available external financing, the debtholder employs this as a governance instrument because the use of debt also causes the manager to share in the downside risk of the firm with the debtholder (Heinrich 2002).

5.6.1 Dynamic Game Issues

A highlight of the developed financial management model is that it observes the strategic interactions of the debtholder and the manager over time and the results provide insights into the interrelationship dynamically. In the dynamic game relationship, the private information held by the manager is revealed through the company's regulatory reporting, such as annual reports, annual corporate governance statements and other forms of external reporting demanded by the debtholder. Additionally, in a dynamic game, the constrained efficiency of the contractual outcome should be affected by the repeated interactions (Bolton & Dewatripont 2005). From the results, due to the repeated interactions in the optimal contract observable from the Pareto-efficient outcome, it is observed that the company enjoys a stable and efficient cost of debt, a greater provision of debt facility, and a higher liquid reserve over time.

Notably, in a dynamic game, there is a tendency that the possibility of renegotiation may limit the efficiency of the dynamic contract, this leads to the discussion on renegotiation proofness.

5.6.2 Renegotiation Proofness

Originally, folk theorem which described Nash equilibrium payoffs employed infinite period punitive measures to discourage cheating and deviation from a cooperative path. Not only are infinite period punishments infeasible and unattainable, but they also have severe repercussions. A refined version of the folk theorem suggests the use of finite period punishments after which cooperation can be reinstated. A finite period punishment is also applicable to the game developed in this study. This is because the debtholder-manager principal-agent relationship is a finite relationship that ends when the debt contract matures if no bankruptcy is reported.

The trigger strategies proposed by the debtholder to ensure cooperation in the finite financial management model include a higher and less efficient cost of debt and a lower debt facility. These serve as incentives and punitive measures to ensure goal congruence throughout the period. However, Van Damme (1989) suggested that with the finite period punishment approach, the manager may renege on cooperation, thereby gaining a higher liquid reserve in one period.

Since the manager is aware that deviating from the optimal path would hurt the debtholder and the trigger strategies would hurt them, the manager can choose to return to the cooperative path in the next period. If the trigger strategies are insufficient to return the manager to the cooperative path and thus require the debtholder to renegotiate the debt contract, then the trigger strategies are deemed to be unstable.

Moreover, in the developed financial management model, the trigger strategies posed by the debtholder have significant consequences on the efficient financial management system of the company. Consequently, they are stable enough to disincentivise the manager, thus proving that the trigger strategies are renegotiation proof. This resolves the underlying information asymmetry and agency problem between the debtholder and the manager.

5.7 Conclusion

Jensen and Meckling (1976) argue that an optimal capital structure is a key corporate governance mechanism to mitigate agency problems as it compels the manager to focus on maximising the wealth of the firm's claimholders. The analysis and results provided in this chapter improve the knowledge and understanding of selecting an optimal capital structure via an optimal debt-equity ratio. This mechanism can minimise the conflict of interest between the debtholder and the manager by minimising the company's marginal agency cost of debt.

The agency relationship based on contract theory has been formalised into a game relationship, which proposes that the objectives of the players partly or totally differ. These objectives have been specified following the systems of the game. The manager who acts on behalf of the shareholder seeks to minimise the company's weighted average cost of capital, while maximising its financial income from asset substitution. The debtholder, on the other hand, seeks to minimise the cost of monitoring their interest payment while maximising the debt face value. The model was designed to specify the contradictory interests of the players in the dynamic to reflect the agency problem.

First, the model analysed the non-cooperative game by providing the optimal capital structure and optimal cost of financing that minimises the company's

marginal agency cost of debt. Additionally, the financial management model, developed as a dynamic game problem, established cooperation by providing incentives to the manager to encourage them to focus on the wealth maximisation goal of the debtholder. These incentives serve as corporate governance mechanisms to mitigate the agency problem between the debtholder and the manager, and thus align the interests of the two parties. These are significant results and provide major contributions to the existing literature in this area. Some existing studies have sought to minimise the negative impact of asset substitution on the debtholder. However, there has been limited analysis of this as a dynamic game problem in which the strategic interactions between the two parties are modelled mathematically over time and the interactions provide self-derived incentives that align the interests of the two parties.

The three solution concepts considered were the open-loop and feedback Nash equilibrium and the Pareto outcome. In each solution concept, the following three encounters (parameter sensitivity) were considered: Encounter 1 - a greater weight on the cost of maximising the firm's financial income than minimising the weighted average cost of capital, and a greater weight on maximising the debtholder's cost of debt; Encounter 2 - a greater weight on maximising the debtholder's debt face value; and Encounter 3 - a greater weight on the cost of minimising the weighted average cost of equity than on maximising the company's financial income, and a greater weight on maximising the debtholder's cost of debt than the debt face value. These costs incorporate transaction and operational costs associated with implementing financial decisions in a firm or by a debtholder.

For an optimal capital structure in the non-cooperative game, the manager adopts a lower cost of equity for the company and avoids capital distribution until the debt matures. The company's cost of debt is also observed to be higher than the cost of equity. These present optimal strategies to minimise the marginal agency cost of debt. The Pareto optimal solution assigns a lower φ to the manager and, deductively, a higher weight to the debtholder's objective function. In addition, if a higher debt-equity ratio implies more financing opportunities for the firm, then the gains of Pareto are also higher than those of the open-loop and feedback cases at the Pareto optimal weight assignment [0.1, 0.9]. Overall, the gains of

Pareto were higher than in the open-loop and feedback cases for the manager, thus inducing them to select the Pareto-efficient outcome. The gains include a lower cost of debt and provision for more debt facility with lower interest rates. Theoretically, other gains may include a longer payback period. The result of the Pareto outcome reduces the opportunistic tendencies of the manager by offering a reduced cost of debt, but with a trade-off for a slightly lower liquid reserve. If the manager shifts from the optimal weight assignment, the debtholder responds with a higher cost of debt and lower debt facility in the future. This resolves the underlying agency problem between the debtholder and the manager in a dynamic continuous-time setting.

In this chapter, the debtholder-manager agency relationship and the emanating conflicts of interest were discussed. In the next chapter, the financial management model is analysed to observe the effect of creative accounting and other financial accounting theories on the model.

Chapter 6 – Corporate Governance, and the Choice of Optimal Capital Structure: The Role of Financial Accounting Theory

6.1 Introduction

6.1.1 Context

Significant interrelationships exist between financial accounting data and strategic decision-making within and across firms. This has shifted the major focus away from the ex-post recording and analysis of financial accounting data to its impacts on strategic interactions within the firm's agency relationships (Migdalas 2002). In principal-agent modelling, it is in fact nearly impossible to contract on actions, also known as a first-best contract (Ross 1973). This gives room for a second-best contract in which the manager's compensation contract is specified as a function of the firm's output processes (Bolton & Dewatripont 2005). Therefore, major management and debt contracts rely significantly on accounting information.

However, the output processes of the company are a function of the manager's actions and inactions and are also subject to the financial reporting methods used (e.g., historical cost and fair value accounting). Thus, different accounting methods may provide different estimations of the outputs of the company. Since principal-agent relationships and the optimal contract are based on the outputs of the company, it is important to consider the impact of different financial accounting theories on the optimal mechanism design and the optimal capital structure. Similarly, a reliance on the manager's reports may give rise to creative accounting actions by the manager. This generates a debate on the accuracy of the output processes reported by the manager. While Chapter 4 and Chapter 5 of this thesis examined information asymmetry with respect to asset substitution, suboptimal effort and suboptimal capital structure, financial accounting theories provide a legal framework for the manager in estimating the company's outputs.

Although pecuniary incentives can align managerial and shareholder interests with respect to overall firm performance, other aspects of managerial activities may no longer be aligned with the claimholder's interests. This is commonly

related to two financial activities of the manager. The first is related to the timing of income recognition. The manager's compensation contract may include certain thresholds which do not reward further effort in the current financial period. This provides the manager with an impetus to delay the reporting of certain financial data, such as profits, until the next financial period to enhance their reward in the that period. The second activity relates to risk management. The manager has incentives to take actions that pose higher or lower risks on the firm's income. Thus, the managerial incentive problem is further complicated by the manager's ability to sift the firm's financial signals via creative accounting (Tirole 2006). The manager also has incentives to apply creative accounting to uphold the financial covenants agreed upon with the debtholder, up to the extent that covenant violation transfers a level of control to the debtholder (Tirole 2006). This further impacts the capital structure reported as it may be over or understated.

Thus, in addition to developing a differential game-based mathematical optimisation model to obtain the optimal capital structure and an optimal mechanism design, it is imperative to study the impact of financial accounting innovations on the outcomes of the financial management and contracting models. To the best of the author's knowledge, no study has examined the impact of financial accounting theory on an optimal capital structure and optimal mechanism design model using a differential game theory framework.

6.1.2 Contributions of the Chapter

This study makes some significant contributions to the literature in this area, including the following: this is a new study on the implications of accounting issues in the specification of optimal capital structures in companies; it provides new insights into the nature of the effects of accounting methods on mechanism design for solving agency problems in specifying optimal capital structure; and it shows the new principles of designing corporate governance when accounting issues and methods have an impact on the identification of optimal capital structure. Finally, the study findings could inform decisions on the choice of accounting theories and enhance the optimal value of a company as such research does not exist in the current literature.

6.1.3 Structure of the Chapter

This chapter is organised as follows. Section 6.2 provides background information on the accounting methods that are the focus of this chapter. Section 6.3 provides the theoretical framework and the propositions to be examined. Section 6.4 re-implements the model presented in Chapter 4 to observe the impact of historical cost, fair value and creative accounting on the optimal outcomes of the differential game between the shareholder and the manager. Section 6.5 re-implements the model presented in Chapter 5 to observe the impact of historical cost, fair value and creative accounting on the optimal outcomes of the differential game between the debtholder and the manager. Sections 6.6 and 6.7 discuss the results and implications of the study. Section 6.8 summarises and concludes the chapter.

6.2 Background Literature

6.2.1 Historical Cost and Fair Value Accounting, Corporate Governance and the Optimal Capital Structure

Cost measurement is a significant part of accounting theory. Cost accounting methods are classified as historical cost and fair value accounting. Historical cost accounting recognises the original cost of an asset on the company's statement of financial position (Tkachuk 2019). Fair value accounting reports the estimated value of assets and liabilities based on market prices, reporting the changes in the fair value as gains and losses in the income statement (Cristea 2017; Laux & Leuz 2010). There has been a sustained debate in the literature for and against these two accounting methods with respect to profit generation, productive assets, and earning management, among others (Godfrey et al. 2010; Marra 2016). This debate has been centred on the trade-off between relevance and reliability (Blecher 2019). For example, the argument for unreliability in fair value accounting stems from the use of estimated value according to current market prices.

Proponents of historical cost accounting argue that fair value estimates are not as reliable as historical cost. Investors would be disinclined to rely on a company valuation based on fair value estimates (Barth 1994). Khurana and Kim (2003) found evidence that historical cost accounting is more informative for loans and

deposits in bank holding companies. Since historical cost is based on actual cost, not estimations, it is found to be admissible in economic decision-making. Moreover, historical cost accounting can be justified by a company's going concern assumptions, since a company is expected to exist indefinitely, hence the non-monetary expectations can be fulfilled. In the principal-agent relationship, historical cost accounting emphasises the manager's accountability for assets provided by claimholders and the outputs on the company's net value of equity. On the other hand, critics have argued that profits recorded under historical cost accounting are retrospective, thus they only reflect a financial capital concept and not an accurate reflection of the company's purchasing power. Furthermore, historical cost accounting largely recognises the past actions of the manager, thus, it lacks objectivity (Godfrey et al. 2010). A fixed asset on a company's financial statement is likely to be worth less in the current market (Kaya 2013).

The arguments against fair value accounting are associated with suboptimal investments decisions, easier manipulation, unreliability, and increased volatility, among others (Ball 2006; Rayman 2007; Ronen 2008; Watts 2006). Advocates for fair value accounting on the other hand argue that it is an improvement on historical cost accounting as it can boost firm efficiency and promotes transparency by providing useful financial information and reliable signals about financial health (Barlev & Haddad 2003; Bleck & Liu 2007; Gigler, Kanodia & Venugopalan 2007). Blecher (2019) suggested fair value accounting as a more appropriate measure of non-financial assets. Since shareholders and debtholders require financial information for decision-making, fair value accounting is recommended for its forward-looking approach.

6.2.2 Mixed Methods

Accounting standard boards have established that a single measurement approach may be unrealistic for business entities. This has made provision for mixed measurement accounting, with different accounting methods being used for different financial reporting. For example, the IFRS outline the use of historical cost accounting in some cases, and fair value in others, thus providing some flexibility. However, researchers have argued that the risks resulting from the flexibility provided by mixed measurement outweigh its gains. Mixed method

accounting reduces the consistency of measurement (Barth 2007). In addition, users of financial statements incur a cost as they are unable to identify line items for which a certain measurement has been applied without reading the additional notes to the financial statements (Kawamura 2015).

Thus, the choice of historical cost or fair value accounting may have an impact on the assessment of the manager's quality of effort and the need for mechanism designs such as capital structuring.

6.2.3 Creative Accounting and Corporate Governance

Copeland (1968) was amongst the earliest researchers to use the term 'accounts manipulation'. He established this by altering figures in a firm's financial statements upward and downward to depict the impact on a firm.

Creative accounting was originally conceived to offer some pliability within regulatory frameworks in order to offer true and fair presentations of an entity's business (Vukoja 2020). The goal was to address different economic and accounting uncertainties arising in an enterprise environment. Over time, it evolved into a manipulative accounting tool (Marilena & Corina 2012), by taking advantage of loopholes in the rules and regulations (Bhasin 2016). The fall of major US firms such as Enron, World Com, and Sahara Group, among others, gave rise to a thorough examination of the impact of creative accounting theory. Other words used as synonyms for creative accounting in the literature include financial engineering, cosmetic accounting, income smoothing, window dressing and innovative accounting (Gupta & Kumar 2013).

The two common types of financial accounting policies heavily impacting on the shareholder-manager principal-agent relationship are conservatism and reliability. Accounting conservatism highlights the probability of reporting profits before they are fully earned or understating liabilities. Accounting reliability refers to the trustworthiness of the financial accounting data reported (Scott 2015). These two accounting policies significantly affect investor-informing usefulness and decision-making in financial reports.

Merchant (1989), and Ronen and Sadan (1981) discussed the many ways in which managers can modify the external evaluation of their firm's performance.

Creative accounting with respect to earnings manipulations has two commonly discussed techniques: accounting methods and operating methods. Accounting methods, also known as 'cooking the books', provide flexibility for managers within the IFRS. This includes flexibility in providing for loan losses, as various permissible assumptions may be made about a customer's ability to reimburse their trade credit, or when decreasing its provision for losses in a certain financial period and improving the firm's profitability assessments. Furthermore, sales that happened in the last month of a financial year may be discretionally reported in the first month of the next financial year, and thus have an impact on the estimation of the firm's outcomes in that year. In addition, a manager may practice balance-sheet window dressing by transferring poor projects and related debts to the firm's non-consolidated subsidiaries (Tirole 2006).

Such manipulations have the potential to misrepresent the firm's performance to its claimholders, altering their assessment of the firm's need for capital restructuring or the optimal mechanism required to maximise the manager's efforts. Direct costs involved include that the manager's efforts may be diverted to practicing creative accounting, and corporate resources may be used in achieving these practices (Tirole 2006).

A second earning manipulations technique is known as the operating method. The manager may distort the firm's operating strategy in order to improve the external assessment of the firm. For instance, the manager may choose to delay property, plant and equipment maintenance, as well as lower the firm's inventory levels, so as to boost current income. Similarly, the firm may run sales in the last few months of the financial period rather than at the start of the next financial period (Tirole 2006). Other means by which creative accounting can be implemented in a firm's financial reporting include: varying inventory methods at different levels of inflation in the economy, different valuations of depreciation techniques, disproportionate alteration of the firm's revenue sources, exaggerated valuation of goodwill, varying methods for lease (Agarwal 2008), and contrived increase in the firm's share value (Akenbor & Ibanichuka 2012).

Financial accounting theory is significant in the design of a dynamic optimal mechanism and optimal capital structure specification. The productive assets required by the company in any period are determined by the measurements of

its assets. According to the Australian Accounting Research Foundation, policies applied in the recognition and measurement of a company's assets and liabilities are a determinant of its capital measurement (Godfrey et al. 2010). Thus, it is important to consider the impact of different accounting measurement practices on the developed differential game models and the outcomes for an optimal capital structure specification in a continuous-time framework.

6.3 Theoretical Framework

As established in the literature, the different accounting methods adopted in a company have a direct impact on contract design and outcomes (Scott 2015). The theoretical framework used to evaluate the impact of historical cost, fair value and creative accounting on a differential game model for financial management is presented in Figure 6.1 below.

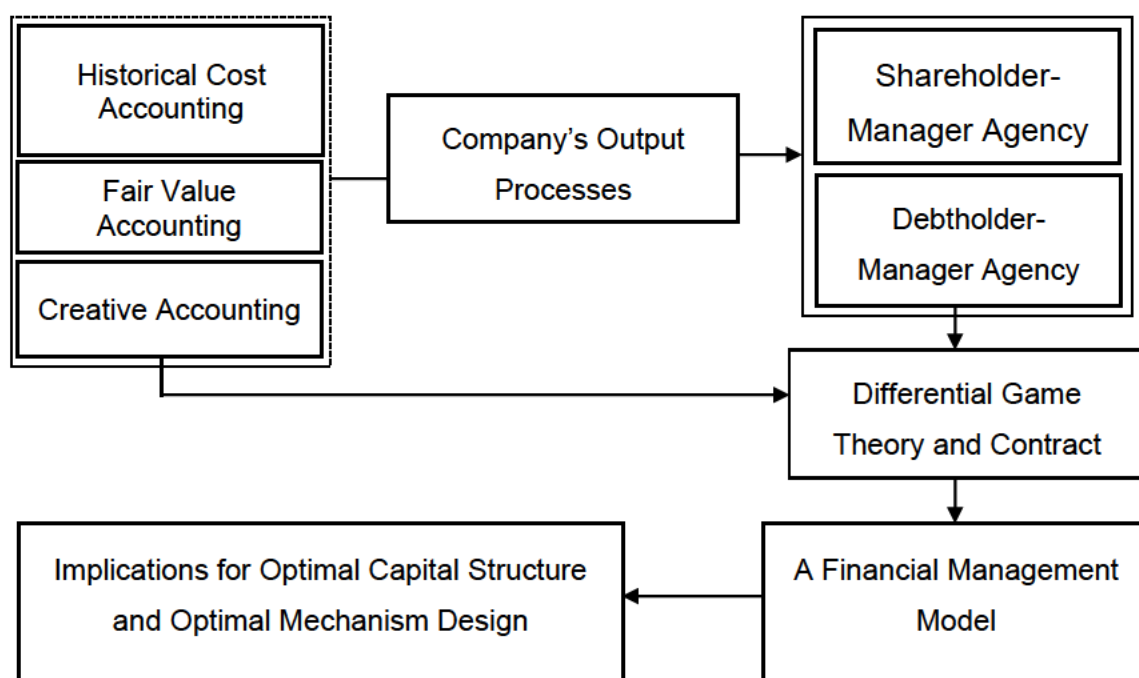


Figure 6.1 Theoretical framework

Managers may select either historical cost or fair value methods in reporting the company's productive assets. In addition, due to the regulatory support provided to creative accounting in terms of flexibility in reporting, the manager may decide upon the timing of income recognition. This may impact the outcomes of the firm, the optimal contract design and the required optimal capital structure (Yadav

2014), thus impacting its operational decision-making (McGuigan, Moyer & Harris 2014).

Financial accounting data drive strategic interactions amongst the stakeholders of the company (Migdalas 2002): the manager, shareholder, and debtholder. Therefore, different accounting methods may derive different dynamic game-theoretic implications in strategic interactions (Scott 2015). Hence, there is a significant relationship between dynamic game theory contract models and financial accounting methods. This study examines this relationship on financial management game-theoretic models. Numerical analysis was carried out to determine the implications for optimal strategies, optimal states and the optimal capital structure.

Based on the above theoretical framework, the following propositions were developed to observe the impact of financial accounting theories on the outcomes of optimal capital structure and the optimal mechanism design in a dynamic framework. These are applied in both a shareholder-manager and debtholder-manager agency relationship.

Proposition 1: *The historical cost accounting method has an impact on optimal mechanism design and optimal capital structure in a dynamic framework.*

Proposition 2: *The fair value accounting method has an impact on optimal mechanism design and optimal capital structure in a dynamic framework.*

Proposition 3: *The creative accounting method can manipulate the dynamic contractual outcomes and the required optimal capital structure in a principal-agent relationship.*

6.4 The Shareholder-Manager Differential Game Model

To evaluate the impact of the three highlighted financial accounting methods on a firm's optimal capital structure and optimal mechanism design, first, the differential game model for the shareholder-manager dynamic agency relationship (presented in Chapter 4) was used. The results of the model provide the optimal and equilibrium strategies required to obtain the optimal payoffs of the two parties. However, the shareholder heavily relies on the company's reported financial information in order to incentive the manager appropriately.

Therefore, the accuracy of the company's output processes is crucial in the dynamic contract setting to derive an optimal capital structure for the company and an optimal mechanism design.

