

Active fault-tolerant control for switched systems with time delay

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Abstract. This paper focuses on the problem of active fault-tolerant control for switched systems with time delay. By utilizing the fault diagnosis observer, an adaptive fault estimate algorithm is proposed, which can estimate the fault signal fast and exactly. Meanwhile, a delay-dependent criteria is obtained with the purpose of reducing the conservatism of the adaptive observer design. Based on the fault estimation information, an observer-based fault-tolerant controller is designed to guarantee the stability of the closed-loop system. In terms of linear matrix inequality, sufficient conditions are derived for the existence of the adaptive observer and fault-tolerant controller. Finally, A numerical example is included to illustrate the efficiency of the proposed approach.

Keywords: Fault-tolerant control; LMI; switched system; time delay.

1 Introduction

In order to increase the safety and reliability of dynamic systems, the issues of fault diagnosis (FD) and fault tolerant control (FTC) have become an attractive topic and been paid much attention in recent years. Many researchers have devoted themselves to these issues, and fruitful results can be found in several excellent papers [1]-[12] and books [13, 14]. As for FTC, which includes two main approaches: passive FTC and active FTC. In passive FTC systems, a single controller with fixed structure/parameters is used to deal with all possible failure scenarios which are assumed to be known a priori. Consequently, the passive controller is usually conservative. Furthermore, if a failure out of those consideration in the design occurs, the stability and performance of the closed-loop system might not be guaranteed. Such potential limitations of passive FTC approaches provide a strong motivation for the development of methods and strategies for active FTC (AFTC) systems.

In contrast with passive FTC systems, AFTC techniques rely on a real-time fault detection and isolation (FDI) scheme and a controller reconfiguration mechanism. Such techniques allow a flexibility to

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select different controllers according to different component failures, and therefore better performance of the closed-loop system can be expected. If an AFTC is designed properly, it will be able to deal with unforeseen faults and maintain the system stability and acceptable level of performance in the presence of fault. Some preliminary results on AFTC can be found in [15]-[17] and references therein.

Compared with the fruitful FTC results for various dynamic systems, relatively few efforts were made to investigate FTC issue for switched systems. [18] and [20] considered the passive FTC issue for discrete-time switched systems. In [19], passive FTC for switched nonlinear systems in lower triangular was studied. Switched system belongs to hybrid system, which consists of several subsystems and a switching signal that specifies which subsystem will be activated along the system trajectory at each instant of time. Many real-world process and systems can be modeled as switched systems, including chemical processes, computer controlled systems, switched circuits, and so on. During the past three decades, fruitful theoretic results have been reported for switched systems, for examples [21]-[25] and references therein. On the other hand, time delays are the inherent features of many physical process and the big sources of instability and poor performances. Meanwhile, switched systems with time delay have strong engineering background, such as in network control systems [29] and power systems [30]. More recently, many theoretical studies were conducted for switched systems with time delays [31]-[33].

Up to date, to the best of the authors' knowledge, the AFTC for switched systems with time delay has not been addressed yet. Research is still under way into the development of an effective solutions for this issue, which motivates us to study this interesting and challenging issue. In this work, this issue will be solved. The contributions of this work can be summarized as the following two aspects.

- 1:) An adaptive estimate law is developed for the time-delay switched system. By constructing a switched Lyapunov function, a novel adaptive fault estimation algorithm is developed for the switched system with time delay. Moreover, the obtained result is delay-dependent, which makes further efforts to degrade the conservativeness. On the other hand, many fault estimate algorithms are confined to estimate constant fault. The advantage of such proposed estimation algorithm can not only estimate the fault fast and exactly, but also is adaptive to estimate two type of fault: constant fault case and time-varying fault case.
- 2:) To review the development of AFTC for switched systems, only the AFTC for nonlinear switched system were investigated in [12, 34]. Until now, the AFTC for switched system with time delay is not considered yet. This paper investigates this issue and efficient results are given.

The paper is organized as follows. Section 2 gives the model description. Section 3 presents the fault detection and fault estimation. The observer-based fault tolerant controller is designed in section 4. A numerical example is illustrated in Section 5 to show the usefulness and applicability of the proposed approaches, and the paper is concluded in Section 6.