6.4.1 A Dynamic Financial Management Model

The model equations used in Chapter 4 are presented here as the numerical analysis provided in this section is based on the proposed dynamic financial management model. As a benchmark, only the best compromise Pareto-efficient model at weight assignment [0.9, 0.1] is incorporated into this analysis.

Notation:

Sets and indexes

t = Time period (1, 2, 3, ..., T)

j = Players (1: Shareholder, 2: Manager)

The Pareto-efficient model is specified as follows:

$$\max \int_0^T \phi e^{-\rho t} (1 - \theta) (Y(t) + u_1(t)^2 - I(t) - u_2(t)^2) + e^{-\rho t} (1 - \phi) (ru_3(t)^2 + I(t) - u_4(t)^2) dt \quad (6.1)$$

Subject to:

$$\dot{Y}(t) = \beta S(t) + (u_4(t) - u_3(t))rY(t) - u_4(t)I(t) - u_1(t)Y(t) + u_2u_4\alpha_2D(t), Y(0) = Y_0; I(0) = I_0 \quad (6.2)$$

$$\dot{D}(t) = (\alpha + u_4(t)\alpha_2)D(t) - u_1(t)Y(t), D(0) = D_0 \quad (6.3)$$

$$\dot{I}(t) = u_4(t)I(t) + \chi Y(t), Y(0) = Y_0; I(0) = I_0 \quad (6.4)$$

$$\dot{S}(t) = ru_3(t)Y(t) + (1 - u_2)\alpha_2u_4(t)D(t) - \mu S(t), S(0) = S_0 \quad (6.5)$$

$$E(t) = Y(t) + S(t) - D(t) \quad (6.6)$$

$$\frac{\dot{D}(t)}{\dot{E}(t)} = \frac{(\alpha + u_4(t)\alpha_2)D(t) - u_1(t)Y(t)}{\beta S(t) + (u_4(t) - u_3(t))rY(t) - u_4(t)I(t) - u_1(t)Y(t) + u_2u_4\alpha_2D(t) + ru_3(t)Y(t) + (1 - u_2)\alpha_2u_4(t)D(t) - \mu S(t) - (\alpha + u_4(t)\alpha_2)D(t) + u_1(t)Y(t)} \quad (6.7)$$

$$I(t) \geq \chi Y(t) \quad (6.8)$$

$$u_1(t), u_2(t), u_3(t), u_4(t) \geq 0 \quad (6.9)$$

6.4.2 Evaluating Propositions

The propositions were evaluated using a numerical analysis of the model described in Chapter 4, with similar data used for testing. To evaluate Propositions 1, 2 and 3, the financial accounting measurement and reporting method of the manager needed to be considered based on the following scenarios:

1. Numerical evaluation 1 (historical cost accounting) – The company's productive assets are reported using the historical cost accounting method.
2. Numerical evaluation 2 (fair value) – The company's productive assets are reported using the fair value accounting method.
3. Numerical evaluation 3 (creative accounting) – The company's cash flow recognises income made from the last quarter of the previous financial period. This represents creative accounting.

6.5 The Debtholder-Manager Differential Game Model

The impact of the creative accounting method on a firm's optimal capital structure and optimal mechanism was also observed for a dynamic game model between the debtholder and manager. The differential game financial management model for the Debtholder-Manager agency relationship (presented in Chapter 5) was used.

6.5.1 A Dynamic Financial Management Model

The model equations used in Chapter 5 are presented here as the numerical analysis provided is based on the proposed dynamic financial management model.

Notation:

Sets and indexes

t = Time period (1, 2, 3, ..., T)

j = Players (1: Manager, 2: Debtholder)

The debtholder-manager Pareto-efficient model is specified as follows:

$$\min \int_0^T \left(\varphi e^{-\rho t} \left(\omega_1 \mu(t) l(t)^2 + \omega_2 c(t)^2 (1 - \theta) (1 - \mu(t)) - \omega_3 r M(t) \right) \right) + e^{-\rho t} (1 - \varphi) (\gamma_1 c(t)^2 - \gamma_2 D(t)) dt \quad (6.22)$$

Subject to:

$$\dot{M}(t) = \beta S(t) + r M(t) - c(t) - l(t), M(0) = M_0; S(0) = S_0 \quad (6.23)$$

$$\dot{D}(t) = \alpha D(t) + c(t), D(0) = D_0 \quad (6.24)$$

$$S(t) = S_0 e^{(r-\beta)t}, S(0) = S_0 \quad (6.25)$$

$$E(t) = M(t) + S(t) - D(t) \quad (6.26)$$

$$\frac{\dot{D}(t)}{\dot{E}(t)} = \frac{\alpha D(t) + c(t)}{\beta S(t) + r M(t) - c(t) - l(t) + S_0 e^{(r-\beta)t} - \alpha D(t) - c(t)} \quad (6.27)$$

Following the definition for $E(t)$, $\mu(t) = 1 - \frac{D(t)}{M(t) + S(t)}$

Only the best compromise Pareto-efficient solution at weight assignment [0.1, 0.9] was considered to observe the impact of creative accounting.

6.5.2 Evaluating Propositions

The propositions were evaluated using a numerical analysis of the model described in Chapter 5. To evaluate Propositions 1, 2 and 3, the financial accounting reporting method of the manager needed to be considered based on the following scenarios:

1. Numerical evaluation 1 – The company's productive assets are reported using the historical cost accounting method.
2. Numerical evaluation 2 – The company's productive assets are reported using the fair value accounting method.
4. Numerical evaluation 3 – The company's liquid reserve recognises estimated credit sales without accurate estimation of the customer's reliability. This represents creative accounting.

6.6 Computational Experiments and Results

The following assumptions were considered for the numerical evaluation.

6.6.1 Assumptions

To observe the impact of fair value, historical and creative accounting methods, the initial values used in the dynamic game were restated according to the specified numerical evaluations:

1. In deploying Numerical evaluation 1, the value of productive assets reported in the case study company's financial statements was used.
2. In deploying Numerical evaluation 2, the initial value of productive assets was restated to be lower. This is because fair value accounting recognises the value of the company's assets at an estimated worth assumed to be less due to greater depreciation and general inflation costs. This is specified as a 30% decrease in the initial value of productive assets $S(0)$. The 1981 Financial Accounting Standards Board (FASB) highlights of financial reporting issues suggests that profit from continuing operations under the fair value/current cost accounting method is often less in most industries than under the historical cost accounting (Godfrey et al. 2010).
3. In deploying Numerical evaluation 3 for the shareholder-manager relationship, cash flow was restated to be higher to recognise the last quarter sales deferred into the initial period. This is specified as a 30% increase in the initial value of cash flow $Y(0)$.
4. Numerical evaluation 3 for the debtholder-manager relationship was deployed by restating the company's initial value of liquid reserves to include estimated credit sales, which may be unrealised. This is specified as a 30% increase in the initial value of liquid reserve $M(0)$.

According to the notes to the financial statements of the case study company, 'the financial statements have been prepared using the historical cost basis except for derivative financial instruments and share-based payments which are measured at fair value' (Metcash Limited 2018, p. 62). Therefore, it is deduced that the company's productive assets have been reported using historical cost accounting. Thus, Numerical test 1 is the result reported for the dynamic game

model in Chapters 4 and 5. The second and third numerical tests were carried out such that the results of the model for historical cost, fair value, and creative accounting were compared. In addition, the financial data used in Chapters 4 and 5 were also used for the numerical analysis.

6.6.2 Results for the Shareholder-Manager Differential Game Model

The basis for parameter sensitivity, as specified in Chapter 4, is as follows:

Encounter 1: no parameter sensitivity, Encounter 2: $\beta = 0.60$, Encounter 3: $\alpha = 0.3, \alpha_2 = 0.2$, Encounter 4: $\beta = 0.60, \alpha = 0.3, \alpha_2 = 0.2$.

6.6.2.1 *Impact on Pareto-efficient strategies*

The figures below show the different outcomes of each accounting method (historical cost, fair value and creative accounting) for the optimal strategies of the shareholder and manager for the best compromise Pareto-efficient cooperative solution.

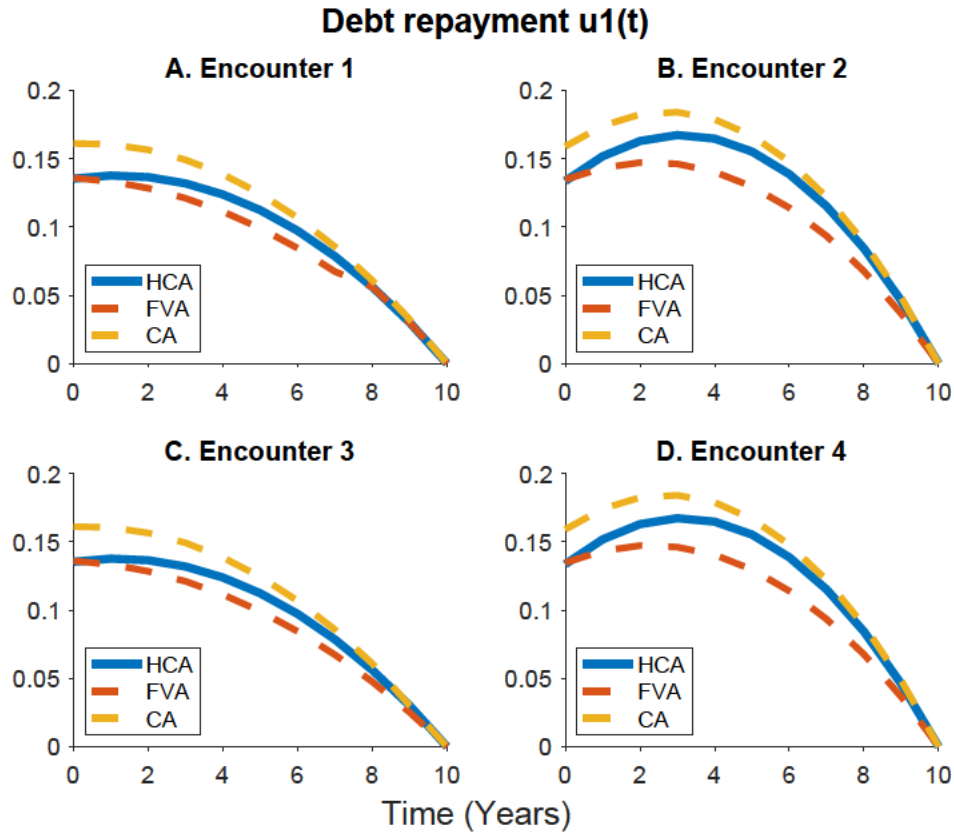


Figure 6.2 Impact of historical cost, fair value and creative accounting on an optimal debt repayment strategy

Figure 6.2 shows a comparison of the optimal debt repayment strategy applicable for historical cost, fair value and creative accounting. The original model was solved with historical cost accounting specified by the blue colour legend. The results show creative accounting requires a higher debt repayment strategy to curb the manager's opportunistic behaviours. This outcome is further heightened by the company's payout rate. With an increase in the company's productive asset payout rate and creative accounting applied, the shareholder adopts a higher repayment strategy. It is also noted that the fair value accounting method requires the shareholder to apply a lower debt repayment strategy. This may be explained by the higher cost of debt associated with fair value accounting (Magnan, Wang & Shi 2016).

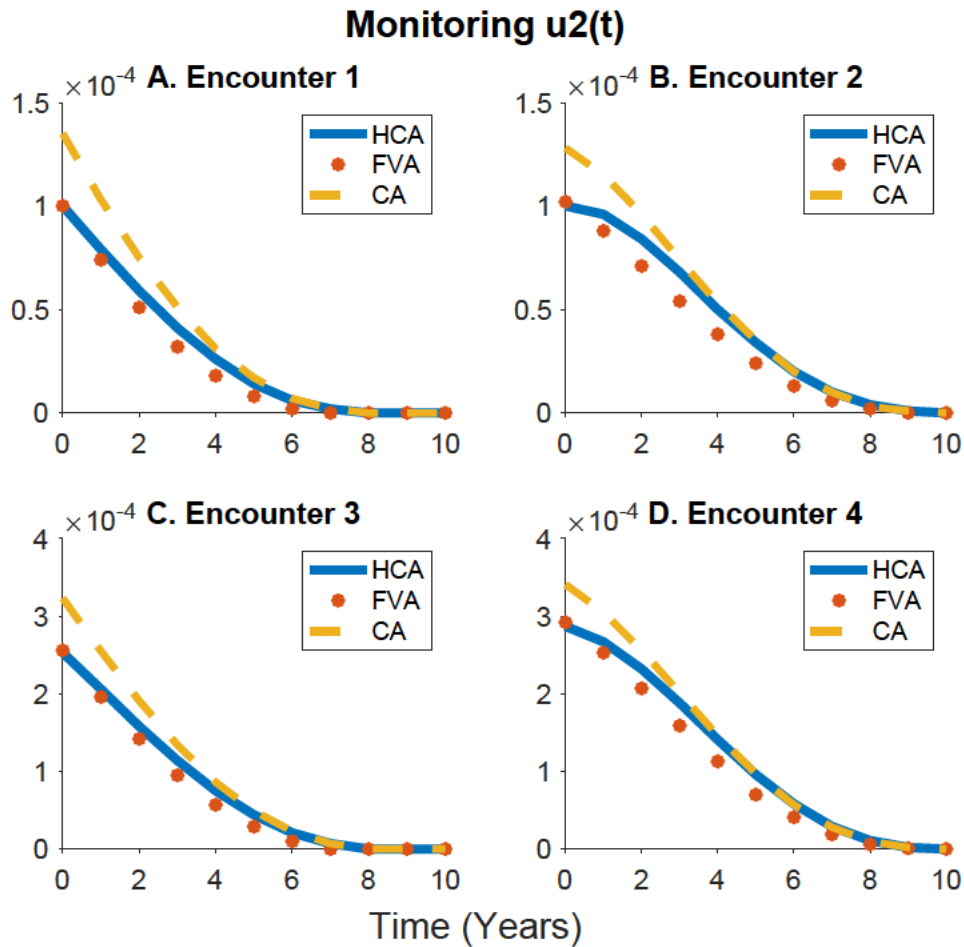


Figure 6.3 Impact of historical cost, fair value and creative accounting on optimal monitoring

Figure 6.3 provides the optimal monitoring required for the different accounting methods. Although the results for the three financial accounting methods are similar, creative accounting requires a slightly higher level of monitoring to protect the shareholder's interests at the start of the management contract. Similar to the previous analysis, higher monitoring is required when the company use of new debt is increased significantly.

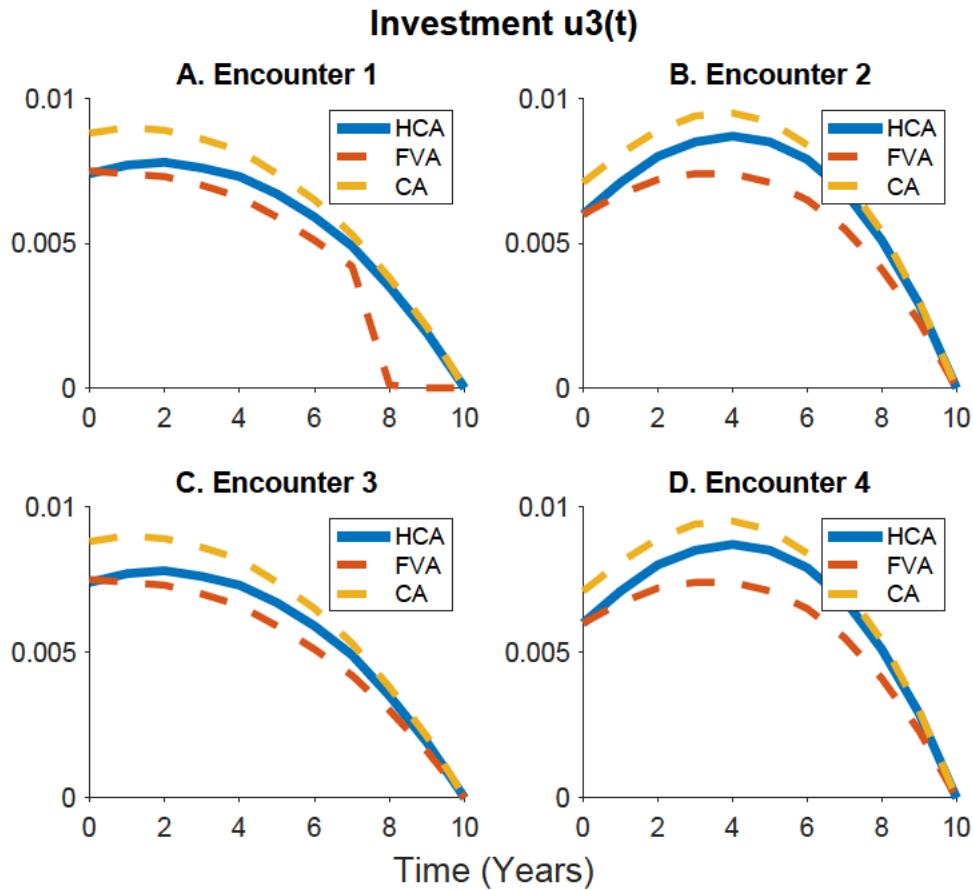


Figure 6.4 Impact of historical cost, fair value and creative accounting on optimal investment

Figure 6.4 represents the impact of historical cost, fair value and creative accounting on the investment strategy of the manager. From the results, it could be seen that creative accounting induces the manager to apply more investment to protect their payoff. This can be explained by the optimal strategy selected by the shareholder. Since the shareholder selects higher debt repayment and monitoring when creative accounting is applied, this may induce the manager to select even higher investment. This suggests that the optimal mechanism designed in the model could disincentivise the manager from creative accounting.

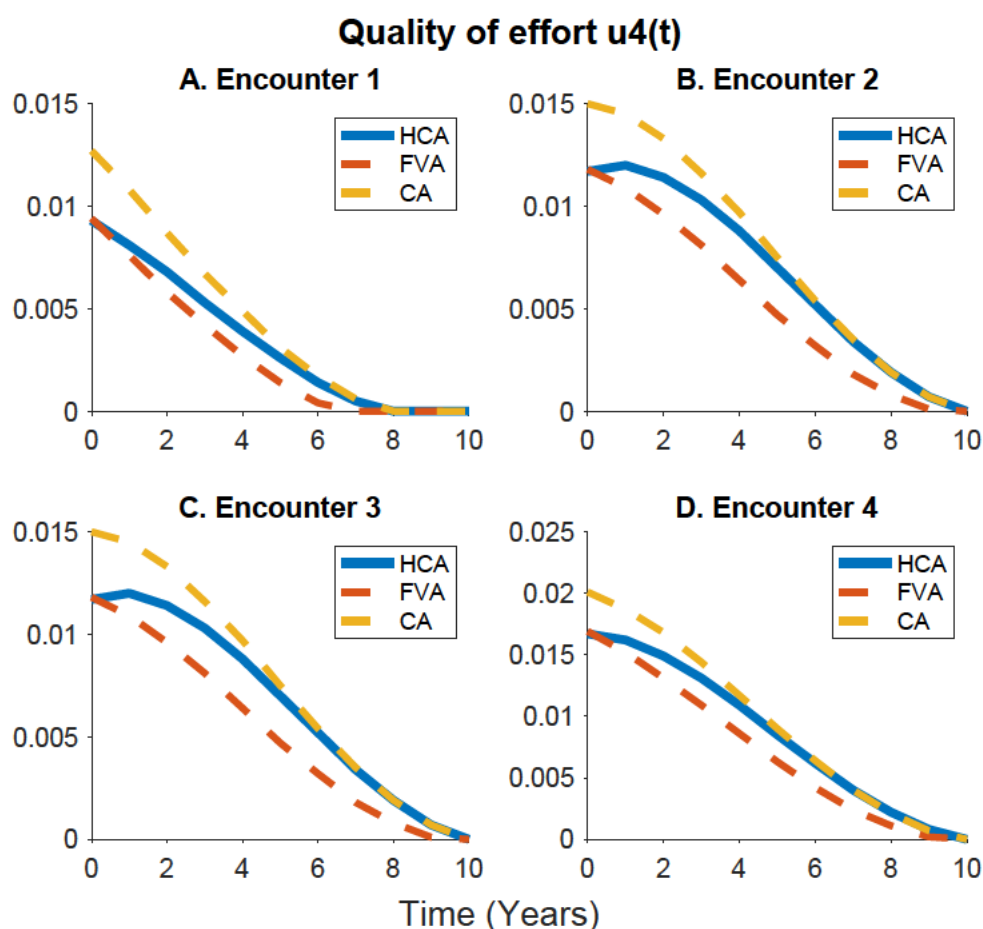


Figure 6.5 Impact of historical cost, fair value and creative accounting on the optimal quality of effort

Figure 6.5 provides the optimal quality of effort of the manager for the three different accounting methods. The manager requires a higher quality of effort at the beginning of the contract if they apply creative accounting. This is also higher when the company uses a significantly greater value of new debt and has a higher productive asset payout rate. The results suggest the fair value accounting method may require a lower quality of effort from the manager.

6.6.2.2 Impact on trajectories of the state variables

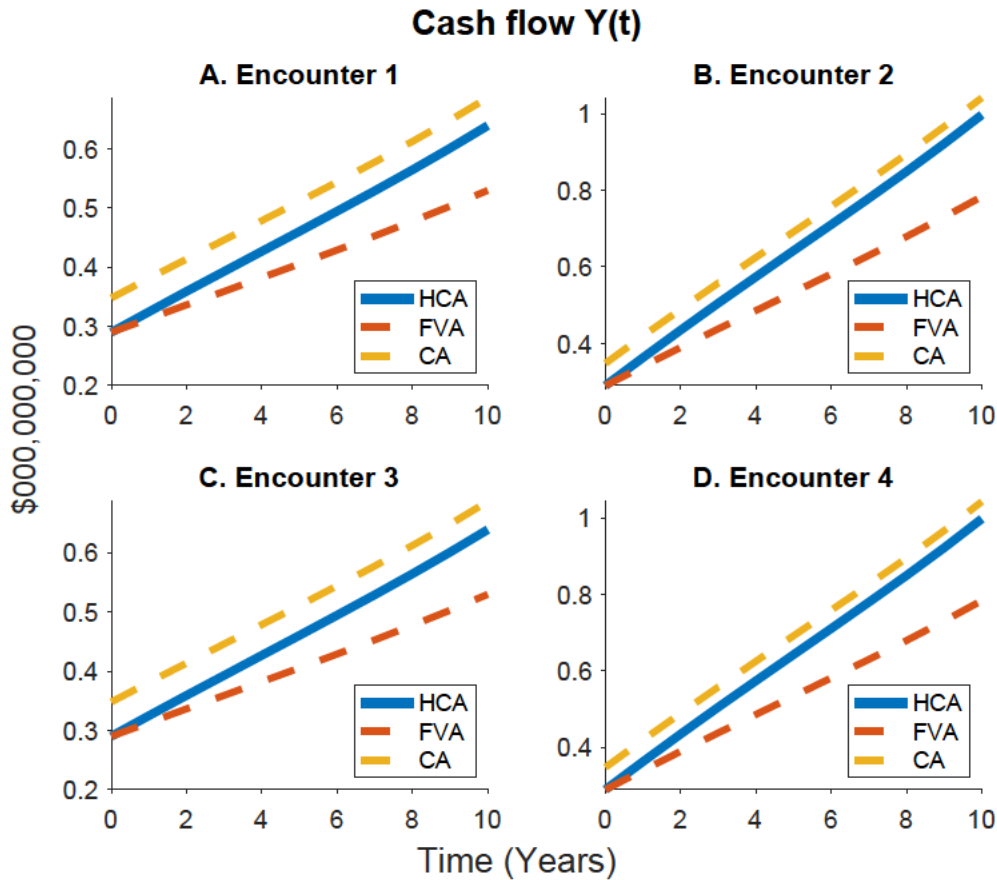


Figure 6.6 Impact of historical cost, fair value and creative accounting on cash flow

Intuitively, creative accounting corresponds to a higher cash flow throughout the contract as different timings for income recognition are selected by the manager to suit the company's output processes. This could give the company a better performance assessment. On the other hand, fair value accounting reduces the cash flow process of the company, as the company's assets are less valued according to current market prices and inflation. This contrasts with historical cost accounting, which adopts an original cost basis. Taplin, Yuan and Brown (2014) suggest that a company's decision to use fair value or historical accounting may be a function of its earnings volatility. For example, companies with above average earnings volatility may be more inclined to use fair value accounting.

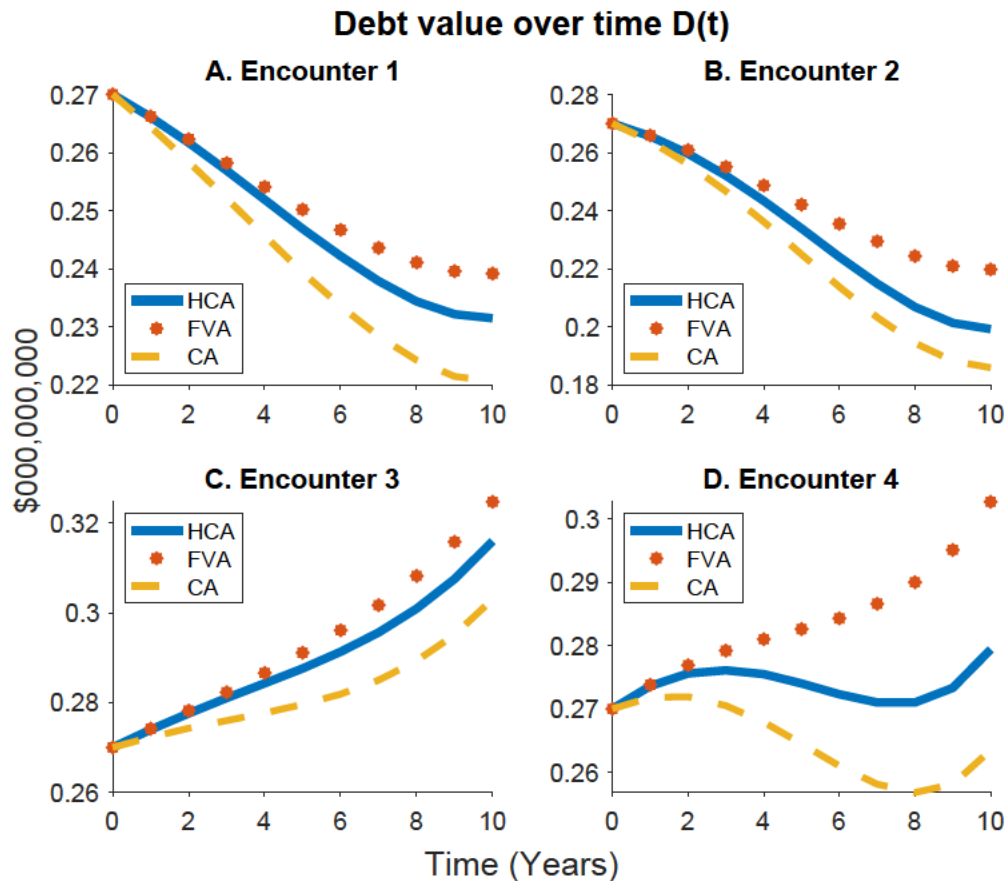


Figure 6.7 Impact of historical cost, fair value and creative accounting on debt value

Figure 6.7 shows that creative accounting can cause a marked decline in the debt value of the company over time when compared to historical cost or fair value accounting. This can be explained by the cash flow reported in each period under creative accounting. A higher cash flow provides the company more capacity to repay its debt in each financial period. Moreover, a higher (but false) estimation of the company's cash flow can reduce the company's estimation of a need for more debt finance. In contrast, the fair value accounting method can cause the debt value of the company to rise more steadily than with historical cost accounting. Due to factors such as a pandemic or inflation, at current market prices, the productive assets are valued less than the original cost, thus declining the income or value derived from the company's assets. This can reduce the company's ability to repay its outstanding debts or increase its need for debt finance. In addition, changes in the fair value of a company's debt could be attributed to other factors, such as credit risk and market interest rates (Scott 2015). In Encounter 4, creative accounting reduces the company's debt further

and fair value accounting causes a sharp increase in the value of debt when the company has a higher asset payout rate and higher use of new debt. This implies that the impact of fair value and creative accounting is observed more when the company's productive asset payout rates are higher, and the company adopts a significantly greater use of new debt.

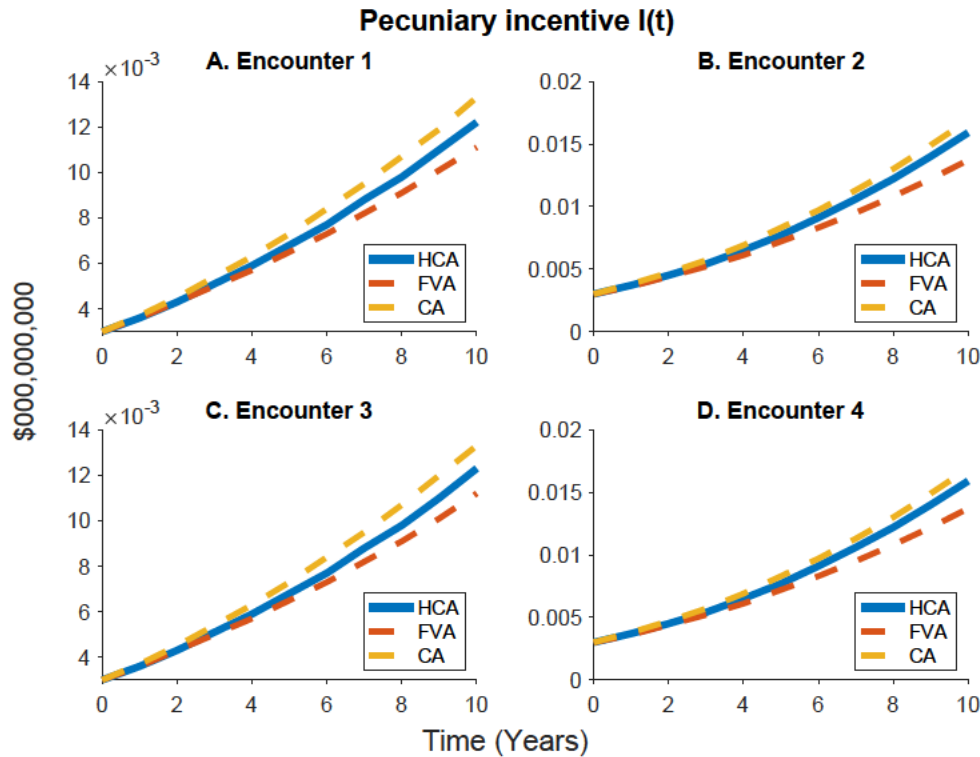


Figure 6.8 Impact of historical cost, fair value and creative accounting on pecuniary incentives

From Figure 6.8, the manager's pecuniary incentive gradually increases with creative accounting, as this method can boost the reported cash flow of the company in any period. However, the optimal mechanism design provides sufficient incentive to maintain the value of the manager's pecuniary incentive in line with their optimal quality of efforts and actions in the strategic interaction. This prevents a striking growth in the manager's income due to creative accounting. In contrast, fair value accounting causes slower growth in the pecuniary incentive.

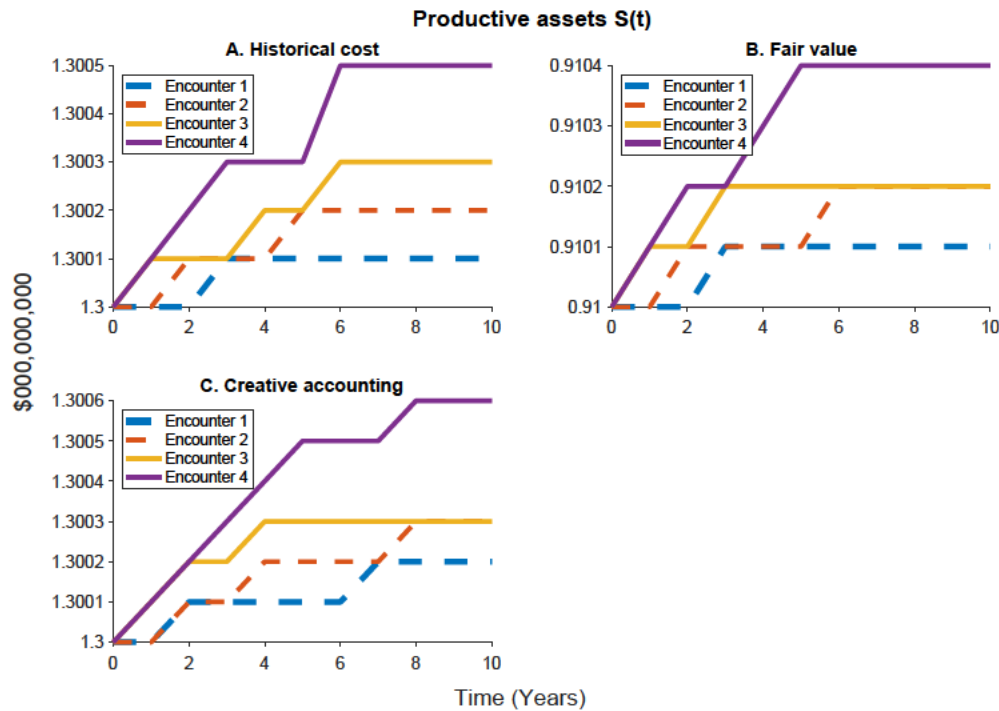


Figure 6.9 Impact of historical cost, fair value and creative accounting on productive assets

As established earlier, fair value accounting can downwardly review the value of a company's productive assets due to current market prices, thus providing a more realistic estimation of the company's assets and its purchasing power. Historical cost accounting reports the original cost of the company's assets in addition to any value derived from investment and the use of debt finance. Although the differences are not significant due to the low riskless rate applied in the model, the results suggest that creative accounting may cause slightly higher growth in the value of the assets reported.

6.6.2.3 Impact on debt-equity ratio

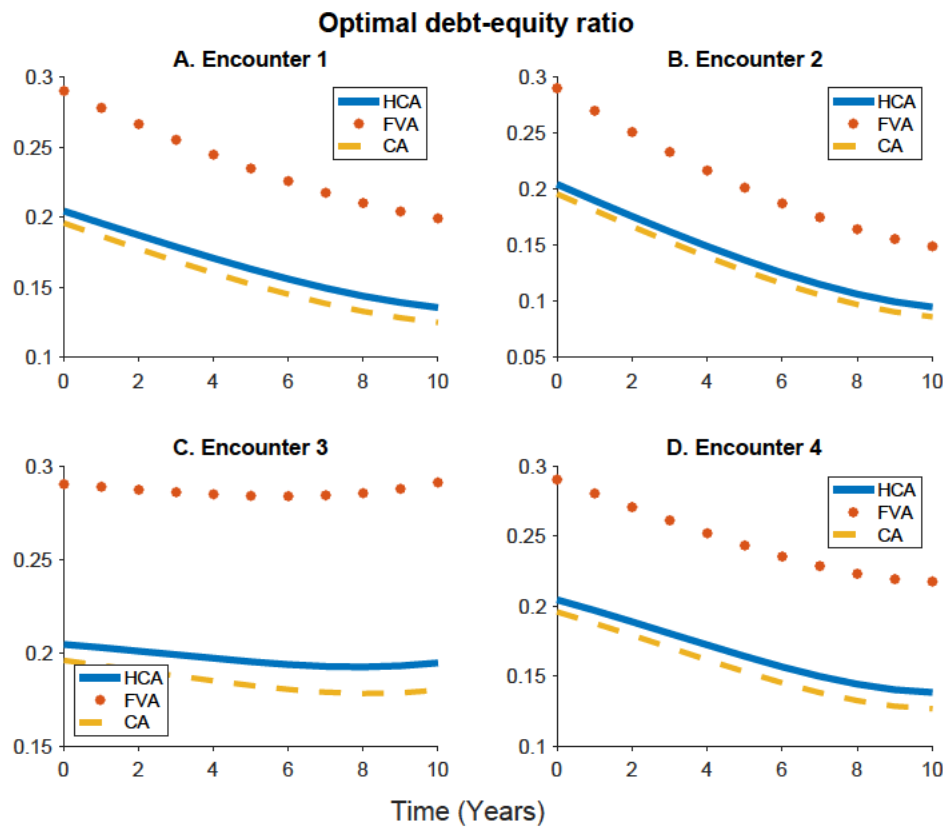


Figure 6.10 Impact of historical cost, fair value and creative accounting on the optimal capital structure

Figure 6.10 shows the impact of historical cost, fair value and creative accounting on the optimal capital structure specification for the dynamic optimal mechanism design for the shareholder-manager agency relationship. It is observed that the fair value accounting method reports a higher optimal debt-equity ratio than historical cost or creative accounting. This outcome could be attributed to the current market pricing estimation of fair value accounting. As the productive assets are valued less than the original cost, the optimal debt level required increases. This is consistent with arguments on the impact of fair value accounting during the 2008 global financial crisis. The argument suggests that fair value accounting contributes to disproportionate leverage in expansion periods and leads to enormous write-downs in contraction periods (Laux & Leuz 2010). On the other hand, creative accounting suggests a slightly lower optimal debt-equity ratio in comparison to the historical cost accounting method. In the

creative accounting method, productive assets are valued at historical cost. However, the cash flow of the company is reported creatively to ensure the discretionary timing of income recognition, thus boosting the company's cash flow in a certain period. This can explain a reduced need for debt financing.

6.6.3 Results for the Debtholder-Manager Differential Game Model

The numerical tests for the fair value and creative accounting propositions were also applied to the debtholder-manager game relationship examined in Chapter 5. The basis for parameter sensitivity in the balancing cost factors is as follows:

Encounter 1: $[\omega_1 = 2, \omega_2 = 2, \omega_3 = 5; \gamma_1 = 5, \gamma_2 = 2]$, Encounter 2: $[\omega_1 = 2, \omega_2 = 2, \omega_3 = 5; \gamma_1 = 5, \gamma_2 = 10]$, Encounter 3: $[\omega_1 = 50, \omega_2 = 2, \omega_3 = 2; \gamma_1 = 5, \gamma_2 = 2]$. Encounter 1 considers a higher balancing cost factor ω_3 for maximising the company's financial income and a higher monitoring cost γ_1 for the debtholder. Encounter 2 considers a higher opportunity cost γ_2 incurred by the debtholder to provide the debt face value. Encounter 3 considers a higher operational cost ω_1 for minimising the company's weighted average cost of equity. The results of the numerical tests are discussed in turn.

6.6.3.1 *Impact on trajectories of the state variables*

The figures below show the impact of the financial accounting measurement and reporting methods on the company's financial variables and performance indicators (i.e., the company's liquid reserve and debt value). The optimal capital structure specification required for the dynamic optimal mechanism design for the debtholder-manager agency relationship at the best compromise Pareto-efficient cooperative solution is also presented.

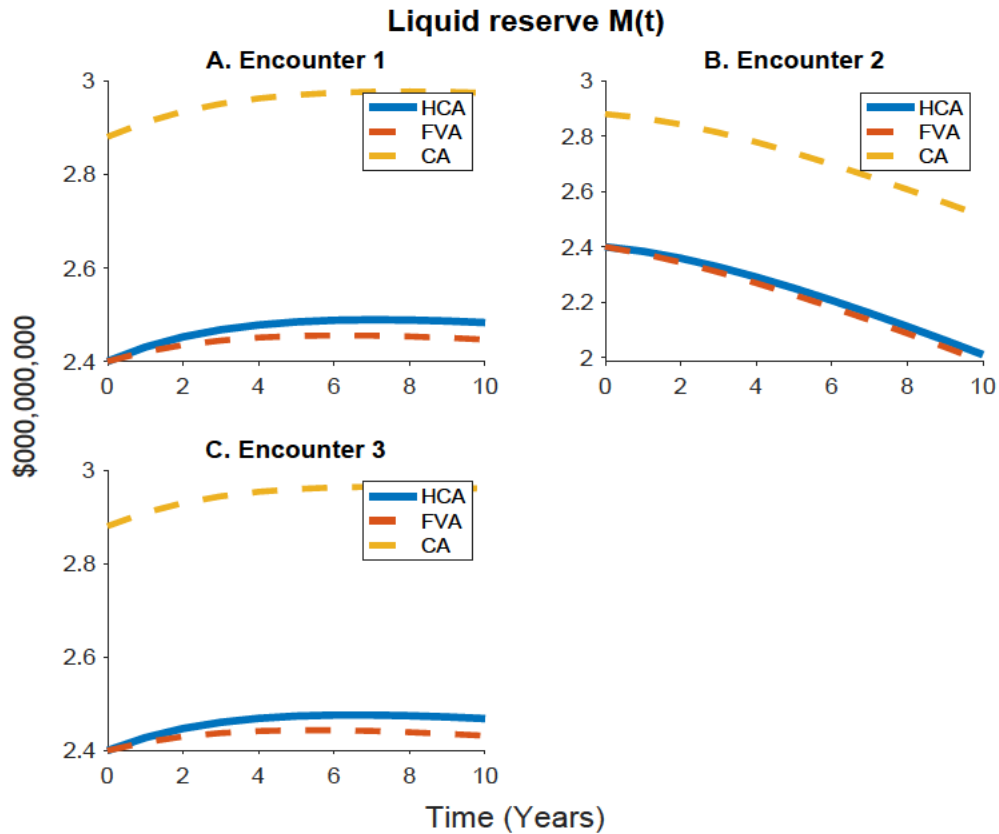


Figure 6.11 Impact of historical cost, fair value and creative accounting on the liquid reserve

Intuitively, creative accounting boosts the liquid reserve of the company due to the manager's discretionary income reporting. Fair value and historical cost accounting suggest a similar trend in the growth or decline of the company's liquid reserves. However, fair value accounting could cause a lower value in the liquid reserve of the company due to the market pricing of the company assets.