2 Model Description

The switched system \mathcal{S} is described as follows :

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + A_{h\sigma(t)}x(t-h) + B_{\sigma(t)}u(t) + B_{\sigma(t)}f(t) \\ y(t) = C_{\sigma(t)}x(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^m$ is the output vector, $u(t) \in \mathbb{R}^p$ is the control input, $f(t) \in \mathbb{R}^l$ is the actuator fault, the time delay h is a known scalar and satisfies $0 < h < \bar{h}$. $\sigma(t) : [0, +\infty) \rightarrow \psi = \{1, \dots, N\}$ is the switching signal, $N > 1$ is the number of subsystems. At an arbitrary continuous time t , $\sigma(t)$, denoted by σ for simplicity, is dependent on t or $x(t)$, or both, or other switching rules. As in [36], we assume that the sequence of subsystems in switching signal $\sigma(t)$ is unknown *a priori*, but its instantaneous value is available in real time. $A_{\sigma(t)}$, $A_{h\sigma(t)}$, $B_{\sigma(t)}$, and $C_{\sigma(t)}$ are constant matrices with appropriate dimensions for all $\sigma(t) \in \psi$. When the i -subsystem is activated, we denote the matrices associated with $\sigma(t) = i$ by $A_{\sigma(t)} = A_i$, $A_{h\sigma(t)} = A_{hi}$, $B_{\sigma(t)} = B_i$, and $C_{\sigma(t)} = C_i$. For the purpose of this work, the following assumptions are given:

Assumption 1 *The matrix B_i is full column rank, i.e. $\text{rank}(B_i) = p$.*

Assumption 2 *The pair (A_i, B_i) is controllable and (A_i, C_i) is observable.*

The failure $f(t) = \nu(t - t_0)f_0(t)$ can be thought of as an additional signal, and the function $\nu(t - t_0)$ is given by

$$\nu(t - t_0) = \begin{cases} 0, & t \leq t_0 \\ 1, & t > t_0 \end{cases} \quad (2)$$

where t_0 is the time of fault occurring. That is, $f(t)$ is zero prior to the failure time $t \leq t_0$ and is $f_0(t)$ after the failure occurs $t > t_0$. It is assumed that the derivation $\dot{f}_0(t)$ with respect to time is norm bounded, i.e. $\|f_0(t)\| \leq f_1$, $\|\dot{f}_0(t)\| \leq f_2$, where $0 \leq f_1 < \infty$, $0 \leq f_2 < \infty$.

Before ending this section, the following lemmas are listed for later use.

Lemma 1 ([26]) *Given matrices \mathcal{W} , \mathcal{X} and \mathcal{Y} of appropriate dimensions and with \mathcal{W} symmetrical, then*

$$\mathcal{W} + \mathcal{X}F(t)\mathcal{Y} + \mathcal{Y}^T F^T(t)\mathcal{X}^T < 0$$

holds for all $F(t)$ satisfying $F^T(t)F(t) \leq I$ if and only if for some $\varepsilon > 0$,

$$\mathcal{W} + \varepsilon\mathcal{X}\mathcal{X}^T + \varepsilon^{-1}\mathcal{Y}^T\mathcal{Y} < 0$$

Lemma 2 ([27]) *For any real vectors a , b and matrix $G > 0$ of compatible dimensions, the following inequality holds:*

$$a^T b + b^T a \leq a^T G a + b^T G^{-1} b, \quad a, b \in R^n$$

3 Fault Detection and Fault Estimation

AFTC procedure includes three steps: fault detection, fault diagnosis and controller reconfiguration (or fault compensation). Fault detection is to decide whether or not a fault has occurred, and then fault diagnosis is to estimate the fault, at last controller reconfiguration compensates the effect of fault and keeps the closed-loop system stable in fault status. To express clearly, the AFTC architecture for switched system is described in Figure 1.

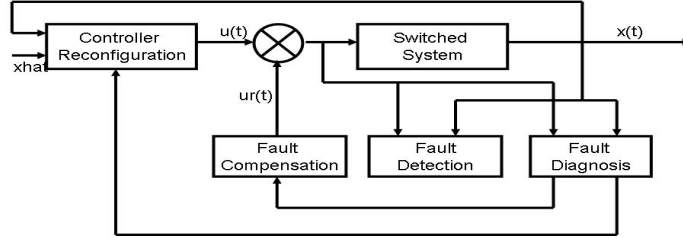


Figure 1. Active fault-tolerant control figure for switched system

3.1 Fault detection

Prior to the design of an adaptive diagnostic observer, the following assumption is made.