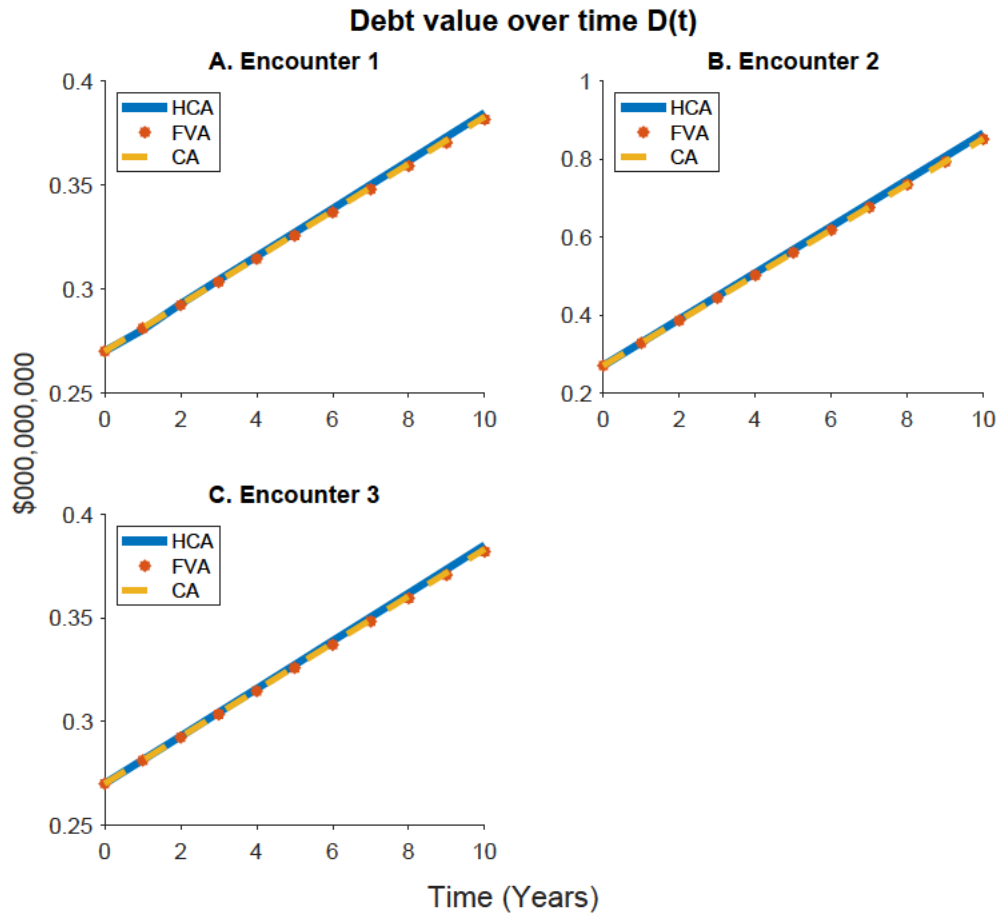


Figure 6.12 Impact of historical cost, fair value and creative accounting on debt value

Figure 6.12 shows no significant difference in the historical cost, fair value and creative accounting of the company. This implies that the debt value of the company is not directly impacted by the choice of accounting theories in a debtholder-manager strategic relationship. This could be attributed to the debtholder's strategic interaction with the company. Although the manager and debtholder may agree on accounting policies, accounting principles provide a substantial opportunity for discretion (Tirole 2006). This places the debtholder as external to the company's accounting policies and, as a result, the debt value provided by the debtholder may remain unaffected by the company's accounting practices.

6.6.3.2 Impact on the optimal capital structure

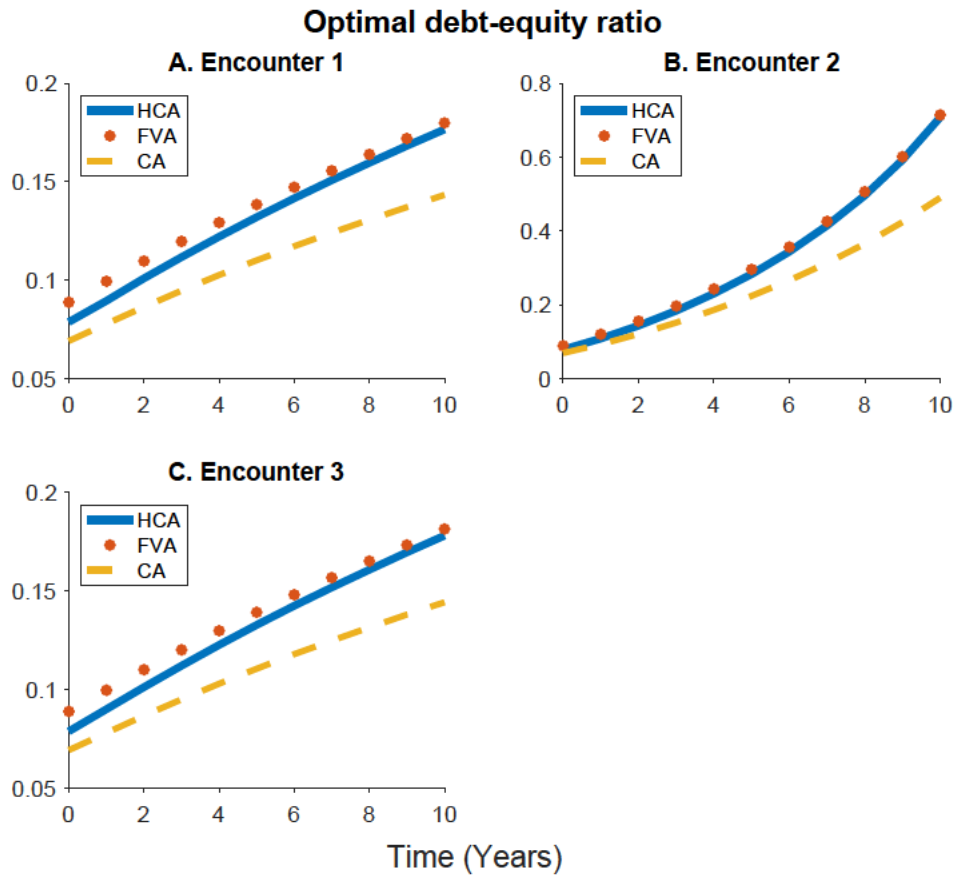


Figure 6.13 Impact of historical cost, fair value and creative accounting on the optimal debt-equity ratio

Figure 6.13 illustrates the impact of historical cost, fair value, and creative accounting on the optimal capital structure in a debtholder-manager agency relationship. The optimal debt-equity is significantly lower when the manager applies creative accounting. This suggests less need for debt in comparison to the historical cost and fair value accounting methods. In contrast, there is no significant difference between the historical and fair value accounting specifications of the optimal capital structure.

6.7 Implications for Corporate Governance and Optimal Capital Structure

1. In the shareholder-manager dynamic game interaction, a higher debt repayment and monitoring strategy is required to curb the manager's

earnings manipulation tendencies when creative accounting is applied. Fair value requires lower-level strategies from the shareholder.

2. The manager requires a higher investment strategy and a greater quality of effort to ensure their payoff in the strategic interaction when creative accounting is applied. This may be explained by the shareholder's optimal actions. The dynamic optimal mechanism design model can disincentivise the manager from creative accounting due to the disciplinary actions applied by the shareholder. Fair value accounting, on the other hand, suggests a lower level of investment and effort from the manager.
3. Creative accounting causes a reported higher cash flow throughout the contract, while fair value causes a lower cash flow than historical cost accounting. This thereby recommends the 'information decay' concept, which states that a company's future evaluation is better forecasted by the latest performance data rather than past performance (Fudenberg & Tirole 1995).
4. Due to the estimation of the company's productive assets at current market prices, fair value suggests a higher optimal debt value for the company over time than historical cost accounting. On the other hand, creative accounting suggests lower optimal debt value for the company. The effect of the financial accounting theories is more defined when the company's productive asset rate and use of new debt is higher.
5. The optimal mechanism design restricts the growth of the manager's pecuniary incentives, irrespective of the choice of historical cost, fair value or creative accounting. However, a moderately higher growth is observed for creative accounting, and a smaller growth is observed when the company selects the fair value accounting method.
6. The impact of the financial accounting theories shows that fair value accounting can increase the optimal leverage of a company more than with historical cost or creative accounting. This may further establish the 'forward-looking' benefit of fair value accounting, as suggested by Godfrey et al. (2010). In contrast, creative accounting causes a slightly lower leverage than in historical cost accounting.
7. In the debtholder-manager dynamic game interaction, creative accounting can boost the reported liquid reserve of the company in each period,

minimising the difficulty in satisfying the debtholder's financial covenants (Tirole 2006). On the other hand, debt value remains unaffected by the financial accounting theories.

8. The optimal capital structure in the debtholder-manager strategic interaction is affected by creative accounting but remains unaffected by the choice of historical cost or fair value accounting.

6.8 Conclusion

Management and debt contracts rely significantly on accounting information. Thus, agency relationships are established based on financial output processes. As these processes are dependent on the accounting methods applied by the company, it is imperative to consider the impact of these methods, classified as historical cost and fair value accounting, on the optimal mechanism design and optimal capital structure. In historical cost accounting, the original cost of productive assets is reported, while fair value accounting considers the current market price estimation of a company's productive assets, assumed to be lower in this study. Creative accounting also gives the manager a reasonable justification to manipulate revenue recognition. Hence, financial accounting can give different estimations of the required optimal capital structure in the design of optimal mechanisms and corporate governance systems. The main goal of this chapter was to examine the impact of historical cost, fair value and creative accounting theories on optimal mechanism design and the specification of the optimal capital structure in a continuous-time dynamic framework.

To achieve the goal of the study, three propositions were stated with respect to historical cost, fair value and creative accounting. The propositions were evaluated using the previously developed financial management model based on the dynamic game strategic interactions between the shareholder and the manager, as well as the debtholder and the manager, using the best compromise Pareto-efficient solutions. The evidence from this study suggests that:

- a. The impact of creative accounting is more significant in the capital structure specification for the debtholder-manager relationship than for the shareholder-manager relationship. This implies that creative accounting can lead to an unreliable estimation of the optimal capital structure

specification in the debtholder-manager agency relationship. On the other hand, the impact of fair value accounting is more significant in the capital structure specification for the shareholder-manager relationship.

- b. Fair value specifies a higher optimal leverage in the shareholder-manager strategic interaction than historical cost accounting.
- c. Creative accounting specifies lower leverage than historical cost accounting in both the shareholder-manager and debtholder-manager dynamic relationships.
- d. The optimal capital structure is largely unaffected by the choice of historical cost or fair value accounting in the debtholder-manager dynamic relationship.

These results can inform decisions on the choice of accounting theories and can enhance the optimal value of a company.

The next chapter presents the summary and conclusion of the thesis.

Chapter 7 – Research Summary and Conclusion

7.1 Introduction

This chapter summarises the main research findings of the study. It also outlines the theoretical and practical implications of the research, and the contributions made to mechanism design and efficient corporate governance systems in a dynamic framework.

The study began with a discussion on the need for good corporate governance practices. Agency problems can complicate decisions around the development of an optimal capital structure. The agency costs associated with conflicts of interest between key stakeholders (the manager, debtholder and shareholder) have significant effects on the firm value and thus require efficient mechanism design. The use of debt as a mechanism, which has been thoroughly discussed, further complicates the specification of an optimal capital structure, as debt can minimise the agency problem and improve firm value. Furthermore, as agency problems are dynamic problems, it is important to resolve the agency problem and the optimal capital structure problem using a dynamic framework.

To address gaps in the literature, the focus of this study was to develop an optimal capital structure and an optimal mechanism using a differential game theory framework in a continuous-time setting. The resulting model was developed using complementary corporate governance mechanisms, corporate finance and accounting principles applied to a non-financial company as a case study. The model was used to examine the significance of mechanism design concepts in resolving the underlying agency and information asymmetry problems, and to support efficient corporate governance systems leading to improved company performance. In addition, the model was used to analyse the impact of financial accounting theories on the optimal capital structure specification and the optimal mechanism design. Therefore, with the aid of a sophisticated multidisciplinary framework for analysing strategic interactions, this study complements the body of literature on financial accounting, mechanism design, corporate governance and dynamic game theory.

7.1.1 Structure of the Chapter

The chapter is structured as follows, Section 7.2 re-presents the key issues and concepts in the study. Sections 7.3 summarises the models, results and implications of the study. Section 7.4 summarises the major findings from the research. Section 7.5 discusses the contributions of the study. Section 7.6 highlights the limitations of the study and opportunities for further research. Finally, Section 7.7 provides the conclusion of this chapter.

7.2 Issues and Key Concepts

The problem of an optimal capital structure complicated by agency problems is a significant corporate governance and corporate finance issue. An optimal capital structure is important in the realisation of the going concern of a company, as poor capital structure decisions can lead to financial distress and bankruptcy. Thus, modern companies consistently seek to develop an optimal capital structure that provides the minimum weighted average cost of capital. Furthermore, due to the disciplinary effects of the use of debt on the manager, it is imperative to optimise jointly the capital structure and the mechanism design required to minimise managerial volition. Therefore, the optimal capital structure specification must embed an optimal mechanism design that motivates the manager to take the best actions in the interests of the shareholder or the debtholder.

In the design of an optimal mechanism design, the use of complementary corporate governance mechanisms can provide more reliable incentive contracts than the use of solely pecuniary incentives or optimal pay. The use of optimal pay as the only element of incentive contracts or mechanism design introduces additional problems in the company's performance as it can change the focus of the manager toward achieving financial thresholds, limiting the manager's productivity. Hence, models that can design optimal mechanisms and specify an optimal capital structure can enhance financial management and improve company performance.

Game theory models have been well advanced due to their proven strength in analysing strategic interactions between two or more parties. As most games are characterised by disproportionate information, game theory has been deployed

as a reliable mathematical modelling tool to integrate information asymmetry problems that can provide insights into unexplained theories.

In addition, as both shareholder-manager and debtholder-manager relationships traverse over a period of time, game-theoretic models that can integrate information asymmetry and analyse strategic interactions between two or more parties are suitable for examining management and debt contracts. In dynamic games, the actions of a player in one period may impact the actions of the other player in the next period. Therefore, it is important to analyse the dynamic relationship between the shareholder and the manager, and between the debtholder and the manager over time. Differential game theory has been deployed rigorously as a suitable framework in the analysis of conflicts of interest in finance and management science. Thus, this study has developed financial management models using the differential game theory as a dynamic strategic interaction framework for designing an optimal mechanism and specifying an optimal capital structure in a dynamic continuous-time setting.

7.3 Models, Results and Implications

As indicated above, the optimal capital structure and optimal mechanism design models in this study have been developed based on the differential game theory framework. The study considers two different strategic interactions with respect to three key stakeholders of a public firm. The first model was specified as a strategic interaction between the shareholder (principal) of a company and the manager (agent). The second model was specified as strategic interaction between the manager (agent) and the debtholder (principal) of the company. The two models were first represented as non-cooperative dynamic games that optimised the two contradicting objectives of the principal and the agent, governed by systems of ordinary differential equations and subject to a certain set of constraints, where applicable. The systems of the game represent the outcome of the game, producing the optimal capital structure for the company. In the first strategic interaction between the shareholder and the manager, the objective functional for the shareholder was to maximise the net cash flow after incentive cost, monitoring and tax accruing. This reflects the utility of the shareholder. On the other hand, the manager in this strategic interaction considered the optimal quality of effort and optimal investment required to

maximise their profit from the contract. In the dynamic game interaction, the shareholder selected debt repayment, monitoring, pecuniary incentives as the corporate governance mechanisms to ensure their optimal utility is derived in the contract. The manager selected quality of effort and investment as their strategies to optimise their utility.

The second strategic interaction assumed that the manager on behalf of the shareholder had opportunistic tendencies to become involved in asset substitution at the expense of the debtholder. In the debtholder-manager strategic interaction, the objective functional for the debtholder was to maximise the principal value of debt issued to the company at the start of the game while minimising the monitoring costs of obtaining their interest payments. In this strategic interaction, the manager had an objective to select the optimal capital structure that produces the lowest weighted average cost of capital and maximise the value derived from asset substitution. As a mechanism, the debtholder decided the cost of debt provided, while the manager considered the optimal cost of equity from other external finance.

The non-cooperative specification was solved using the Nash equilibrium solution concepts. Following this, assumptions for cooperation were made, and the Pareto-efficient optimal solutions were derived.

The interrelationships between the objective functionals and the systems of the game established an incentive-compatible contract between the shareholder and the manager, as well as between the manager and the debtholder. These align the interests of both parties, resolve the underlying information asymmetry and agency problems and provide the optimal capital structure to improve firm value.

Finally, the impact of historical cost, fair value and creative accounting on the dynamic game models was investigated. The models were simulated using real-life data from a sample company (the case study) to obtain equilibrium and optimal solutions for the two strategic interactions, and to analyse the dynamic game-theoretic, information economics and financial accounting perspectives. The main findings and implications of the study are discussed below.

7.3.1 Differential Game Framework for Developing Dynamic Corporate Governance Mechanisms

The differential game theory embeds the strategic interactions between the shareholder and the manager, and an optimal incentive contract that motivates the manager to improve their quality effort on behalf of the shareholder. Hence, differential game theory aligns the interests of both parties, mitigates the underlying agency and information problems and enhances company performance. The solutions of the model simulation discussed in Chapter 4 (Section 4.5) suggest that the model based on the differential game framework motivates the manager to provide an optimal quality of effort and select the optimal choice of investment. The model also recommends the optimal debt repayment strategy, optimal level of monitoring, and the manager's optimal pecuniary incentives to the shareholder throughout the dynamic contract. This leads to higher financial outcomes. The improved financial performance indicators are shown by an increasing cash flow $Y(t)$, a decrease in debt value $D(t)$, an increase in productive assets $S(t)$, and an increase in the manager's pecuniary incentive $I(t)$, consistent with the productivity of the company. Moreover, the dynamic game model also produces an optimum capital structure that can derive the optimal tax benefits from the use of debt financing as an additional mechanism and minimise the risk of financial distress. This minimises major financial risks.

These results imply that the incentive contract embedded in the dynamic game mechanism design model induces the manager to exert optimal strategies. These can improve the company's efficiency while rewarding the manager proportionately throughout the dynamic process. This ensures renegotiation proofness. The findings of the study also suggest a decrease in the manager's discretionary spending of company resources, shown by the increased debt repayment when the company's cash flow increases. These results indicate that the mechanism design embedded in the dynamic game model can discourage and prevent managerial opportunistic actions, such as cash diversion for empire building.

Furthermore, the solutions of the model simulation discussed in Chapter 5 (Section 5.5) suggest that the differential game-based model framework

disincentivises the manager from asset substitution by recommending an optimal cost of debt to the debtholder if the manager reneges on the terms of cooperation. This leads to better and stable financial outcomes. The enhanced financial performance indicators for the debt contract are shown by a reduced and consistent cost of debt $c(t)$, and a consistent provision of debt facility $D(t)$ to the company. Thus, the dynamic incentive contract embedded in the differential game model establishes an appropriate and efficient mechanism. This minimises and resolves the underlying dynamic agency and information asymmetry problems between the debtholder and the manager in a company.

7.3.2 The Impact of Financial Accounting Theories on Differential Game-Based Models

The choice of different accounting theories can have different impacts on the evaluation of a manager's behaviour and the company's performance assessment (Godfrey et al. 2010). Thus, the company's output processes for which an optimal capital structure and an optimal mechanism design was specified must be thoroughly considered. The argument for and against historical cost, fair value and creative accounting in the financial accounting literature has emphasised the need to examine the impact of the different accounting methods on financial management models. This can provide significant insights into the model applications. The solutions of the numerical tests discussed in Chapter 6 (Section 6.6) suggest that the optimal capital structure specification of the debtholder-manager dynamic strategic interaction is impacted more by creative accounting than is the case in the shareholder-manager strategic interaction. This is shown by a significantly lower optimal debt-equity ratio for creative accounting in the debtholder-manager strategic interaction than in the shareholder-manager interaction. This implies that creative accounting can provide an unreliable estimate of the optimal capital structure in the debtholder-manager agency relationship. Contrarily, the impact of fair value accounting on the optimal capital structure specification is more notable in the shareholder-manager dynamic strategic interaction than in the debtholder-manager interaction. This is shown by a significantly higher optimal debt-equity ratio for fair value accounting in the shareholder-manager strategic interaction than in the debtholder-manager interaction. However, the model based on the differential game framework

developed in Chapter 4 consistently motivates the manager to provide a higher quality of effort and investment levels when the manager applies creative accounting. This establishes the robustness of the models. It also recommends higher levels of debt repayment and monitoring to the shareholder when creative accounting is applied. This can disincentivise the manager from creative accounting, leading to greater financial performance.

7.4 Summary of Major Findings

The major findings with respect to the aims of the study are discussed as follows:

- 1. Develop an optimal capital structure via an optimal debt-equity ratio and dynamic optimal mechanism design framework to resolve the conflicts of interest between the shareholder and the manager.*

The differential game-based mechanism design model presented in Chapter 4 provides optimal strategies for both the shareholder and the manager. The results recommend a best compromise Pareto-efficient outcome as a centralised or cooperative solution. Thus, the model can resolve the underlying agency and information asymmetry problems between the shareholder and the manager in a modern corporation. This is shown by the reduced agency costs and the optimal outcomes of the model. Hence, the model can improve organisational design and enhance company value. It also provides the specification for an optimal capital structure that can improve the company's performance in a dynamic framework.