Assumption 3 *There exist positive definite matrices P_i and Q_i , $i, j \in \psi$, such that*

$$\begin{bmatrix} P_i(A_i - L_{di}C_i) + (A_i - L_{di}C_i)^T P_i + Q_i & P_i A_{hi} \\ * & -Q_j \end{bmatrix} < 0 \quad (3)$$

Under the condition (3), the fault detection observer can be designed as follows:

$$\begin{cases} \dot{x}_m(t) = A_{\sigma(t)}x_m(t) + A_{h\sigma(t)}x_m(t-h) + B_{\sigma(t)}u(t) + L_{d\sigma(t)}(y(t) - y_m(t)) \\ y_m(t) = C_{\sigma(t)}x_m(t) \end{cases} \quad (4)$$

where $x_m(t) \in \mathbb{R}^n$ is the state vector of the observer, $y_m(t) \in \mathbb{R}^m$ is the output vector of the observer. Define

$$e_m(t) = x(t) - x_m(t); \quad r_d(t) = y(t) - y_m(t) \quad (5)$$

Then the observation error and output error equations are given by

$$\begin{cases} \dot{e}_m(t) = (A_{\sigma(t)} - L_{d\sigma(t)}C_{\sigma(t)})e_m(t) + A_{h\sigma(t)}e_m(t-h) + B_{\sigma(t)}f(t) \\ r_d(t) = C_{\sigma(t)}e_m(t) \end{cases} \quad (6)$$

If no fault occurs, then from (6), one obtains that $\lim_{t \rightarrow \infty} r_d(t) = 0$. However, if there is a fault $f(t)$ and $\lim_{t \rightarrow \infty} f(t) \neq 0$, then $\lim_{t \rightarrow \infty} r_d(t) \neq 0$. Therefore the fault detection can be readily carried out as

$$\begin{cases} \lim_{t \rightarrow \infty} r_d(t) = 0, & \text{no fault occurs} \\ \lim_{t \rightarrow \infty} r_d(t) \neq 0, & \text{fault has occurred} \end{cases} \quad (7)$$

and the observer given by (4) is referred to as a fault detection observer for the switched system (1).

Remark 1 *Since the system (6) is a stable system, therefore we can successfully detect whether a fault occurs according to the logistic rule (7). For more details, one can refer to [2].*

3.2 Observer-based fault estimation

To diagnose the actuator fault after the alarm (7) has been generated, the following fault diagnosis observer is designed:

$$\begin{cases} \dot{\hat{x}}(t) = A_{\sigma(t)}\hat{x}(t) + A_{h\sigma(t)}\hat{x}(t-h) + B_{\sigma(t)}u(t) + B_{\sigma(t)}\hat{f}(t) + L_{\sigma(t)}(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C_{\sigma(t)}\hat{x}(t) \end{cases} \quad (8)$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state vector of the observer, $\hat{y}(t) \in \mathbb{R}^m$ is the output vector of the observer, $\hat{f}(t)$ is an estimate of $f(t)$, $L_{\sigma(t)}$ is the observer gain.

Denote

$$e(t) = \hat{x}(t) - x(t); \quad r(t) = \hat{y}(t) - y(t); \quad \tilde{f}(t) = \hat{f}(t) - f(t); \quad (9)$$

then the error dynamics is described by

$$\begin{cases} \dot{e}(t) = \bar{A}_{\sigma(t)}e(t) + A_{h\sigma(t)}e(t-h) + B_{\sigma(t)}\tilde{f}(t) \\ r(t) = C_{\sigma(t)}e(t) \end{cases} \quad (10)$$

where $\bar{A}_{\sigma(t)} = A_{\sigma(t)} - L_{\sigma(t)}C_{\sigma(t)}$. As a result, the purpose of fault estimation is to find a diagnostic algorithm for $\hat{f}(t)$ such that

$$\lim_{t \rightarrow \infty} e(t) = 0; \quad \lim_{t \rightarrow \infty} \hat{f}(t) = f(t) \quad (11)$$

In the sequel, a convergent adaptive diagnostic algorithm to estimate the fault $f(t)$ is given, which is obtained from the residual $r(t)$.

Theorem 1 *If there exist positive definite matrices P_i , Q_i , X_i , G , matrices F_i , Y_i , Z_i , W_i , and a scalar $\eta > 0$, for all $i, j, k \in \psi$, such that the following conditions hold*

$$\begin{bmatrix} P_i & I \\ I & X_i \end{bmatrix} \geq 0, \quad P_i X_i = I \quad (12)$$

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & -\bar{h}Y_i & (1,5) & 0 & C_i^T W_i^T \\ * & (2,2) & (2,3) & -\bar{h}Z_i & \bar{h}A_{hi}^T P_i & 0 & 0 \\ * & * & (3,3) & 0 & \bar{h}B_i^T P_i & F_i C_i & 0 \\ * & * & * & -\bar{h}P_j & 0 & 0 & 0 \\ * & * & * & * & -\bar{h}P_i & 0 & 0 \\ * & * & * & * & * & -\eta I & 0 \\ * & * & * & * & * & * & -\eta I \end{bmatrix} < 0 \quad (13)$$

where

$$(1,1) = P_i A_i + A_i^T P_i - W_i C_i - C_i^T W_i^T + Y_i + Y_i^T + Q_i;$$

$$(1,2) = P_i A_{hi} - Y_i + Z_i^T;$$

$$(1,3) = P_i B_i - C_i^T F_i^T - A_i^T C_i^T F_i^T;$$

$$(1,5) = \bar{h}A_i^T P_i - \bar{h}C_i^T W_i^T;$$

$$(2,2) = -Q_k - Z_i - Z_i^T;$$

$$(2,3) = -A_{hi}^T C_i^T F_i^T;$$

$$(3,3) = -2F_i C_i B_i + G;$$

then the following diagnosis algorithm

$$\dot{\hat{f}}(t) = -\Gamma F_i(\dot{r}(t) + r(t)) \quad (14)$$

can realize

$$\lim_{t \rightarrow \infty} e(t) = 0; \quad \lim_{t \rightarrow \infty} \hat{f}(t) = f(t)$$

where $W_i = P_i L_i$, $*$ denotes the symmetric elements in a symmetric matrix, and $\Gamma = \Gamma^T > 0$ is a given weighting matrix.

Proof. Suppose the conditions in (12) and (13) hold. Construct the following switched Lyapunov function:

$$V = V_1 + V_2 + V_3 + V_4 \quad (15)$$

where

$$\begin{cases} V_1 = e^T(t)P_{\sigma(t)}e(t) \\ V_2 = \int_{-h}^0 \int_{t+l}^t \dot{e}^T(s)P_{\sigma(s)}\dot{e}(s)dsdl \\ V_3 = \int_{t-h}^t e^T(s)Q_{\sigma(s)}e(s)ds \\ V_4 = \tilde{f}^T(t)\Gamma^{-1}\tilde{f}(t) \end{cases} \quad (16)$$

Then, by the Newton-Leibniz formula,

$$e(t-h) = e(t) - \int_{t-h}^t \dot{e}(s)ds \quad (17)$$

we have

$$\begin{aligned} \dot{V}_1 &= 2e^T(t)P_{\sigma(t)}(\bar{A}_{\sigma(t)}e(t) + A_{h\sigma(t)}e(t-h) + B_{\sigma(t)}\tilde{f}(t)) \\ &= 2e^T(t)P_{\sigma(t)}(\bar{A}_{\sigma(t)} + A_{h\sigma(t)})e(t) - 2e^T(t)P_{\sigma(t)}A_{h\sigma(t)} \int_{t-h}^t \dot{e}(s)ds + 2e^T(t)P_{\sigma(t)}B_{\sigma(t)}\tilde{f}(t) \\ &= 2e^T(t)P_{\sigma(t)}(\bar{A}_{\sigma(t)} + A_{h\sigma(t)})e(t) + 2e^T(t)(Y_{\sigma(t)} - P_{\sigma(t)}A_{h\sigma(t)}) \int_{t-h}^t \dot{e}(s)ds \\ &\quad + 2e^T(t-h)Z_{\sigma(t)} \int_{t-h}^t \dot{e}(s)ds + 2e^T(t)P_{\sigma(t)}B_{\sigma(t)}\tilde{f}(t) \\ &\quad - \left[2e^T(t)Y_{\sigma(t)} \int_{t-h}^t \dot{e}(s)ds + 2e^T(t-h)Z_{\sigma(t)} \int_{t-h}^t \dot{e}(s)ds \right] \\ &= \frac{1}{h} \int_{t-h}^h \left[2e^T(t)(P_{\sigma(t)}\bar{A}_{\sigma(t)} + Y_{\sigma(t)})e(t) + 2e^T(t)(P_{\sigma(t)}A_{h\sigma(t)} - Y_{\sigma(t)} + Z_{\sigma(t)}^T)e(t-h) \right. \\ &\quad \left. - 2e^T(t-h)Z_{\sigma(t)}e(t-h) - 2e^T(t)hY_{\sigma(t)}\dot{e}(s) \right. \\ &\quad \left. - 2e^T(t-h)hZ_{\sigma(t)}\dot{e}(s) + 2e^T(t)P_{\sigma(t)}B_{\sigma(t)}\tilde{f}(t) \right] ds \end{aligned} \quad (18)$$