- 2. Resolve the agency problem of asset substitution and minimise the conflicts of interest between the debtholder and the manager via mechanism design, as well specify the optimal capital structure over time.*

The dynamic game model presented in Chapter 5 provides an analysis of the conflicts of interest and strategic interactions between the debtholder and the manager acting on behalf of the shareholder. The analysis produces an incentive contract model to motivate the manager to take the best actions in the interests of the debtholder. This resolves the agency

problems between the debtholder and the manager in an over-time framework. This is shown by improved debt facilities and the reduced cost of debts to the manager. This can boost the company's ability to compete for debt finance in the capital market. In addition, the specification of the optimal capital structure in the debtholder-manager relationship can reduce the company's aggregate shocks and its risks of financial distress.

3. *Develop dynamic corporate governance models based on differential game theory that offer a firm optimal financing and maximise its value in a dynamic framework.*

The dynamic optimal mechanism design and the optimal capital structure models developed in this study show that corporate governance issues can be modelled as dynamic mechanism design problems. The use of differential game theory models provides a methodical framework to analyse and investigate the strategic relationships between the shareholder and the manager, and the debtholder and the manager. Corporate governance mechanisms are specified as the strategies of the parties in the dynamic strategic interaction. Differential game theory deploys the descriptive ability of differential equations to analyse the situations of conflict in a dynamic process. Hence, it provides the optimal strategies and outcomes for coordination. The results of this study show that complementary corporate governance mechanisms embedded in the models provide the manager with incentives to protect the interests of the shareholder and debtholder and ensure renegotiation proofness in the dynamic process.

4. *Examine the impact of financial accounting theories on the corporate governance models.*

The analysis and discussion of financial accounting theories in Chapter 6 provides insights into the effects of accounting methods on corporate governance and the optimal capital structure. This study shows that the dynamic differential game-based model motivates the manager to avoid creative accounting. It also formulates the optimal strategies and

outcomes when the fair value accounting method is deployed in a company. Hence, it provides a comprehensive corporate governance framework that can improve the efficiency of management and debt contract. This supports the shareholder and debtholder's value creation and maintains the firm's sustainability over time with respect to the different choices of accounting methods.

7.5 Contributions of the Study: A Re-Appraisal

7.5.1 Theoretical Contributions

The primary research focus of this study was to develop time-dependent optimal corporate governance frameworks and mechanisms for choosing a company's optimal capital structure. The research findings provide several contributions to the existing body of knowledge. Firstly, this study contributes to the financial accounting, corporate governance, corporate finance, dynamic mechanism design theory and differential game theory literature. It does so by examining the economic benefits of a differential game-based framework in specifying an optimal capital structure, achieving good corporate governance and improving company value in a dynamic integrated analysis. Through an integrated dynamic game theory based on the mechanism design framework, this study investigates and encompass the principles and concepts of mechanism design, financial and managerial accounting, complementary corporate governance and corporate finance into the objective functionals and dynamic systems of the financial management models. The results of the study reveal the economic benefits of a differential game theory framework in providing credible and robust analyses of situations of conflict. The study also provides reliable financial strategies required for decision-making processes, hence reducing the information asymmetry problem.

In addition, notwithstanding the in-depth literature on the design of optimal incentives to minimise managerial volition, this study provides a new insight into the use of complementary corporate governance instruments via the mechanism design approach as a framework to integrate the issues in a continuous-time setting.

Furthermore, this study contributes to the current debate on suitable mathematical models that can specify dynamic capital structure models by providing models for the specification of an optimal capital structure when the problem is characterised by agency problems. This provides insights into benefits of jointly optimising the capital structure and minimising agency problems. The analysis in this study was conducted by developing mechanism design models based on differential game theory and information economics perspectives as a dynamic framework to integrate the research issues. This has never been done in the previous research. Additionally, the models developed in this study were operationalised via a simulation of real-life data, which has also not been done in previously conducted research. The real-life simulation of the models is important for revealing the economic benefits of the model in practice. Hence, this study advances the knowledge and methods for designing differential game-based models in financial management.

7.5.2 Practical Contributions

The main goal of this study was to analyse the interrelationships between corporate governance, financial accounting theories, differential game theory, capital structure and company value. It also aimed to investigate the degree to which the relationships could produce real benefits for achieving good corporate governance, efficient capital structure design, and improved company value. To achieve these goals, this study developed integrated mechanism design models using complementary corporate governance instruments as a dynamic game problem. The models incorporate corporate finance and accounting principles in a differential game model, which has not been done in previous studies. The results of the model simulations reveal the equilibrium and optimal solutions offered by the differential game modelling. It provides the optimal strategies required for the key stakeholders (shareholder, debtholder, manager) of modern corporations to ensure cooperation and coordination in management and debt contracts. The recommendations from the results of the models provide insights into the strategies and requirements for the alignment of interests among the parties throughout the dynamic contract. The study has proposed the optimal actions required by the shareholder and the debtholder to protect their interests in financial contracts. In addition, the study has specified the optimal capital

structure that serves as a corporate governance mechanism and minimises the agency problem. In this way, this study provides practical ideas on the design of efficient complementary mechanism designs to improve the managers quality of effort over time. This maximises the value to the shareholder and the debtholder in the strategic relationship.

This study sought to develop practical models applicable in real-life financial management situations. It used real financial data from a publicly listed company to obtain solutions from the dynamic mechanism design embedded in the models. This study fills the gap between the theoretical dynamic models for mechanism design, capital structure and real-world application of such models by making the models designed in this study valuable in real-life scenarios. The verification and validity tests and the sensitivity analyses carried out in the study prove the plausibility of the models. Thus, the proposed models can be applied as practical guides for designing policies related to incentives, management and debt contracting, and corporate governance that can improve company value and protect the shareholder and debtholder interests in multiple periods. Overall, this study provides practical contributions in terms of specifying a framework for financial management modelling based on agency relationships, and the integrated analysis of incentives, accounting methods, corporate governance and information economics in a dynamic context based on a differential game theory framework.

7.6 Limitations and Opportunities for Further Research

The study was limited in the following ways:

- 1.) First, in designing a bond contract between the debtholder and the manager of a company, the use of debt covenants is often seen as insurance for the interests of the debtholder. This study did not engage the use of debt covenants in the differential game model as various studies have argued for and against the impacts of covenants on company value.
- 2.) Additionally, it was beyond the scope of this study to examine the signalling use of the state variables of the financial management model (i.e., what information the outcomes of the optimal state variables may carry and convey for the financial markets and other stakeholders).

- 3.) The scope of the study was also limited in the number of incentives used to minimise the agency problem. Future studies may evaluate the use of other complementary corporate governance mechanisms in minimising the highlighted agency cost of debt.
- 4.) Due to practical constraints, the computational studies were limited to one case study company. This limits the numerical results to those applicable to the case study company. Therefore, the model may give different interpretations when applied to other case study companies.
- 5.) There have been notable differences between corporate governance practices across the world, such as in the US, Japan, Germany and other countries. This can add significant complexities to the research on corporate governance. However, this study did not address a comparative analysis of corporate governance practices internationally. This represents an area for future research.
- 6.) The current research deployed the use of Nash equilibrium as a robust solution strategy to analyse the incongruity of interests in the models. Future research may deploy Stackelberg equilibrium as an alternative solution concept to evaluate and compare the outcomes.
- 7.) This study holds an assumption that the manager acts on behalf of the shareholder in the debtholder-manager relationship due to indebtedness. This assumption may be relaxed in future research to develop a shareholder-debtholder-manager cooperative model.
- 8.) Following the significant corporate governance reforms being undertaken around the world, future research can investigate the impact of such reforms on the optimal capital structure design. Currently, a positive relationship has been established between audit and board independence and firm value (Fauver et al. 2017), as well as board reforms and higher dividend payout rate (Bae et al. 2020). Thus, the value derivable from board reforms can be implemented in the corporate governance model developed in this study via the monitoring strategy variable in the shareholder-manager game relationship. This can be reflected by appropriating greater weight to the parameter in a future work. Similarly, the positive impact of board reforms on dividend payout can be implemented in the design of the debtholder-manager agency relationship, as the dividend payout rate was specified as the

manager's strategy. This has a potential impact of increasing the company's liquid reserve, thus, improving the value derivable to the firm.

- 9.) Finally, the model was only applied to a non-financial company and thus gives limited insights to financial companies that have different capital structure designs.

Future research could be directed at incorporating the debt overhang problem and examining the impact of credit spread on the model results.

7.7 Conclusion

This study has developed new dynamic mechanism models for determining the optimal capital structure of a company based on information economics using the differential game theory model. The mechanism design models in this study characterise non-cooperative games or contracts between two key financial stakeholder relationships (the shareholder and manager; and the debtholder and the manager) of a modern corporation. The model considered the objectives of both the principal (shareholder/debtholder) and the agent (manager). Based on a differential game theory framework, the model was designed to achieve cooperation between the principal and the agent, by providing incentives to the manager to act on behalf of the principal's interests. Thus, the models engage the objectives of mechanism design, which coordinate the interests of the principal and the agent.

The simulated results from the study show that the developed models are applicable to the design of optimal incentive contracts. These can resolve agency and information asymmetry problems in organisations characterised by agency relationships. The financial management dynamic game models have been developed based on rigorous accounting and corporate finance practices, thus, they can operationalise significant issues in corporate governance and corporate finance. This study has examined the integrated issues of corporate governance, mechanism design, capital structure, efficient contracting and differential game theory. Hence, it provides several original contributions to the existing body of knowledge in numerous areas.

Although this study is limited in several ways, as discussed previously, the research has been carried out using sophisticated methodical procedures. The

thoroughness of this study is corroborated by the conduct of meticulous data analysis, an extensive literature review and a rigorous methodology. This ensures the plausibility of the results and implications, and the generalisation of the models. The dynamic mechanism design of capital structure models developed in this study are operational and satisfy robust mathematical verification and validation tests.

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Appendix

Numerical Codes

Due to word limit constraints, the appendix includes only MATLAB numerical codes and outputs for the Pareto-efficient outcome discussed in Chapter 4, and the codes to plot the subplots for the optimal strategies/control variables, the optimal states, and the optimal capital structure results.

```
function y = opdatedparetochpt4
%It is a forward backward sweep optimal control approach
%Pareto case with 3 encounter, for chapter 4 of
%thesis (shareholder-manager problem)
clc;
global alpha beta theta r rho p chi mu phi
test = -1;

delta=0.001;
N=100;
t=linspace(0,10,N+1);
h=1/N;
h2=h/2;
%Initialization of parameter
u1= zeros(1,N+1);%u1 stands for control 1, cost of debt
u2= zeros(1,N+1);%u2 stands for control 2, monitoring cost
u3= zeros(1,N+1);%u3 stands for control 3, investment
u4= zeros(1,N+1);%u4 stands for control 4, effort
Y= zeros(1,N+1);%x1 stands for state 1, cash flow
D= zeros(1,N+1);%x2 stands for state 2, company debt
I= zeros(1,N+1);%x3 stands for state 3, pecuniary incentive
S= zeros(1,N+1);%x4 stands for state 4, productive assets
L1= zeros(1,N+1); % for adjoints
L2= zeros(1,N+1);
L3= zeros(1,N+1);
L4= zeros(1,N+1);

Y(1)=0.29; %financial accounting theory
D(1)=0.27;
I(1)=0.003;
S(1)=2;

alpha=0.01; beta=0.30; theta=0.28; r=0.02; rho=0.001; p=0.1;...
chi=0.02; mu = 0.00001; phi=0.9;
%Encounter, beta 0.30, E2 beta = 0.60, E3, alpha= 0.3, p 0.2
```

```

% alpha = ratio of new debt to existing market value of company debt
% beta = payout rate of company's productive assets
% theta = effective tax rate
% r = riskless rate
% rho = discount rate
% p = cost of debt
% chi = proportion of management fee
% mu = asset depreciation rate

%Starting forward runge kutta for the states
while (test<0)
    oldu1 = u1; oldu2 = u2; oldu3 = u3; oldu4 = u4;
    oldY = Y; oldD = D; oldI = I; oldS = S;
    oldL1=L1; oldL2=L2; oldL3=L3; oldL4=L4;

    for i=1:N
        %k1 step
        k11 = beta*S(i) + (u4(i)-u3(i))*r.*Y(i) + u2(i)*u4(i)*p*D(i) -
u4(i).*I(i) - u1(i)*Y(i);
        k12 = (alpha + u4(i)*p).*D(i) - u1(i)*Y(i);
        k13 = u4(i).*I(i) + chi*Y(i);
        k14 = r*u3(i).*Y(i) + (1-u2(i))*p*u4(i)*D(i) - mu*S(i);
        %k2 step
        k21 = beta*(S(i)+h2*k14) + (0.5*((u4(i)+u4(i+1)) -
(u3(i)+u3(i+1))))*r.*(Y(i)+h2*k11)...
+ (0.5*(u2(i)+u2(i+1)))*(0.5*(u4(i)+u4(i+1)))*p*(D(i)+h2*k12) -
(0.5*(u4(i)+u4(i+1)))*(I(i)+h2*k13)...
- (0.5*(u1(i)+u1(i+1)))*(Y(i)+h2*k11);
        k22 = (alpha + (0.5*(u4(i)+u4(i+1)))*p).*(D(i)+h2*k12) -
(0.5*(u1(i)+u1(i+1)))*(Y(i)+h2*k11);
        k23 = (0.5*(u4(i)+u4(i+1)))*(I(i)+h2*k13) + chi*(Y(i)+h2*k11);
        k24 = r*(0.5*(u3(i)+u3(i+1)))*(Y(i)+h2*k11) + (1-
(0.5*(u2(i)+u2(i+1)))*p*(0.5*(u4(i)+u4(i+1)))*(D(i)+h2*k12)...
- mu*(S(i)+h2*k14);

        %k3 step
        k31 = beta*(S(i)+h2*k24) + (0.5*(u4(i)+u4(i+1)))-
(u3(i)+u3(i+1)))*r.*(Y(i)+h2*k21)...
+ (0.5*(u2(i)+u2(i+1)))*(0.5*(u4(i)+u4(i+1)))*p*(D(i)+h2*k22) -
(0.5*(u4(i)+u4(i+1)))*(I(i)+h2*k23)...
- (0.5*(u1(i)+u1(i+1)))*(Y(i)+h2*k21);
        k32 = (alpha + (0.5*(u4(i)+u4(i+1)))*p).*(D(i)+h2*k22) -
(0.5*(u1(i)+u1(i+1)))*(Y(i)+h2*k21);
        k33 = (0.5*(u4(i)+u4(i+1)))*(I(i)+h2*k23) + chi*(Y(i)+h2*k21);
        k34 = r*(0.5*(u3(i)+u3(i+1)))*(Y(i)+h2*k21) + (1-
(0.5*(u2(i)+u2(i+1)))*p*(0.5*(u4(i)+u4(i+1)))*(D(i)+h2*k22)...
- mu*(S(i)+h2*k24);

        % k4 step
        k41 = beta*(S(i)+h*k34) + (u4(i+1)-u3(i+1))*r.*(Y(i)+h*k31) +
u2(i+1)*u4(i+1)*p*(D(i)+h*k32)...
- u4(i+1).*(I(i)+h*k33) - u1(i+1)*(Y(i)+h*k31);
        k42 = (alpha + u4(i+1)*p).*(D(i)+h*k32) - u1(i+1)*(Y(i)+h*k31);
        k43 = u4(i+1).*(I(i)+h*k33) + chi*(Y(i)+h*k31);
        k44 = r*u3(i+1).*(Y(i)+h*k31) + (1-u2(i+1))*p*u4(i+1)*(D(i)+h*k32) -
mu*(S(i)+h*k34);

        %State solution
        Y(i+1) = Y(i)+(h/6)*(k11 + 2*k21 + 2*k31 + k41);
        D(i+1) = D(i)+(h/6)*(k12 + 2*k22 + 2*k32 + k42);
        I(i+1) = I(i)+(h/6)*(k13 + 2*k23 + 2*k33 + k43);
        S(i+1) = S(i)+(h/6)*(k14 + 2*k24 + 2*k34 + k44);
    end

    for i=1:N
        j=N+2-i;
        %k1 step

```

```

    m11= (rho + u1(j) - ((u4(j) - u3(j))*r))*L1(j) - phi*(1-theta) +
    u1(j)*L2(j) - chi*L3(j) - u3(j)*r*L4(j);

    m12= (rho - (alpha + u4(j)*p))*L2(j) - u2(j)*u4(j)*p*L1(j) - (1-
    u2(j))*p*u4(j)*L4(j);
    m13= (rho - u4(j))*L3(j) + u4(j)*L1(j) + phi*(1-theta) - (1-phi);
    m14= (rho+mu)*L4(j) - beta*L1(j);

    %k2 step
    m21= (rho + (0.5*(u1(j)+u1(j-1))) - ((0.5*((u4(j)+u4(j-1)) -
    (u3(j)+u3(j-1))))*r))*L1(j)-h2*m11)...
    - phi*(1-theta) + (0.5*(u1(j)+u1(j-1)))*(L2(j)-h2*m12) - chi*(L3(j)-
    h2*m13) - (0.5*(u3(j)+u3(j-1)))*r*(L4(j)-h2*m14);

    m22 = (rho - (alpha + (0.5*(u4(j)+u4(j-1)))*p))*(L2(j)-h2*m12) - ...
    (0.5*(u2(j)+u2(j-1)))*(0.5*(u4(j)+u4(j-1)))*p*(L1(j)-h2*m11)...
    - (1-(0.5*(u2(j)+u2(j-1))))*p*(0.5*(u4(j)+u4(j-1)))*(L4(j)-h2*m14);

    m23= (rho - (0.5*(u4(j)+u4(j-1))))*(L3(j)-h2*m13) + (0.5*(u4(j)+u4(j-
    1)))*(L1(j)-h2*m11) + phi*(1-theta) - (1-phi);

    m24= (rho+mu)*(L4(j)-h2*m14) - beta*(L1(j)-h2*m11);

    %k3 step
    m31= (rho + (0.5*(u1(j)+u1(j-1))) - ((0.5*((u4(j)+u4(j-1)) -
    (u3(j)+u3(j-1))))*r))*L1(j)-h2*m21)...
    - phi*(1-theta) + (0.5*(u1(j)+u1(j-1)))*(L2(j)-h2*m23) - chi*(L3(j)-
    h2*m23) - (0.5*(u3(j)+u3(j-1)))*r*(L4(j)-h2*m24);

    m32 = (rho - (alpha + (0.5*(u4(j)+u4(j-1)))*p))*(L2(j)-h2*m22) - ...
    (0.5*(u2(j)+u2(j-1)))*(0.5*(u4(j)+u4(j-1)))*p*(L1(j)-h2*m21)...
    - (1-(0.5*(u2(j)+u2(j-1))))*p*(0.5*(u4(j)+u4(j-1)))*(L4(j)-h2*m24);

    m33= (rho - (0.5*(u4(j)+u4(j-1))))*(L3(j)-h2*m23) + (0.5*(u4(j)+u4(j-
    1)))*(L1(j)-h2*m21) + phi*(1-theta) - (1-phi);

    m34= (rho+mu)*(L4(j)-h2*m24) - beta*(L1(j)-h2*m21);

    %4th step
    m41= (rho + u1(j-1) - ((u4(j-1) - u3(j-1))*r))*L1(j)-h*m31) - phi*(1-
    theta)...
    + u1(j-1)*(L2(j)-h*m32) - chi*(L3(j)-h*m33) - u3(j-1)*r*(L4(j)-
    h*m34);

    m42= (rho - (alpha + u4(j-1)*p))*(L2(j)-h*m32) - u2(j-1)*u4(j-
    1)*p*(L1(j)-h*m31) - (1-u2(j-1))*p*u4(j-1)*(L4(j)-h*m34);
    m43= (rho - u4(j-1))*(L3(j)-h*m33) + u4(j-1)*(L1(j)-h*m31) + phi*(1-
    theta) - (1-phi);
    m44= (rho+mu)*(L4(j)-h*m34) - beta*(L1(j)-h*m31);

    %Adjoint solution
    L1(j-1)= L1(j) - (h/6)*(m11 + 2*m21 + 2*m31 + m41);
    L2(j-1)= L2(j) - (h/6)*(m12 + 2*m22 + 2*m32 + m42);
    L3(j-1)= L3(j) - (h/6)*(m13 + 2*m23 + 2*m33 + m43);
    L4(j-1)= L4(j) - (h/6)*(m14 + 2*m24 + 2*m34 + m44);

end

%Update control
U1= max(0, ((Y.*(L1+L2))/(2*phi*(1-theta))));
U2= max(0, ((u4.*p.*D.*(L1-L4))/(2*phi*(1-theta))));
U3= max(0, ((Y.*(L1-L4))/2*(1-phi)));
U4= max(0, ((1/(2*(1-phi)))*(u2.*p.*D + r*Y - I).*L1 + p*D.*L2 + I.*L3 +
(1-u2).*p.*D.*L4));

```

```

u1= 0.5*(U1+oldu1);
u2= 0.5*(U2+oldu2);
u3= 0.5*(U3+oldu3);
u4= 0.5*(U4+oldu4);%convex combination of the optimal control

%%Checking for convergence
temp1 = delta*sum(abs(u1)) - sum(abs(oldu1 - u1));
temp2 = delta*sum(abs(u2)) - sum(abs(oldu2 - u2));
temp3 = delta*sum(abs(u3)) - sum(abs(oldu3 - u3));
temp4 = delta*sum(abs(u4)) - sum(abs(oldu4 - u4));
Semp1 = delta*sum(abs(Y)) - sum(abs(oldY - Y));
Semp2 = delta*sum(abs(D)) - sum(abs(oldD - D));
Semp3 = delta*sum(abs(I)) - sum(abs(oldI - I));
Semp4 = delta*sum(abs(S)) - sum(abs(oldS - S));
Vemp1 = delta*sum(abs(L1)) - sum(abs(oldL1 - L1));
Vemp2 = delta*sum(abs(L2)) - sum(abs(oldL2 - L2));
Vemp3 = delta*sum(abs(L3)) - sum(abs(oldL3 - L3));
Vemp4 = delta*sum(abs(L4)) - sum(abs(oldL4 - L4));

test = min(temp1,min(temp2,min(temp3,min(temp4,min(Semp1,min(Semp2,...
min(Semp3,min(Semp4,min(Vemp1,min(Vemp2,min(Vemp3,Vemp4))))))));

end

y(1,:)=t;
y(2,:)=u1;
y(3,:)=u2;
y(4,:)=u3;
y(5,:)=u4;
y(6,:)=Y;
y(7,:)=D;
y(8,:)=I;
y(9,:)=S;
y(10,:)=L1;
y(11,:)=L2;
y(12,:)=L3;
y(13,:)=L4;

figure(11) %plotting

subplot(2,2,1);
plot(y(1,:),y(2,:), '--', 'linewidth', 5)
legend('Cost debt')
grid on
ylabel('Cost of Debt - u1(t)')
title ('Cost of Debt')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,2);
plot(y(1,:),y(3,:), '-', 'linewidth', 5)
legend('Monitoring')
grid on
ylabel('Monitoring - u2(t)')
title ('Monitoring')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

```

```

subplot(2,2,3);
plot(y(1,:),y(4,:), '-', 'linewidth', 5)
legend('Investment')
grid on
ylabel('Investment -  $u_3(t)$ ')
title ('Investment')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,4);
plot(y(1,:),y(5,:), '-', 'linewidth', 5)
legend('Effort')
grid on
ylabel('Quality of effort -  $u_4(t)$ ')
title ('Effort')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

figure(21)

subplot(2,2,1);
plot(y(1,:),y(6,:), '--', 'linewidth', 5)
legend('Cash flow')
grid on
ylabel('Cash flow  $Y(t)$ ')
title ('Cash flow')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,2);
plot(y(1,:),y(7,:), '--', 'linewidth', 5)
legend('Value of Debt  $D(t)$ ')
grid on
ylabel('Value of Debt  $D(t)$ ')
title ('Debt over time')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,3);
plot(y(1,:),y(8,:), '--', 'linewidth', 5)
legend('Pecuniary Incentive  $I(t)$ ')
grid on
ylabel('Pecuniary Incentive  $I(t)$ ')
title ('Pecuniary Incentive')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,4);
plot(y(1,:),y(9,:), '--', 'linewidth', 5)
legend('Productive Assets')
grid on

```



```

ylabel('Productive Assets S(t)')
title ('Productive Assets S(t)')
set(gca,'FontSize',15)
set(gca,'XTick', (0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

figure(31)

subplot(2,2,1);
plot(y(1,:),y(10,:), '--', 'linewidth', 5)
legend('L1')
grid on
ylabel('Lambda')
set(gca,'FontSize',15)
set(gca,'XTick', (0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,2);
plot(y(1,:),y(7,:), '--', 'linewidth', 5)
legend('L2')
grid on
ylabel('Lambda')
set(gca,'FontSize',15)
set(gca,'XTick', (0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,3);
plot(y(1,:),y(8,:), '--', 'linewidth', 5)
%legend('D(t)')
legend('L3')
grid on
ylabel('Lambda')
set(gca,'FontSize',15)
set(gca,'XTick', (0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,4);
plot(y(1,:),y(9,:), '--', 'linewidth', 5)
legend('L4')
grid on
ylabel('Lambda')
set(gca,'XTick', (0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

han=axes(figure(21),'visible','off');
han.XLabel.Visible='on';
xlabel(han,'Time(Years)', 'FontSize',18);
xlh=xlabel(han,'Time(Years)', 'FontSize',20);
xlh.Position(2) =xlh.Position(2) - abs(xlh.Position(2) * 0.4);

pos = get(gca, 'Position');
pos(1) = pos(1) - 0.03;
set(gca, 'Position',pos)
%}

```

ans =

Columns 1 through 7

0	0.1000	0.2000	0.3000	0.4000	0.5000	0.6000
0.1344	0.1357	0.1370	0.1382	0.1394	0.1405	0.1416
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0074	0.0074	0.0075	0.0076	0.0077	0.0078	0.0078
0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093
0.2900	0.2956	0.3012	0.3068	0.3123	0.3179	0.3235
0.2700	0.2696	0.2693	0.2689	0.2685	0.2681	0.2676
0.0030	0.0031	0.0031	0.0032	0.0032	0.0033	0.0034
2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
0.6006	0.5951	0.5895	0.5839	0.5784	0.5728	0.5673
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.5516	-0.5460	-0.5404	-0.5348	-0.5292	-0.5237	-0.5181
0.0928	0.0910	0.0892	0.0875	0.0857	0.0840	0.0823

Columns 8 through 14

0.7000	0.8000	0.9000	1.0000	1.1000	1.2000	1.3000
0.1426	0.1435	0.1444	0.1453	0.1461	0.1468	0.1475
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0079	0.0080	0.0080	0.0081	0.0082	0.0082	0.0083
0.0092	0.0092	0.0092	0.0091	0.0091	0.0090	0.0090
0.3290	0.3345	0.3400	0.3455	0.3510	0.3565	0.3620
0.2672	0.2668	0.2663	0.2658	0.2654	0.2649	0.2644
0.0034	0.0035	0.0036	0.0036	0.0037	0.0038	0.0039
2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
0.5617	0.5561	0.5505	0.5449	0.5394	0.5338	0.5282

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.5125	-0.5069	-0.5014	-0.4958	-0.4902	-0.4847	-0.4791
0.0806	0.0789	0.0773	0.0756	0.0740	0.0724	0.0708

Columns 15 through 21

1.4000	1.5000	1.6000	1.7000	1.8000	1.9000	2.0000
0.1482	0.1487	0.1493	0.1497	0.1502	0.1505	0.1509
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0083	0.0084	0.0084	0.0085	0.0085	0.0085	0.0086
0.0089	0.0089	0.0088	0.0087	0.0086	0.0086	0.0085
0.3674	0.3729	0.3783	0.3838	0.3892	0.3946	0.4000
0.2639	0.2633	0.2628	0.2623	0.2617	0.2611	0.2606
0.0039	0.0040	0.0041	0.0042	0.0042	0.0043	0.0044
2.0000	2.0000	2.0000	2.0000	2.0000	2.0001	2.0001
0.5226	0.5169	0.5113	0.5057	0.5001	0.4944	0.4888
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.4735	-0.4680	-0.4624	-0.4568	-0.4513	-0.4457	-0.4402
0.0692	0.0676	0.0661	0.0646	0.0631	0.0616	0.0601

Columns 22 through 28

2.1000	2.2000	2.3000	2.4000	2.5000	2.6000	2.7000
0.1511	0.1513	0.1515	0.1516	0.1516	0.1516	0.1516
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.0086	0.0086	0.0087	0.0087	0.0087	0.0087	0.0087
0.0084	0.0083	0.0082	0.0081	0.0080	0.0079	0.0078
0.4054	0.4108	0.4161	0.4215	0.4269	0.4322	0.4375

0.2600	0.2594	0.2588	0.2582	0.2576	0.2570	0.2563
0.0045	0.0046	0.0046	0.0047	0.0048	0.0049	0.0050
2.0001	2.0001	2.0001	2.0001	2.0001	2.0001	2.0001
0.4831	0.4775	0.4718	0.4661	0.4604	0.4547	0.4490
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.4346	-0.4291	-0.4235	-0.4180	-0.4124	-0.4069	-0.4013
0.0587	0.0572	0.0558	0.0544	0.0530	0.0516	0.0503

Columns 29 through 35

2.8000	2.9000	3.0000	3.1000	3.2000	3.3000	3.4000
0.1515	0.1513	0.1511	0.1508	0.1505	0.1501	0.1497
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000
0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087
0.0077	0.0075	0.0074	0.0073	0.0072	0.0071	0.0069
0.4429	0.4482	0.4535	0.4588	0.4641	0.4694	0.4747
0.2557	0.2550	0.2544	0.2537	0.2531	0.2524	0.2517
0.0051	0.0052	0.0052	0.0053	0.0054	0.0055	0.0056
2.0001	2.0001	2.0001	2.0001	2.0001	2.0001	2.0001
0.4433	0.4376	0.4318	0.4261	0.4203	0.4145	0.4087
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.3958	-0.3903	-0.3847	-0.3792	-0.3736	-0.3681	-0.3626
0.0489	0.0476	0.0463	0.0450	0.0437	0.0425	0.0413

Columns 36 through 42

3.5000	3.6000	3.7000	3.8000	3.9000	4.0000	4.1000
0.1492	0.1487	0.1481	0.1475	0.1468	0.1460	0.1452

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0087	0.0087	0.0087	0.0087	0.0086	0.0086	0.0086
0.0068	0.0067	0.0065	0.0064	0.0063	0.0061	0.0060
0.4800	0.4853	0.4906	0.4958	0.5011	0.5064	0.5116
0.2510	0.2503	0.2496	0.2489	0.2482	0.2475	0.2468
0.0057	0.0058	0.0059	0.0060	0.0061	0.0062	0.0063
2.0001	2.0001	2.0001	2.0001	2.0001	2.0001	2.0001
0.4029	0.3971	0.3913	0.3855	0.3796	0.3738	0.3679
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.3571	-0.3515	-0.3460	-0.3405	-0.3350	-0.3294	-0.3239
0.0400	0.0388	0.0377	0.0365	0.0354	0.0342	0.0331

Columns 43 through 49

4.2000	4.3000	4.4000	4.5000	4.6000	4.7000	4.8000
0.1444	0.1435	0.1425	0.1415	0.1404	0.1393	0.1381
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0085	0.0085	0.0084	0.0084	0.0084	0.0083	0.0082
0.0058	0.0057	0.0056	0.0054	0.0053	0.0051	0.0050
0.5169	0.5221	0.5274	0.5326	0.5379	0.5431	0.5484
0.2461	0.2454	0.2446	0.2439	0.2432	0.2424	0.2417
0.0064	0.0065	0.0066	0.0067	0.0068	0.0069	0.0071
2.0001	2.0001	2.0001	2.0001	2.0001	2.0001	2.0001
0.3620	0.3561	0.3502	0.3443	0.3383	0.3324	0.3264
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.3184	-0.3129	-0.3074	-0.3019	-0.2963	-0.2908	-0.2853
0.0320	0.0309	0.0299	0.0288	0.0278	0.0268	0.0258

Columns 50 through 56

4.9000	5.0000	5.1000	5.2000	5.3000	5.4000	5.5000
0.1369	0.1356	0.1342	0.1329	0.1314	0.1299	0.1283
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0082	0.0081	0.0081	0.0080	0.0079	0.0078	0.0077
0.0049	0.0047	0.0046	0.0044	0.0043	0.0041	0.0040
0.5536	0.5588	0.5641	0.5693	0.5746	0.5798	0.5851
0.2410	0.2402	0.2395	0.2388	0.2381	0.2373	0.2366
0.0072	0.0073	0.0074	0.0075	0.0076	0.0077	0.0079
2.0001	2.0001	2.0001	2.0001	2.0001	2.0001	2.0001
0.3204	0.3145	0.3085	0.3024	0.2964	0.2904	0.2843
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.2798	-0.2743	-0.2688	-0.2633	-0.2578	-0.2523	-0.2468
0.0248	0.0239	0.0230	0.0220	0.0211	0.0203	0.0194

Columns 57 through 63

5.6000	5.7000	5.8000	5.9000	6.0000	6.1000	6.2000
0.1267	0.1251	0.1233	0.1216	0.1197	0.1178	0.1159
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0077	0.0076	0.0075	0.0074	0.0073	0.0072	0.0071
0.0039	0.0037	0.0036	0.0034	0.0033	0.0032	0.0030
0.5903	0.5956	0.6008	0.6061	0.6113	0.6166	0.6219
0.2359	0.2352	0.2344	0.2337	0.2330	0.2323	0.2316
0.0080	0.0081	0.0082	0.0083	0.0085	0.0086	0.0087
2.0001	2.0001	2.0001	2.0001	2.0001	2.0001	2.0001
0.2782	0.2722	0.2661	0.2599	0.2538	0.2477	0.2415

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.2413	-0.2358	-0.2303	-0.2248	-0.2193	-0.2138	-0.2083
0.0186	0.0177	0.0169	0.0161	0.0154	0.0146	0.0139

Columns 64 through 70

6.3000	6.4000	6.5000	6.6000	6.7000	6.8000	6.9000
0.1139	0.1118	0.1097	0.1076	0.1053	0.1031	0.1007
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0070	0.0069	0.0067	0.0066	0.0065	0.0064	0.0062
0.0029	0.0028	0.0026	0.0025	0.0024	0.0023	0.0021
0.6272	0.6325	0.6378	0.6431	0.6484	0.6537	0.6590
0.2309	0.2302	0.2295	0.2289	0.2282	0.2275	0.2269
0.0088	0.0090	0.0091	0.0092	0.0093	0.0095	0.0096
2.0001	2.0001	2.0001	2.0001	2.0002	2.0002	2.0002
0.2354	0.2292	0.2230	0.2168	0.2106	0.2044	0.1981
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.2028	-0.1974	-0.1919	-0.1864	-0.1809	-0.1754	-0.1699
0.0132	0.0125	0.0118	0.0111	0.0105	0.0099	0.0093

Columns 71 through 77

7.0000	7.1000	7.2000	7.3000	7.4000	7.5000	7.6000
0.0984	0.0959	0.0934	0.0909	0.0882	0.0856	0.0828
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0061	0.0059	0.0058	0.0056	0.0055	0.0053	0.0052
0.0020	0.0019	0.0018	0.0017	0.0016	0.0015	0.0013
0.6644	0.6697	0.6751	0.6805	0.6858	0.6912	0.6967

0.2263	0.2256	0.2250	0.2244	0.2238	0.2233	0.2227
0.0097	0.0099	0.0100	0.0101	0.0103	0.0104	0.0105
2.0002	2.0002	2.0002	2.0002	2.0002	2.0002	2.0002
0.1919	0.1856	0.1793	0.1730	0.1667	0.1604	0.1541
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.1644	-0.1589	-0.1535	-0.1480	-0.1425	-0.1370	-0.1315
0.0087	0.0081	0.0076	0.0070	0.0065	0.0060	0.0056

Columns 78 through 84

7.7000	7.8000	7.9000	8.0000	8.1000	8.2000	8.3000
0.0801	0.0772	0.0743	0.0714	0.0684	0.0653	0.0622
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0050	0.0048	0.0047	0.0045	0.0043	0.0041	0.0039
0.0012	0.0011	0.0011	0.0010	0.0009	0.0008	0.0007
0.7021	0.7075	0.7130	0.7185	0.7240	0.7295	0.7350
0.2222	0.2216	0.2211	0.2206	0.2201	0.2197	0.2192
0.0107	0.0108	0.0110	0.0111	0.0113	0.0114	0.0115
2.0002	2.0002	2.0002	2.0002	2.0002	2.0002	2.0002
0.1478	0.1414	0.1351	0.1287	0.1224	0.1160	0.1096
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.1260	-0.1206	-0.1151	-0.1096	-0.1041	-0.0986	-0.0932
0.0051	0.0047	0.0043	0.0039	0.0035	0.0031	0.0028

Columns 85 through 91

8.4000	8.5000	8.6000	8.7000	8.8000	8.9000	9.0000
0.0590	0.0557	0.0524	0.0491	0.0456	0.0422	0.0386

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0037	0.0035	0.0033	0.0031	0.0029	0.0027	0.0025
0.0006	0.0006	0.0005	0.0004	0.0004	0.0003	0.0003
0.7406	0.7461	0.7517	0.7574	0.7630	0.7687	0.7744
0.2188	0.2184	0.2180	0.2176	0.2173	0.2170	0.2167
0.0117	0.0118	0.0120	0.0121	0.0123	0.0125	0.0126
2.0002	2.0002	2.0002	2.0002	2.0002	2.0002	2.0002
0.1032	0.0968	0.0904	0.0840	0.0775	0.0711	0.0647
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0877	-0.0822	-0.0767	-0.0712	-0.0658	-0.0603	-0.0548
0.0025	0.0022	0.0019	0.0016	0.0014	0.0012	0.0010

Columns 92 through 98

9.1000	9.2000	9.3000	9.4000	9.5000	9.6000	9.7000
0.0350	0.0314	0.0277	0.0239	0.0201	0.0162	0.0122
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0022	0.0020	0.0018	0.0015	0.0013	0.0010	0.0008
0.0002	0.0002	0.0001	0.0001	0.0001	0.0000	0.0000
0.7801	0.7858	0.7916	0.7974	0.8032	0.8090	0.8149
0.2164	0.2162	0.2160	0.2158	0.2157	0.2155	0.2154
0.0128	0.0129	0.0131	0.0132	0.0134	0.0136	0.0137
2.0002	2.0002	2.0002	2.0002	2.0002	2.0002	2.0002
0.0582	0.0518	0.0453	0.0388	0.0324	0.0259	0.0194
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0493	-0.0438	-0.0384	-0.0329	-0.0274	-0.0219	-0.0164
0.0008	0.0006	0.0005	0.0003	0.0002	0.0002	0.0001

Columns 99 through 101

9.8000	9.9000	10.0000
0.0082	0.0041	0
0.0000	0.0000	0
0.0005	0.0003	0
0.0000	0.0000	0
0.8208	0.8268	0.8328
0.2154	0.2153	0.2153
0.0139	0.0140	0.0142
2.0002	2.0002	2.0002
0.0130	0.0065	0
0.0000	0.0000	0
-0.0110	-0.0055	0
0.0000	0.0000	0

%Chapter 4 - comparing control variables in all solution concepts

```

t =      [0      1      2      3      4      5      6      7
8        9      10];

%Encounter 1
%Openloop
u111 =    [0.1352    0.1375    0.1363    0.1318    0.1237    0.1122    0.0971
0.0785    0.0561    0.0300    0];

u121 =    [0.000032    0.000032    0.000030    0.000027    0.000023    0.000019
0.000014    0.000009    0.000004    0.000001    0];

u131 =    [0.0012    0.0012    0.0010    0.0009    0.0007    0.0005    0.0004
0.0002    0.0001    0.0000    0];

u141 =    [0.0030    0.0033    0.0035    0.0036    0.0035    0.0034    0.0031
0.0026    0.0020    0.0011    0];

%Feedback
u112 =    [0.1352    0.1375    0.13630    0.1318    0.1237    0.1122
0.0971    0.0785    0.0561    0.03000    0];

u122 =    [0.000032    0.000032    0.000030    0.000027    0.000023    0.000019
0.0000138 0.000009    0.000004    0.000001    0];

```

```

u132 = [0.0012    0.0011    0.0010    0.00080    0.0007    0.0005
0.0004    0.0002    0.0001    0.0000    0];

u142 = [0.0030    0.0033    0.0035    0.0036    0.0035    0.0034
0.0031    0.0026    0.0020    0.0011    0];

%Pareto (0.9, 0.1)
%phi = 0.9
u113 = [0.1354    0.1377    0.1365    0.1319    0.1238    0.1123    0.0972
0.0785    0.0561    0.0300    0];

u123 = [0.0001    0.000079 0.000059 0.000041 0.000026 0.000014 0.000006
0.000002    0        0        0];

u133 = [0.0074    0.0077    0.0078    0.0076    0.0073    0.0067    0.0059
0.0049    0.0035    0.0019    0];

u143 = [0.0093    0.0081    0.0068    0.0053    0.0039    0.0026    0.0014
0.0005    0.0000    0        0];

%Pareto(0.1, 0.9)

u114 = [0.1515    0.1522    0.1492    0.1425    0.1322    0.1185    0.1014
0.0810    0.0572    0.030    0];

u124 = [0.000035 0.000033 0.000030 0.000026 0.000022 0.000017 0.000012
0.000008 0.000004 0.000001 0];

u134 = [0.0084    0.0085    0.0085    0.0083    0.0078    0.0071    0.0062
0.0050    0.00360 0.0019    0];

u144 = [0.0029    0.0030    0.0031    0.0031    0.0031    0.0029    0.0026
0.0022    0.0016    0.0009    0];

%Encounter 2
%openloop
u211 = [0.1334    0.1516    0.1628    0.1672    0.1646    0.1552    0.1387
0.1151    0.0842    0.0459    0];

u221 = [0.000026    0.000027    0.000026    0.000025    0.000022    0.000018
0.000014    0.000009    0.000005    0.000001 0];

u231 = [0.0011    0.0011    0.0011    0.0010    0.0009    0.0007    0.0005
0.0003    0.0002    0.0000    0];

u241 = [0.0031    0.0033    0.0036    0.0038    0.0039    0.0039    0.0036
0.0032    0.0024    0.0014    0];

%feedback
u212 = [0.1333    0.1515    0.1628    0.1671    0.1646    0.1551
0.1387    0.1151    0.0842    0.0459 0];

u222 = [0.000026 0.000027 0.000026 0.000025 0.000022 0.000018
0.000014 0.000009 0.000005 0.000001 0];

u232 = [0.0010    0.0011    0.0011    0.0010    0.0008    0.0007
0.0005    0.0003    0.0002    0.00004 0];

u242 = [0.0031    0.0033    0.0036    0.0038    0.0039    0.0039
0.0036    0.0032    0.0024    0.0014 0];

%Pareto (0.9, 0.1)
%phi = 0.9