$$\begin{aligned} \dot{V}_2 &= \int_{-h}^0 [\dot{e}^T(t)P_{\sigma(t)}\dot{e}(t) - \dot{e}^T(t+l)P_{\sigma(t+l)}\dot{e}(t+l)] dl \\ &= \int_{t-h}^t [\dot{e}^T(t)P_{\sigma(t)}\dot{e}(t) - \dot{e}^T(s)P_{\sigma(s)}\dot{e}(s)] ds \\ &= \int_{t-h}^t \left[(\bar{A}_{\sigma(t)}e(t) + A_{h\sigma(t)}e(t-h) + B_{\sigma(t)}\tilde{f}(t))^T P_{\sigma(t)}(\bar{A}_{\sigma(t)}e(t) \right. \\ &\quad \left. + A_{h\sigma(t)}e(t-h) + B_{\sigma(t)}\tilde{f}(t)) - \dot{e}^T(s)P_{\sigma(s)}\dot{e}(s) \right] ds \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{h} \int_{t-h}^t \left[e^T(t) h \bar{A}_{\sigma(t)}^T P_{\sigma(t)} \bar{A}_{\sigma(t)} e(t) + 2e^T(t) h \bar{A}_{\sigma(t)}^T P_{\sigma(t)} A_{h\sigma(t)} e(t-h) \right. \\
&\quad + 2e^T(t) h \bar{A}_{\sigma(t)}^T P_{\sigma(t)} B_{\sigma(t)} \tilde{f}(t) + e^T(t-h) h A_{h\sigma(t)}^T P_{\sigma(t)} A_{h\sigma(t)} e(t-h) \\
&\quad + 2e^T(t-h) h A_{h\sigma(t)}^T P_{\sigma(t)} B_{\sigma(t)} \tilde{f}(t) + \tilde{f}^T(t) h B_{\sigma(t)}^T P_{\sigma(t)} B_{\sigma(t)} \tilde{f}(t) \\
&\quad \left. - \dot{e}^T(s) h P_{\sigma(s)} \dot{e}(s) \right] ds \tag{19}
\end{aligned}$$

$$\begin{aligned}
\dot{V}_3 &= e^T(t) Q_{\sigma(t)} e(t) - e^T(t-h) Q_{\sigma(t-h)} e(t-h) \\
&= \frac{1}{h} \int_{t-h}^t \left[e^T(t) Q_{\sigma(t)} e(t) - e^T(t-h) Q_{\sigma(t-h)} e(t-h) \right] ds \tag{20}
\end{aligned}$$

$$\begin{aligned}
\dot{V}_4 &= 2\tilde{f}^T(t) \Gamma^{-1} \dot{\tilde{f}}(t) \\
&= -2\tilde{f}^T(t) F_{\sigma(t)} \dot{r}(t) - 2\tilde{f}^T(t) F_{\sigma(t)} r(t) - 2\tilde{f}^T(t) \Gamma^{-1} \dot{f}(t) \\
&= -2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} \bar{A}_{\sigma(t)} e(t) - 2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} A_{h\sigma(t)} e(t-h) \\
&\quad - 2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} B_{\sigma(t)} \tilde{f}(t) - 2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} e(t) - 2\tilde{f}^T(t) \Gamma^{-1} \dot{f}(t) \tag{21}
\end{aligned}$$

By Lemma 2, one can get that

$$\begin{aligned}
-2\tilde{f}^T(t) \Gamma^{-1} \dot{f}(t) &\leq \tilde{f}^T(t) G \tilde{f}(t) + \dot{f}^T(t) \Gamma^{-1} G^{-1} \Gamma^{-1} \dot{f}(t) \\
&\leq \tilde{f}^T(t) G \tilde{f}(t) + f_2^2 \lambda_{max}(\Gamma^{-1} G^{-1} \Gamma^{-1}) \tag{22}
\end{aligned}$$

By (22), (21) is transformed into

$$\begin{aligned}
\dot{V}_4 &\leq -2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} \bar{A}_{\sigma(t)} e(t) - 2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} A_{h\sigma(t)} e(t-h) \\
&\quad - 2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} B_{\sigma(t)} \tilde{f}(t) - 2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} e(t) + \tilde{f}^T(t) G \tilde{f}(t) \\
&\quad + f_2^2 \lambda_{max}(\Gamma^{-1} G^{-1} \Gamma^{-1}) \\
&= \frac{1}{h} \int_{t-h}^t \left[-2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} \bar{A}_{\sigma(t)} e(t) - 2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} A_{h\sigma(t)} e(t-h) \right. \\
&\quad \left. - 2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} B_{\sigma(t)} \tilde{f}(t) - 2\tilde{f}^T(t) F_{\sigma(t)} C_{\sigma(t)} e(t) + \tilde{f}^T(t) G \tilde{f}(t) \right] ds \\
&\quad + f_2^2 \lambda_{max}(\Gamma^{-1} G^{-1} \Gamma^{-1}) \tag{23}
\end{aligned}$$

Utilizing (18)-(20), (23), and for the particular case $\sigma(t) = i$, $\sigma(s) = j$, and $\sigma(t-h) = k$, one can get

$$\dot{V} \leq \frac{1}{h} \int_{t-h}^t \xi^T(t) \Xi_{ijk} \xi(t) ds + \beta \tag{24}$$

where

$$\begin{aligned}\xi(t) &= [e^T(t) \quad e^T(t-h) \quad \tilde{f}^T(t) \quad e^T(s)]^T, \\ \beta &= f_2^2 \lambda_{max}(\Gamma^{-1}G^{-1}\Gamma^{-1}), \\ \Xi_{ijk} &= \begin{bmatrix} \Xi_{ijk11} & \Xi_{ijk12} & \Xi_{ijk13} & \Xi_{ijk14} \\ * & \Xi_{ijk22} & \Xi_{ijk23} & \Xi_{ijk24} \\ * & * & \Xi_{ijk33} & 0 \\ * & * & * & \Xi_{ijk44} \end{bmatrix} \\ \Xi_{ijk11} &= P_i \bar{A}_i + \bar{A}_i^T P_i + Y_i + Y_i^T + h \bar{A}_i^T P_i \bar{A}_i + Q_i; \\ \Xi_{ijk12} &= P_i A_{hi} - Y_i + Z_i^T + h \bar{A}_i^T P_i A_{hi}; \\ \Xi_{ijk13} &= P_i B_i + h \bar{A}_i^T P_i B_i - \bar{A}_i^T C_i^T F_i^T - C_i^T F_i^T; \\ \Xi_{ijk14} &= -h Y_i; \\ \Xi_{ijk22} &= -Q_k - Z_i - Z_i^T + h A_{hi}^T P_i A_{hi}; \\ \Xi_{ijk23} &= h A_{hi}^T P_i B_i - A_{hi}^T C_i^T F_i^T; \\ \Xi_{ijk24} &= -h Z_i; \\ \Xi_{ijk33} &= h B_i^T P_i B_i - 2 F_i C_i B_i + G; \\ \Xi_{ijk44} &= -h P_j\end{aligned}$$

Set $W_i = P_i L_i$ and use Schur complement lemma, one can get that matrix Ξ_{ijk} is equivalent to the following formula

$$\left. \begin{aligned} \bar{\Xi}_{ijk} + \begin{bmatrix} 0 \\ 0 \\ F_i C_i \\ 0 \\ 0 \end{bmatrix} P_i^{-1} [0 \quad 0 \quad W_i C_i \quad 0 \quad 0] + \begin{bmatrix} C_i^T W_i^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} P_i^{-1} [0 \quad 0 \quad C_i^T F_i^T \quad 0 \quad 0] \\ \bar{\Xi}_{ijk} = \begin{bmatrix} (1,1) & (1,2) & (1,3) & -h Y_i & h A_{hi}^T P_i - h C_i^T W_i^T \\ * & (2,2) & (2,3) & -h Z_i & h A_{hi}^T P_i \\ * & * & (3,3) & 0 & h B_i^T P_i \\ * & * & * & -h P_j & 0 \\ * & * & * & * & -h P_i \end{bmatrix} \end{aligned} \right\} \quad (25)$$