```

```

u213 = [0.1336 0.1518 0.1630 0.1673 0.1647 0.1552 0.1387
0.1151 0.0842 0.0459 0];

u223 = [0.000100 0.000096 0.000084 0.000068 0.000050 0.000034 0.00002
0.000010 0.000004 0.000001 0];

u233 = [0.0060 0.0071 0.0080 0.0085 0.0087 0.0085 0.0079
0.0068 0.0051 0.0029 0];

u243 = [0.0117 0.0120 0.0114 0.0103 0.0088 0.0070 0.0052
0.0034 0.0019 0.0007 0];

%Pareto(0.1, 0.9)

%phi = 0.1

u214 = [0.1494 0.1677 0.1779 0.1805 0.1757 0.1637 0.1446
0.1186 0.0858 0.0463 0];

u224 = [0.000031 0.000030 0.000028 0.000025 0.000021 0.000017 0.000013
0.000008 0.000004 0.000001 0];

u234 = [0.0068 0.0079 0.0088 0.0092 0.0093 0.0090 0.0083
0.0070 0.0052 0.0029 0];

u244 = [0.0032 0.0033 0.0035 0.0035 0.0035 0.0034 0.0032
0.0027 0.0021 0.0012 0];

%Encounter 3
%Openloop
u311 = [0.1352 0.1375 0.1363 0.1318 0.1237 0.1122 0.0971
0.0785 0.0561 0.0300 0];

u321 = [0.000065 0.000066 0.000064 0.000060 0.000053 0.000044
0.000033 0.000022 0.000011 0.000003 0];

u331 = [0.0012 0.0012 0.0010 0.0009 0.0007 0.0005 0.0004
0.0002 0.0001 0.0000 0];

u341 = [0.0031 0.0033 0.0035 0.0036 0.0035 0.0034 0.0031
0.0026 0.0020 0.0011 0];

%feedback
u312 = [0.1352 0.1375 0.1363 0.1318 0.1237 0.1122 0.0971
0.0785 0.0561 0.0300 0];

u322 = [0.000065 0.000066 0.000064 0.000060 0.000053 0.000044
0.000033 0.000022 0.000011 0.000003 0];

u332 = [0.0012 0.0011 0.0010 0.0008 0.0007 0.0005 0.0004
0.0002 0.0001 0.00003 0];

u342 = [0.0031 0.0033 0.0035 0.0035 0.0035 0.0034 0.0031
0.0026 0.0020 0.0011 0];

%Pareto (0.9, 0.1)
%phi = 0.9
u313 = [0.1354 0.1377 0.1365 0.1319 0.1238 0.1123 0.0972
0.0785 0.0561 0.0300 0];

u323 = [0.000253 0.000206 0.000158 0.000114 0.000075 0.000044 0.000021
0.000007 0.000000 0.00000 0];

u333 = [0.0074 0.0077 0.0078 0.0076 0.0073 0.0067 0.0059
0.0049 0.0035 0.0019 0];

```

```

u343 = [0.0119 0.0103 0.0086 0.0068 0.0050 0.0034 0.0020
0.0008 0.0001 0.0000 0];

%Pareto(0.1, 0.9)
u314 = [0.1515 0.1522 0.1492 0.1425 0.1322 0.1185 0.1014
0.0810 0.0572 0.0302 0];

u324 = [0.000077 0.000073 0.000068 0.000060 0.0000513 0.000041 0.000030
0.000019 0.000010 0.000003 0];

u334 = [0.0084 0.0085 0.0085 0.0083 0.0078 0.0071 0.0062
0.0050 0.0036 0.0019 0];

u344 = [0.0032 0.0033 0.0033 0.0033 0.0032 0.0030 0.0027
0.0022 0.0017 0.0009 0];

%Encounter 4
%Openloop
u411 = [0.1334 0.1516 0.1628 0.1672 0.1646 0.1552 0.1387
0.1151 0.0842 0.0459 0];

u421 = [0.000053 0.000056 0.000056 0.000055 0.000050 0.000043
0.000034 0.000023 0.000012 0.000004 0];

u431 = [0.0011 0.0011 0.0011 0.0010 0.0009 0.0007 0.0005
0.0003 0.0002 0.0000 0];

u441 = [0.0031 0.0034 0.0036 0.0038 0.0039 0.0039 0.0036
0.0032 0.0024 0.0014 0];

%feedback
u412 = [0.1333 0.1515 0.1628 0.1671 0.1646 0.1551 0.1387
0.1151 0.0842 0.0459 0];

u422 = [0.000053 0.000056 0.000056 0.000055 0.000050 0.000043
0.000034 0.000023 0.000012 0.000004 0];

u432 = [0.0010 0.0011 0.0011 0.0010 0.0008 0.0007 0.0005
0.0003 0.0002 0.00004 0];

u442 = [0.0031 0.0034 0.0036 0.0038 0.0039 0.0039 0.0036
0.0032 0.0024 0.0014 0];

%Pareto (0.9, 0.1)
%phi = 0.9
u413 = [0.1336 0.1518 0.1630 0.1673 0.1648 0.1553 0.1387
0.1151 0.0842 0.0459 0];

u423 = [0.000287 0.000267 0.000232 0.000188 0.000141 0.000096 0.000058
0.000029 0.000011 0.000002 0];

u433 = [0.0060 0.0071 0.0080 0.0085 0.0087 0.0085 0.0079
0.0068 0.0051 0.0029 0];

u443 = [0.0167 0.0162 0.0149 0.0131 0.0109 0.0085 0.0062
0.0040 0.0022 0.0008 0];

%Pareto (0.1, 0.9)phi = 0.1
u414 = [0.1494 0.1677 0.1779 0.1805 0.1757 0.1637 0.1446
0.1187 0.0858 0.0463 0];

```

```

u424 = [0.000074 0.000071 0.000066 0.000060 0.000052 0.000042 0.000032
0.000021 0.000011 0.000003 0];

u434 = [0.0068 0.0079 0.0088 0.0092 0.0093 0.0090 0.0083
0.0070 0.0052 0.0029 0];

u444 = [0.0038 0.0038 0.0039 0.0039 0.0038 0.0036 0.0033
0.0028 0.0021 0.0012 0];

%plotting subplots for each state variable on 4 figure

%figure(11);
%figure for u1 in each encounter
fig1 = figure(11);
subplot(2,2,1);
plot(t, u111, '--', 'linewidth', 5)
hold on
plot(t, u112, '--', 'linewidth', 4)
hold on
plot(t, u113, '*', 'linewidth', 4)
hold on
plot(t, u114, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1,
0.9]','Location','southwest')
grid on
ylabel('Mkt Value of Liquid Reserve M(t) ($b)');
xlabel('Time(Years)')
title('A. Encounter 1')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
%saveas(ffigure(1),'Olanike Adeoye, 11, open-loop Nash equilibrium, feedback
Nash equilibrium, Pareto Trajectories for the company liquid reserve
(Encounter 1).tif')
box off

subplot(2,2,2);
plot(t, u211, '--', 'linewidth', 5)
hold on
plot(t, u212, '--', 'linewidth', 4)
hold on
plot(t, u213, '*', 'linewidth', 4)
hold on
plot(t, u214, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1,
0.9]','Location','southwest')
grid on
ylabel('Mkt Value of Liquid Reserve M(t) ($b)')
xlabel('Time(Years)')
title('B. Encounter 2')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
%saveas(ffigure(1),'Olanike Adeoye, 11, open-loop Nash equilibrium, feedback
Nash equilibrium, Pareto Trajectories for the company liquid reserve
(Encounter 1).tif')
box off

subplot(2,2,3);
plot(t, u311, '--', 'linewidth', 5)
hold on
plot(t, u312, '--', 'linewidth', 4)

```

```

hold on
plot(t, u313, '*', 'linewidth', 4)
hold on
plot(t, u314, 'linewidth', 4)
hold on
legend('Open-loop', 'Feedback', 'Pareto [0.9, 0.1]', 'Pareto [0.1, 0.9]', 'Location', 'southwest')
grid on
title ('C. Encounter 3')
set(gca, 'FontSize', 15)
set(gca, 'XTick', (0:2:10))
set(gca, 'TickDir', 'out')
xlim([0 10])
box off

subplot(2,2,4);
plot(t, u411, '--', 'linewidth', 5)
hold on
plot(t, u412, '--', 'linewidth', 4)
hold on
plot(t, u413, '*', 'linewidth', 4)
hold on
plot(t, u414, 'linewidth', 4)
hold on
legend('Open-loop', 'Feedback', 'Pareto [0.9, 0.1]', 'Pareto [0.1, 0.9]', 'Location', 'southwest')
grid on
title ('D. Encounter 4')
set(gca, 'FontSize', 15)
set(gca, 'XTick', (0:2:10))
set(gca, 'TickDir', 'out')
xlim([0 10])

sgt = sgtitle('Debt repayment u1(t)', 'Color', 'Black', 'FontWeight', 'bold');
sgt.FontSize = 20;
%
han=axes(fig1, 'visible', 'off');
han.XLabel.Visible='on';
han.YLabel.Visible='on';
xlabel(han, 'Time (Years)', 'FontSize', 18);
xlh=xlabel(han, 'Time (Years)', 'FontSize', 20);
xlh.Position(2) =xlh.Position(2) - abs(xlh.Position(2) * 0.4);

pos = get(gca, 'Position');
pos(1) = pos(1) - 0.03;
set(gca, 'Position', pos)

%figure(21);
%plotting u2 in each encounter
fig2 = figure(21);
subplot(2,2,1);
plot(t, u121, '--', 'linewidth', 6)
hold on
plot(t, u122, '*', 'linewidth', 7)
hold on
plot(t, u123, 'linewidth', 4)
hold on
plot(t, u124, 'linewidth', 4)
hold on
legend('Open-loop', 'Feedback', 'Pareto [0.9, 0.1]', 'Pareto [0.1, 0.9]', 'Location', 'northeast')
grid on
title ('A. Encounter 1')
set(gca, 'FontSize', 15)
set(gca, 'XTick', (0:2:10))
set(gca, 'TickDir', 'out')
xlim([0 10])

```



```

box off

subplot(2,2,2);
plot(t, u221, '--', 'linewidth', 6)
hold on
plot(t, u222, '*', 'linewidth', 7)
hold on
plot(t, u223, 'linewidth', 4)
hold on
plot(t, u224, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','northeast')
grid on
title ('B. Encounter 2')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])

subplot(2,2,3);
plot(t, u321, '--', 'linewidth', 6)
hold on
plot(t, u322, '*', 'linewidth', 7)
hold on
plot(t, u323, 'linewidth', 4)
hold on
plot(t, u324, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','northeast')
grid on
title ('C. Encounter 3')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,4);
plot(t, u421, '--', 'linewidth', 6)
hold on
plot(t, u422, '*', 'linewidth', 7)
hold on
plot(t, u423, 'linewidth', 4)
hold on
plot(t, u424, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','northeast')
grid on
title ('D. Encounter 4')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

%sgt gives common title to the subplots
sgt = sgtitle('Monitoring u2(t)', 'Color','Black', 'FontWeight', 'bold', 'FontSize', 20);
han=axes(fig2,'visible','off');
%han.Title.Visible='on';
han.XLabel.Visible='on';

```

```

han.YLabel.Visible='on';
xlabel(han,'Time (Years)', 'FontSize',18);
xlh=xlabel(han,'Time (Years)', 'FontSize',20);
xlh.Position(2) =xlh.Position(2) - abs(xlh.Position(2) * 0.4);

pos = get(gca, 'Position');
pos(1) = pos(1) - 0.03;
set(gca, 'Position',pos)

%plotting figure(31)
fig3 = figure(31);
subplot(2,2,1);
plot(t, u131, '--', 'linewidth', 5)
hold on
plot(t, u132, '--','linewidth', 4)
hold on
plot(t, u133,'linewidth', 4)
hold on
plot(t, u134,'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','northeast')
grid on
title ('A. Encounter 1')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,2);
plot(t, u231, '--', 'linewidth', 5)
hold on
plot(t, u232, '--','linewidth', 4)
hold on
plot(t, u233,'linewidth', 4)
hold on
plot(t, u234,'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','northeast')
grid on
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,3);
plot(t, u331, '--', 'linewidth', 5)
hold on
plot(t, u332, '--','linewidth', 4)
hold on
plot(t, u333,'linewidth', 4)
hold on
plot(t, u334,'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','northeast')
grid on
title ('C. Encounter 3')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

```

```

subplot(2,2,4);
plot(t, u431, '--', 'linewidth', 5)
hold on
plot(t, u432, '--', 'linewidth', 4)
hold on
plot(t, u433, 'linewidth', 4)
hold on
plot(t, u434, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1,
0.9]','Location','northeast')
grid on
title ('D. Encounter 4')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

%sgt gives common title to the subplots
sgt = sgtitle('Investment u3(t)', 'Color','Black', 'FontWeight', 'bold');
sgt.FontSize = 20;
%
han=axes(fig3,'visible','off');
han.XLabel.Visible='on';
han.YLabel.Visible='on';
xlabel(han,'Time (Years)', 'FontSize',18);
xlh=xlabel(han,'Time (Years)', 'FontSize',20);
xlh.Position(2) =xlh.Position(2) - abs(xlh.Position(2) * 0.4);

pos = get(gca, 'Position');
pos(1) = pos(1) - 0.03;
set(gca, 'Position',pos)

%plotting figure(31)
fig4 = figure(41);
subplot(2,2,1);
plot(t, u141, '--', 'linewidth', 5)
hold on
plot(t, u142, '--', 'linewidth', 4)
hold on
plot(t, u143, 'linewidth', 4)
hold on
plot(t, u144, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1,
0.9]','Location','northeast')
grid on
title ('A. Encounter 1')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,2);
plot(t, u241, '--', 'linewidth', 5)
hold on
plot(t, u242, '--', 'linewidth', 4)
hold on
plot(t, u243, 'linewidth', 4)
hold on
plot(t, u244, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1,
0.9]','Location','northeast')
grid on

```

```

title ('B. Encounter 2')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,3);
plot(t, u241, '--', 'linewidth', 5)
hold on
plot(t, u242, '--', 'linewidth', 4)
hold on
plot(t, u243, 'linewidth', 4)
hold on
plot(t, u244, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','northeast')
grid on
title ('C. Encounter 3')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,4);
plot(t, u441, '--', 'linewidth', 5)
hold on
plot(t, u442, '--', 'linewidth', 4)
hold on
plot(t, u443, 'linewidth', 4)
hold on
plot(t, u444, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','northeast')
grid on
title ('D. Encounter 4')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])

%sgt gives common title to the subplots
sgt = sgtitle('Quality of effort u4(t)', 'Color','Black', 'FontWeight',
'bold');
sgt.FontSize = 20;
%
han=axes(fig4,'visible','off');
han.XLabel.Visible='on';
han.YLabel.Visible='on';
xlabel(han,'Time (Years)', 'FontSize',18);
xlh=xlabel(han,'Time (Years)', 'FontSize',20);
xlh.Position(2) =xlh.Position(2) - abs(xlh.Position(2) * 0.4);

pos = get(gca, 'Position');
pos(1) = pos(1) - 0.03;
set(gca, 'Position',pos)

```

```
%Chapter 4 - comparing state variables in all solution concepts
```

```
t =      [0      1      2      3      4      5      6      7
8        9      10];
```

```
%Encounter 1
```

```
%Openloop
```

```
Y11 =      [0.2900    0.3248    0.3591    0.3931    0.4268    0.4606    0.4946
0.5291    0.5644    0.6009    0.6389];
```

```
D11 =      [0.2700    0.2661    0.2617    0.2569    0.2519    0.2469    0.2421
0.2379    0.2344    0.2322    0.2315];
```

```
I11 =      [0.0030    0.0036    0.0043    0.0051    0.0059    0.0068    0.0077
0.0087    0.0098    0.0110    0.0123];
```

```
S11 =      [1.3000    1.3000    1.3000    1.3000    1.3000    1.3000    1.3000
1.3001    1.3001    1.3001    1.3001];
```

```
%Feedback
```

```
Y12 =      [0.2900    0.3248    0.3591    0.3931    0.4268    0.4606    0.4946
0.5291    0.5644    0.6009    0.6389];
```

```
D12 =      [0.2700    0.2661    0.2617    0.2569    0.2519    0.2469    0.2422
0.2379    0.2344    0.2322    0.2315];
```

```
I12 =      [0.0030    0.0036    0.0043    0.0051    0.0059    0.0068    0.0077
0.0087    0.0098    0.0110    0.0123];
```

```
S12 =      [1.3000    1.3000    1.3000    1.3000    1.3000    1.3000    1.3000
1.3000    1.3001    1.3001    1.3001];
```

```
%Pareto (0.9, 0.1)
```

```
%phi = 0.9
```

```
Y13 =      [0.2900    0.3248    0.3591    0.3930    0.4268    0.4605    0.4945
0.5290    0.5643    0.6008    0.6389];
```

```
D13 =      [0.2700    0.2661    0.2617    0.2569    0.2519    0.2469    0.2422
0.2379    0.2344    0.2322    0.2315];
```

```
I13 =      [0.0030    0.0036    0.0043    0.0051    0.0059    0.0068    0.0077
0.0088    0.0098    0.0110    0.0122];
```

```
S13 =      [1.3000    1.3000    1.3000    1.3001    1.3001    1.3001    1.3001
1.3001    1.3001    1.3001    1.3001];
```

```
%Pareto(0.1, 0.9)
```

```
Y14 =      [0.2900    0.3243    0.3582    0.3917    0.4251    0.4585    0.4923
0.5266    0.5618    0.5983    0.6364];
```

```
D14 =      [0.2700    0.2656    0.2607    0.2555    0.2502    0.2449    0.2399
0.2355    0.2319    0.2296    0.2289];
```

```
I14 =      [0.0030    0.0036    0.0043    0.0051    0.0059    0.0068    0.0077
0.0087    0.0098    0.0110    0.0122];
```

```
S14 =      [1.3000    1.3000    1.3000    1.3000    1.3001    1.3001    1.3001
1.3001    1.3001    1.3001    1.3001];
```

```
%Encounter 2
```

```
%openloop
```

```
Y21 =      [0.2900    0.3633    0.4350    0.5052    0.5742    0.6425    0.7105
0.7790    0.8489    0.9211    0.9969];
```

```

D21 = [0.2700 0.2656 0.2596 0.2520 0.2433 0.2338 0.2241
0.2148 0.2069 0.2013 0.1993];

I21 = [0.0030 0.0037 0.0045 0.0054 0.0065 0.0077 0.0091
0.0105 0.0122 0.0140 0.0159];

S21 = [1.3000 1.3000 1.3000 1.3000 1.3000 1.3000 1.3001
1.3001 1.3001 1.3001 1.3001];

%feedback
Y22 = [0.2900 0.3633 0.4350 0.5052 0.5742 0.6425 0.7105
0.7791 0.8489 0.9211 0.9969];

D22 = [0.2700 0.2656 0.2596 0.2520 0.2433 0.2338 0.2241
0.2148 0.2069 0.2013 0.1993];

I22 = [0.0030 0.0037 0.0045 0.0054 0.0065 0.0077 0.0091
0.0105 0.0122 0.0140 0.0159];

S22 = [1.3000 1.3000 1.3000 1.3000 1.3000 1.3000 1.3001
1.3001 1.3001 1.3001 1.3001];

%Pareto (0.9, 0.1)
%phi = 0.9
Y23 = [0.2900 0.3633 0.4350 0.5052 0.5742 0.6424 0.7105
0.7790 0.8488 0.9211 0.9968];

D23 = [0.2700 0.2656 0.2596 0.2521 0.2434 0.2339 0.2241
0.2149 0.2069 0.2014 0.1993];

I23 = [0.0030 0.0037 0.0045 0.0054 0.0065 0.0077 0.0091
0.0106 0.0122 0.0140 0.0159];

S23 = [1.3000 1.3000 1.3001 1.3001 1.3001 1.3002 1.3002
1.3002 1.3002 1.3002 1.3002];

%Pareto(0.1, 0.9)
%phi = 0.1
Y24 = [0.2900 0.3628 0.4339 0.5034 0.5718 0.6395 0.7071
0.7753 0.8450 0.9171 0.9929];

D24 = [0.2700 0.2651 0.2584 0.2503 0.2409 0.2308 0.2207
0.2111 0.2030 0.1974 0.1953];

I24 = [0.0030 0.0037 0.0045 0.0054 0.0065 0.0077 0.0090
0.0105 0.0121 0.0139 0.0158];

S24 = [1.3000 1.3000 1.3000 1.3000 1.3001 1.3001 1.3001
1.3001 1.3001 1.3001 1.3001];

%Encounter 3
%Openloop
Y31 = [0.2900 0.3248 0.3591 0.3931 0.4268 0.4606 0.4946
0.5291 0.5644 0.6009 0.6389];

D31 = [0.2700 0.2740 0.2776 0.2809 0.2842 0.2875 0.2912
0.2955 0.3008 0.3074 0.3158];

I31 = [0.0030 0.0036 0.0043 0.0051 0.0059 0.0068 0.0077
0.0087 0.0098 0.0110 0.0123];

S31 = [1.3000 1.3000 1.3000 1.3001 1.3001 1.3001 1.3001
1.3001 1.3001 1.3001 1.3001];