Now, applying Schur complement equivalence to (13) gives

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & 0 & C_i^T W_i^T \\ * & (2,2) & (2,3) & 0 & 0 \\ * & * & (3,3) & F_i C_i & 0 \\ * & * & * & -\eta I & 0 \\ * & * & * & * & -\eta I \end{bmatrix}$$

$$+\bar{h} \begin{bmatrix} -Y_i & \bar{A}_i^T P_i \\ -Z_i & A_{hi}^T P_i \\ 0 & B_i^T P_i \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -P_j^{-1} & 0 \\ 0 & -P_i^{-1} \end{bmatrix} \begin{bmatrix} -Y_i^T & -Z_i^T & 0 & 0 & 0 \\ P_i \bar{A}_i & P_i A_{hi} & P_i B_i & 0 & 0 \end{bmatrix} < 0 \quad (26)$$

for all $0 < h \leq \bar{h}$, we have

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & 0 & C_i^T W_i^T \\ * & (2,2) & (2,3) & 0 & 0 \\ * & * & (3,3) & F_i C_i & 0 \\ * & * & * & -\eta I & 0 \\ * & * & * & * & -\eta I \end{bmatrix} + h \begin{bmatrix} -Y_i & \bar{A}_i^T P_i \\ -Z_i & A_{hi}^T P_i \\ 0 & B_i^T P_i \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_j^{-1} & 0 \\ 0 & P_i^{-1} \end{bmatrix} \begin{bmatrix} -Y_i^T & -Z_i^T & 0 & 0 & 0 \\ P_i \bar{A}_i & P_i A_{hi} & P_i B_i & 0 & 0 \end{bmatrix} \leq \begin{bmatrix} (1,1) & (1,2) & (1,3) & 0 & C_i^T W_i^T \\ * & (2,2) & (2,3) & 0 & 0 \\ * & * & (3,3) & 0 & 0 \\ * & * & * & -\eta I & 0 \\ * & * & * & * & -\eta I \end{bmatrix} + \bar{h} \begin{bmatrix} -Y_i & \bar{A}_i^T P_i \\ -Z_i & A_{hi}^T P_i \\ 0 & B_i^T P_i \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_j^{-1} & 0 \\ 0 & P_i^{-1} \end{bmatrix} \begin{bmatrix} -Y_i^T & -Z_i^T & 0 & 0 & 0 \\ P_i \bar{A}_i & P_i A_{hi} & P_i B_i & 0 & 0 \end{bmatrix} < 0 \quad (27)$$

Set $\eta = \min\{\varepsilon, \varepsilon^{-1}\}$ and use Lemma 1, if the matrix P_i satisfies $P^{-1}P^{-1} \leq I$, one can see that (27) implies (25) < 0 , which means $\dot{V}(t) < -\alpha\|\vartheta(t)\|^2 + \beta$, where $\alpha = \lambda_{\min}(\Xi_{ijk})$. It follows that $\dot{V}(t) < 0$ for $\alpha\|\vartheta(t)\|^2 > \beta$, which means that $e(t)$ and $\tilde{f}(t)$ converges to a small set according to Lyapunov theory. Therefore, estimation errors of the fault and the state are uniformly bounded. Then the proof is concluded.

Remark 2 From the adaptive fault estimation algorithm, one can get that it contains the derivative of $r(t)$ and $\dot{r}(t)$. It is feasible when $\dot{r}(t)$ can be obtained. But if the signal $\dot{r}(t)$ can not be easily obtained from certain systems, we should resort to other alternative methods. In order to deal with this problem, $\dot{r}_f(t)$ is introduced to be a substitute for $\dot{r}(t)$ [9]. The relationship is defined as follows:

$$\dot{r}_f(t) = -\frac{1}{\varepsilon}(r_f(t) - r(t)) \quad (28)$$

From (28), one can get that under zero initial condition, using Laplace transform yields

$$\dot{r}_f(t) = \frac{1}{\varepsilon s + 1} \dot{r}(t) \quad (29)$$

Therefore, it is easy to show that the substitute $\dot{r}_f(t)$ can approximate to $\dot{r}(t)$ with any desired accuracy as $\varepsilon \rightarrow 0$. Meanwhile, when $s \rightarrow 0$, that is $t \rightarrow \infty$, $\dot{r}_f(t)$ asymptotically converges to $\dot{r}(t)$.

Remark 3 By utilizing the novel switched Lyapunov function formation in (15), the derivational result in Theorem 1 is delay-dependent. If we employ the following Lyapunov function formation: $V = e^T(t)P_{\sigma(t)}e(t) + \int_{t-h}^t e^T(s)Q_{\sigma(s)}e(s)ds + \tilde{f}^T(t)\Gamma^{-1}\tilde{f}(t)$, the obtained result will be delay-independent. Comparing the two cases, one can see that the result in Theorem 1 is less conservativeness.