```

```

%feedback
Y32 = [0.2900 0.3248 0.3591 0.3931 0.4268 0.4606 0.4946
0.5291 0.5644 0.6009 0.6389];

D32 = [0.2700 0.2740 0.2776 0.2809 0.2842 0.2875 0.2912
0.2955 0.3008 0.3074 0.3158];

I32 = [0.0030 0.0036 0.0043 0.0051 0.0059 0.0068 0.0077
0.0087 0.0098 0.0110 0.0123];

S32 = [1.3000 1.3000 1.3000 1.3001 1.3001 1.3001 1.3001
1.3001 1.3001 1.3001 1.3001];

%Pareto (0.9, 0.1)
%phi = 0.9
Y33 = [0.2900 0.3248 0.3591 0.3930 0.4268 0.4605 0.4945
0.5290 0.5643 0.6008 0.6389];

D33 = [0.2700 0.2740 0.2776 0.2810 0.2843 0.2876 0.2913
0.2956 0.3009 0.3075 0.3159];

I33 = [0.0030 0.0036 0.0043 0.0051 0.0059 0.0068 0.0077
0.0088 0.0098 0.0110 0.0123];

S33 = [1.3000 1.3001 1.3001 1.3001 1.3002 1.3002 1.3003
1.3003 1.3003 1.3003 1.3003];

%Pareto(0.1, 0.9)
Y34 = [0.2900 0.3243 0.3582 0.3917 0.4251 0.4585 0.4923
0.5266 0.5618 0.5983 0.6364];

D34 = [0.2700 0.2735 0.2766 0.2795 0.2823 0.2853 0.2887
0.2928 0.2979 0.3044 0.3127];

I34 = [0.0030 0.0036 0.0043 0.0051 0.0059 0.0068 0.0077
0.0087 0.0098 0.0110 0.0122];

S34 = [1.3000 1.3000 1.3000 1.3001 1.3001 1.3001 1.3001
1.3002 1.3002 1.3002 1.3002];

%Encounter 4
%Openloop
Y41 = [0.2900 0.3633 0.4350 0.5052 0.5742 0.6425 0.7105
0.7791 0.8489 0.9211 0.9969];

D41 = [0.2700 0.2735 0.2754 0.2759 0.2753 0.2738 0.2720
0.2707 0.2707 0.2730 0.2791];

I41 = [0.0030 0.0037 0.0045 0.0054 0.0065 0.0077 0.0091
0.0105 0.0122 0.0140 0.0159];

S41 = [1.3000 1.3000 1.3000 1.3001 1.3001 1.3001 1.3001
1.3001 1.3002 1.3002 1.3002];

%feedback
Y42 = [0.2900 0.3633 0.4350 0.5052 0.5743 0.6425 0.7105
0.7791 0.8489 0.9212 0.9969];

D42 = [0.2700 0.2735 0.2754 0.2760 0.2753 0.2738 0.2720
0.2707 0.2707 0.2730 0.2791];

I42 = [0.0030 0.0037 0.0045 0.0054 0.0065 0.0077 0.0091
0.0105 0.0122 0.0140 0.0159];

S42 = [1.3000 1.3000 1.3000 1.3001 1.3001 1.3001 1.3001
1.3001 1.3002 1.3002 1.3002];

```

```

%Pareto (0.9, 0.1)
%phi = 0.9

Y43 = [0.2900    0.3633    0.4350    0.5052    0.5742    0.6425    0.7105
0.7791    0.8489    0.9212    0.9970];

D43 = [0.2700    0.2736    0.2756    0.2761    0.2755    0.2740    0.2723
0.2710    0.2710    0.2733    0.2794];

I43 = [0.0030    0.0037    0.0045    0.0054    0.0065    0.0077    0.0091
0.0106    0.0122    0.0140    0.0159];

S43 = [1.3000    1.3001    1.3002    1.3003    1.3003    1.3003    1.3005
1.3005    1.3005    1.3005    1.3005];

%Pareto (0.1, 0.9)phi = 0.1
Y44 = [0.2900    0.3628    0.4339    0.5034    0.5718    0.6395    0.7071
0.7753    0.8450    0.9172    0.9929];

D44 = [0.2700    0.2730    0.2743    0.2741    0.2727    0.2706    0.2683
0.2665    0.2662    0.2684    0.2743];

I44 = [0.0030    0.0037    0.0045    0.0054    0.0065    0.0077    0.0090
0.0105    0.0121    0.0139    0.0158];

S44 = [1.3000    1.3000    1.3001    1.3001    1.3001    1.3001    1.3002
1.3002    1.3002    1.3002    1.3002];

```



```

%plotting subplots for each state variable on 4 figures

%figure(11);
fig1 = figure(11);
subplot(2,2,1);
plot(t, Y11, '--', 'linewidth', 5)
hold on
plot(t, Y12, '--', 'linewidth', 4)
hold on
plot(t, Y13, '*', 'linewidth', 4)
hold on
plot(t, Y14, '--', 'linewidth', 3)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','southeast')
grid on
title ('A. Encounter 1')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,2);
plot(t, Y21, '--', 'linewidth', 5)
hold on
plot(t, Y22, '--', 'linewidth', 4)
hold on
plot(t, Y23, '*', 'linewidth', 4)
hold on
plot(t, Y24, '--', 'linewidth', 3)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','southeast')
grid on
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,3);
plot(t, Y31, '--', 'linewidth', 5)
hold on
plot(t, Y32, '--', 'linewidth', 4)
hold on
plot(t, Y33, '*', 'linewidth', 4)
hold on
plot(t, Y34, '--', 'linewidth', 3)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','southeast')
grid on
title ('C. Encounter 3')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,4);
plot(t, Y41, '--', 'linewidth', 5)
hold on
plot(t, Y42, '--', 'linewidth', 4)
hold on

```

```

plot(t, Y43, '*', 'linewidth', 4)
hold on
plot(t, Y44, '--', 'linewidth', 3)
hold on
legend('Open-loop', 'Feedback', 'Pareto [0.9, 0.1]', 'Pareto [0.1, 0.9]', 'Location', 'southeast')
grid on
title ('D. Encounter 4')
set(gca, 'FontSize', 15)
set(gca, 'XTick', (0:2:10))
set(gca, 'TickDir', 'out')
xlim([0 10])
box off

%sgt gives common title to the subplots
sgt = sgtitle('Cash flow Y(t)', 'Color', 'Black', 'FontWeight', 'bold');
sgt.FontSize = 20;
%
han=axes(fig1, 'visible', 'off');
han.XLabel.Visible='on';
han.YLabel.Visible='on';
ylabel(han, '$000,000,000', 'FontSize', 18);
xlabel(han, 'Time (Years)', 'FontSize', 18);
xlh=xlabel(han, 'Time (Years)', 'FontSize', 20);
xlh.Position(2) =xlh.Position(2) - abs(xlh.Position(2) * 0.4);

pos = get(gca, 'Position');
pos(1) = pos(1) - 0.03;
set(gca, 'Position', pos)

%figure(21);
fig2 = figure(21);
subplot(2,2,1);
plot(t, D11, '--', 'linewidth', 6)
hold on
plot(t, D12, '*', 'linewidth', 7)
hold on
plot(t, D13, 'linewidth', 4)
hold on
plot(t, D14, 'linewidth', 4)
hold on
legend('Open-loop', 'Feedback', 'Pareto [0.9, 0.1]', 'Pareto [0.1, 0.9]', 'Location', 'southwest')
grid on
title ('A. Encounter 1')
set(gca, 'FontSize', 15)
set(gca, 'XTick', (0:2:10))
set(gca, 'TickDir', 'out')
xlim([0 10])
box off

subplot(2,2,2);
plot(t, D21, '--', 'linewidth', 6)
hold on
plot(t, D22, '*', 'linewidth', 7)
hold on
plot(t, D23, 'linewidth', 4)
hold on
plot(t, D24, 'linewidth', 4)
hold on
legend('Open-loop', 'Feedback', 'Pareto [0.9, 0.1]', 'Pareto [0.1, 0.9]', 'Location', 'southwest')
grid on
title ('B. Encounter 2')
set(gca, 'FontSize', 15)
set(gca, 'XTick', (0:2:10))
set(gca, 'TickDir', 'out')
xlim([0 10])

```

```

box off

subplot(2,2,3);
plot(t, D31, '--', 'linewidth', 6)
hold on
plot(t, D32, '*', 'linewidth', 7)
hold on
plot(t, D33, 'linewidth', 4)
hold on
plot(t, D34, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','southeast')
grid on
title ('C. Encounter 3')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,4);
plot(t, D41, '--', 'linewidth', 6)
hold on
plot(t, D42, '*', 'linewidth', 7)
hold on
plot(t, D43, 'linewidth', 4)
hold on
plot(t, D44, 'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','southwest')
grid on
title ('D. Encounter 4')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

%sgt gives common title to the subplots
sgt = sgtitle('Debt value over time D(t)', 'Color','Black', 'FontWeight',
'bold', 'FontSize', 20);
han=axes(fig2,'visible','off');
han.XLabel.Visible='on';
han.YLabel.Visible='on';
ylabel(han,'$000,000,000','FontSize',18);
xlabel(han,'Time (Years)', 'FontSize',18);
xlh=xlabel(han,'Time (Years)', 'FontSize',20);
xlh.Position(2) =xlh.Position(2) - abs(xlh.Position(2) * 0.4);

pos = get(gca, 'Position');
pos(1) = pos(1) - 0.03;
set(gca, 'Position',pos)

%plotting figure(31)
fig3 = figure(31);
subplot(2,2,1);
plot(t, I11, '--', 'linewidth', 5)
hold on
plot(t, I12, '--', 'linewidth', 4)
hold on
plot(t, I13, '*', 'linewidth', 4)

```

```

hold on
plot(t, I14, '--', 'linewidth', 3)
hold on
legend('Open-loop', 'Feedback', 'Pareto [0.9, 0.1]', 'Pareto [0.1, 0.9]', 'Location', 'southeast')
grid on
title ('A. Encounter 1')
set(gca, 'FontSize', 15)
set(gca, 'XTick', (0:2:10))
set(gca, 'TickDir', 'out')
xlim([0 10])
box off

subplot(2,2,2);
plot(t, I21, '--', 'linewidth', 5)
hold on
plot(t, I22, '--', 'linewidth', 4)
hold on
plot(t, I23, '*', 'linewidth', 4)
hold on
plot(t, I24, '--', 'linewidth', 3)
hold on
legend('Open-loop', 'Feedback', 'Pareto [0.9, 0.1]', 'Pareto [0.1, 0.9]', 'Location', 'southeast')
grid on
title ('B. Encounter 2')
set(gca, 'FontSize', 15)
set(gca, 'XTick', (0:2:10))
set(gca, 'TickDir', 'out')
xlim([0 10])
box off

subplot(2,2,3);
plot(t, I31, '--', 'linewidth', 5)
hold on
plot(t, I32, '--', 'linewidth', 4)
hold on
plot(t, I33, '*', 'linewidth', 4)
hold on
plot(t, I34, '--', 'linewidth', 3)
hold on
legend('Open-loop', 'Feedback', 'Pareto [0.9, 0.1]', 'Pareto [0.1, 0.9]', 'Location', 'southeast')
grid on
title ('C. Encounter 3')
set(gca, 'FontSize', 15)
set(gca, 'XTick', (0:2:10))
set(gca, 'TickDir', 'out')
xlim([0 10])
box off

subplot(2,2,4);
plot(t, I41, '--', 'linewidth', 5)
hold on
plot(t, I42, '--', 'linewidth', 4)
hold on
plot(t, I43, '*', 'linewidth', 4)
hold on
plot(t, I44, '--', 'linewidth', 3)
hold on
legend('Open-loop', 'Feedback', 'Pareto [0.9, 0.1]', 'Pareto [0.1, 0.9]', 'Location', 'southeast')
grid on
title ('D. Encounter 4')
set(gca, 'FontSize', 15)
set(gca, 'XTick', (0:2:10))
set(gca, 'TickDir', 'out')

```

```

xlim([0 10])
box off

%sgt gives common title to the subplots
sgt = sgtitle('Pecuniary incentive I(t)', 'Color','Black', 'FontWeight',
'bold');
sgt.FontSize = 20;
%
han=axes(fig3,'visible','off');
%han.Title.Visible='on';
han.XLabel.Visible='on';
han.YLabel.Visible='on';
ylabel(han,'$000,000,000','FontSize',18);
xlabel(han,'Time (Years)', 'FontSize',18);
xlh=xlabel(han,'Time (Years)', 'FontSize',20);
xlh.Position(2) =xlh.Position(2) - abs(xlh.Position(2) * 0.4);

pos = get(gca, 'Position');
pos(1) = pos(1) - 0.03;
set(gca, 'Position',pos)

%plotting figure(31)
fig4 = figure(41);
subplot(2,2,1);
plot(t, S11, '--', 'linewidth', 5)
hold on
plot(t, S12, '--','linewidth', 4)
hold on
plot(t, S13,'linewidth', 4)
hold on
plot(t, S14,'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1,
0.9]','Location','northwest')
grid on
title ('A. Encounter 1')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,2);
plot(t, S21, '--', 'linewidth', 5)
hold on
plot(t, S22, '--','linewidth', 4)
hold on
plot(t, S23, 'linewidth', 4)
hold on
plot(t, S24,'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1,
0.9]','Location','northwest')
grid on
title ('B. Encounter 2')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,3);
plot(t, S31, '--', 'linewidth', 5)
hold on
plot(t, S32, '--','linewidth', 4)
hold on
plot(t, S33,'linewidth', 4)

```

```

hold on
plot(t, S34,'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','northwest')
grid on
title ('C. Encounter 3')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,4);
plot(t, S41, '--', 'linewidth', 5)
hold on
plot(t, S42, '--','linewidth', 4)
hold on
plot(t, S43,'linewidth', 4)
hold on
plot(t, S44,'linewidth', 4)
hold on
legend('Open-loop','Feedback','Pareto [0.9, 0.1]','Pareto [0.1, 0.9]','Location','northwest')
grid on
title ('D. Encounter 4')
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

%sgt gives common title to the subplots
sgt = sgtitle('Productive assets S(t)', 'Color','Black', 'FontWeight', 'bold');
sgt.FontSize = 20;
%
han=axes(fig4,'visible','off');
han.XLabel.Visible='on';
han.YLabel.Visible='on';
ylabel(han,'$000,000,000','FontSize',18);
xlabel(han,'Time (Years)', 'FontSize',18);
xlh=xlabel(han,'Time (Years)', 'FontSize',20);
xlh.Position(2) =xlh.Position(2) - abs(xlh.Position(2) * 0.4);

ylh=ylabel(han,'$000,000,000', 'FontSize',20);
ylh.Position(2) =ylh.Position(2) - abs(ylh.Position(2) * 0.01);

pos = get(gca, 'Position');
pos(1) = pos(1) - 0.03;
set(gca, 'Position',pos)

```

```
%Chapter 4 - Plotting debt-equity ratio for all cases, at phi = 0.1 and phi =0.9
```

```
t =      [0      1      2      3      4      5      6      7
8      9     10];
```

```
%Encounter 1
```

```
DE11 = [0.2045    0.1955    0.1865    0.1779    0.1696    0.1618    0.1545
0.1480    0.1423    0.1376    0.1340];
```

```
DE91 = [0.2045    0.1958    0.1873    0.1789    0.1708    0.1631    0.1560
0.1495    0.1438    0.1392    0.1356];
```

```
%openloop case
```

```
DE21 = [0.2045    0.1958    0.1873    0.1789    0.1708    0.1631    0.1559
0.1495    0.1438    0.1391    0.1356];
```

```
%Feedback
```

```
DE31 = [0.2045    0.1958    0.1873    0.1789    0.1708    0.1631    0.1560
0.1495    0.1438    0.1391    0.1356];
```

```
%Encounter 2
```

```
DE12 = [0.2045    0.1897    0.1751    0.1612    0.1477    0.1351    0.1235
0.1132    0.1045    0.0977    0.0931];
```

```
DE92 = [0.2045    0.1900    0.1759    0.1623    0.1492    0.1369    0.1254
0.1153    0.1065    0.0997    0.0950];
```

```
%openloop case
```

```
DE22 = [0.2045    0.1900    0.1760    0.1622    0.1492    0.1368    0.1254
0.1152    0.1065    0.0997    0.0950];
```

```
%Feedback
```

```
DE32 = [0.2045    0.1900    0.1760    0.1622    0.1492    0.1368    0.1254
0.1152    0.1065    0.0997    0.0950];
```

```
%Encounter 3
```

```
DE13 = [0.2045    0.2025    0.2002    0.1979    0.1956    0.1936    0.1920
0.1909    0.1905    0.1910    0.1926];
```

```
DE93 = [0.2045    0.2028    0.2009    0.1990    0.1971    0.1952    0.1937
0.1927    0.1924    0.1930    0.1946];
```

```
%openloop
```

```
DE23 = [0.2045    0.2028    0.2009    0.1989    0.1970    0.1952    0.1937
0.1927    0.1924    0.1929    0.1946];
```

```
%Feedback
```

```
DE33 = [0.2045    0.2028    0.2009    0.1989    0.1970    0.1952    0.1937
0.1927    0.1924    0.1929    0.1946];
```

```
%Encounter 4
```

```
DE14 = [0.2045    0.1964    0.1879    0.1792    0.1705    0.1621    0.1543
0.1473    0.1417    0.1377    0.1359];
```

```
DE94 = [0.2045    0.1969    0.1888    0.1805    0.1723    0.1642    0.1566
0.1498    0.1443    0.1403    0.1384];
```

```
%openloop
```

```
DE24 = [0.2045    0.1968    0.1887    0.1804    0.1722    0.1641    0.1564
0.1497    0.1441    0.1401    0.1383];
```

```
%feedback
```

```
DE34 = [0.2045    0.1968    0.1887    0.1805    0.1722    0.1641    0.1564
0.1497    0.1441    0.1401    0.1383];
```

```
fig = figure;
```

```

subplot(2,2,1);
plot(t, DE21, '*', 'linewidth', 10)
hold on
plot(t, DE22, 'linewidth', 5)
hold on
plot(t, DE23, 'linewidth', 5)
hold on
plot(t, DE24, 'linewidth', 5)
legend('Encounter 1','Encounter 2','Encounter 3', 'Encounter 4',
'Location','southwest')
title ('A. Open-loop')
grid on
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,2);
plot(t, DE31, '*', 'linewidth', 10)
hold on
plot(t, DE32, 'linewidth', 5)
hold on
plot(t, DE33, 'linewidth', 5)
hold on
plot(t, DE34, 'linewidth', 5)
legend('Encounter 1','Encounter 2','Encounter 3', 'Encounter 4',
'Location','southwest')
title ('B. Feedback')
grid on
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

subplot(2,2,3);
plot(t, DE91, '*', 'linewidth', 10)
hold on
plot(t, DE92, 'linewidth', 5)
hold on
plot(t, DE93, 'linewidth', 5)
hold on
plot(t, DE94, 'linewidth', 5)
legend('Encounter 1','Encounter 2','Encounter 3', 'Encounter 4',
'Location','southwest')
title ('C. Pareto (0.9, 0.1)')
grid on
set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

% plotting the feedback case
subplot(2,2,4);
plot(t, DE11, '*', 'linewidth', 10)
hold on
plot(t, DE12, 'linewidth', 5)
hold on
plot(t, DE13, 'linewidth', 5)
hold on
plot(t, DE14, 'linewidth', 5)
legend('Encounter 1','Encounter 2','Encounter 3', 'Encounter 4',
'Location','southwest')
title('D. Pareto (0.1, 0.9)')
grid on

```



```

set(gca,'FontSize',15)
set(gca,'XTick',(0:2:10))
set(gca,'TickDir','out')
xlim([0 10])
box off

%sgt gives common title to the subplots
sgt = sgtitle('Optimal debt-equity ratio', 'Color','Black', 'FontWeight',
'bold');
sgt.FontSize = 20;
%
han=axes(fig,'visible','off');
han.XLabel.Visible='on';
han.YLabel.Visible='on';
xlabel(han,'Time(Years)', 'FontSize',18);
xlh=xlabel(han,'Time(Years)', 'FontSize',20);
xlh.Position(2) =xlh.Position(2) - abs(xlh.Position(2) * 0.4);

pos = get(gca, 'Position');
pos(1) = pos(1) - 0.03;
set(gca, 'Position',pos)

```

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