Remark 4 In the proving process, we set $\check{f}(t) = \dot{\tilde{f}}(t) - \dot{f}(t)$. If the fault is a constant, which means that $\dot{f}(t) = 0$. Under this case, the fault estimate algorithm will be changed into $\dot{\check{f}}(t) = -\Gamma F_i \dot{r}(t)$, which is only adaptive to estimate constant faults. From the above discussion, one can get that the estimate algorithm in (14) is not only adaptive to estimate constant faults, but also adaptive to estimate time-varying faults.

Remark 5 It can be seen that the condition (12) is not a strict LMI formation due to the equation $P_i X_i = I$, which can not be solved directly by Matlab linear matrix inequality Control Toolbox. However, we can solve this nonconvex feasibility problem by formulating it into a special sequential optimization problem subject to LMI constraints. In the following, a specific algorithm is given by utilizing the result in [28].

Now using a cone complementarity approach [28], we present the following algorithm to solve the problem formulated in Theorem 1. $\min tr(\sum_{i=1}^N (P_i X_i))$ subject to (12) and (13). According to [28], if the solution of the above minimization problem is $2n$, that is, $\min tr(P_i X_i) = 2n$, then the conditions in Theorem 2 are solvable. We can modify Algorithm 1 in [28] to solve the above problem formulated in Theorem 1.

Algorithm 1:

Step 1: Find a feasible set $\{P_i^{(0)}, X_i^{(0)}, Q_i^{(0)}, F_i^{(0)}, Y_i^{(0)}, Z_i^{(0)}, W_i^{(0)}, G^{(0)}, \eta^{(0)}, \}$ satisfying (12) and (13). Set $k = 0$.

Step 2: Solve the following LMI problem

$$\min tr \left(\sum_{i=1}^N \left(P_i X_i^{(k)} + P_i^{(k)} X_i \right) \right)$$

subject to (12) and (13).

Step 3: Substitute the obtained matrix variables $\{P_i, X_i, Q_i, F_i, Y_i, Z_i, W_i, G, \eta\}$ into (12) and (13). If the condition (12) is satisfied with

$$|\text{tr}(\sum_{i=1}^N P_i X_i) - (N+1)n| < \delta$$

for some sufficient small scalar $\delta > 0$, then output the feasible solution $\{P_i, X_i, Q_i, F_i, Y_i, Z_i, W_i, G, \eta\}$, **EXIT**.

Step 4: If $k > N$ where N is the maximum number of iterations allowed, **EXIT**.

Step 5: Set $k = k + 1$, $\{P_i^{(k)}, X_i^{(k)}, Q_i^{(k)}, F_i^{(k)}, Y_i^{(k)}, Z_i^{(k)}, W_i^{(k)}, G^{(k)}, \eta^{(k)}\} = \{P_i, X_i, Q_i, F_i, Y_i, Z_i, W_i, G, \eta\}$, and go to *Step 2*.

4 Fault Accommodation

Since the state $x(t)$ is unavailable, the estimation value $\hat{x}(t)$ is substituted for $x(t)$. Therefore, the observer-based normal controller is given

$$u_r(t) = -K_{\sigma(t)}\hat{x}(t) + d(t) \quad (30)$$

where $K_{\sigma(t)}$ is the feedback gain matrix and $d(t)$ is the reference input.

Once a fault occurs, based on the accurate and rapid estimation of the fault, the following observer-based fault-tolerant controller is activated to compensate for the fault

$$u(t) = u_r(t) - \hat{f}(t) \quad (31)$$

Assuming $d(t) = 0$ and substituting (31) into (1), one obtains

$$\begin{cases} \dot{x}(t) = (A_{\sigma(t)} - B_{\sigma(t)}K_{\sigma(t)})x(t) + A_{h\sigma(t)}x(t-h) + \rho(t) \\ y(t) = C_{\sigma(t)}x(t) \end{cases} \quad (32)$$

where $\rho(t) = -B_{\sigma(t)}K_{\sigma(t)}e(t) - B_{\sigma(t)}\tilde{f}(t)$.

From the result of Theorem 1, one can get that $e(t) \rightarrow 0$ and $\tilde{f}(t) \rightarrow 0$ when $t \rightarrow \infty$. The signal $\rho(t)$ can be treated as a disturbance of the system (32). So, if only the feedback gain K_i can ensure that the following system is asymptotically stable.

$$\begin{cases} \dot{x}(t) = (A_{\sigma(t)} - B_{\sigma(t)}K_{\sigma(t)})x(t) + A_{h\sigma(t)}x(t-h) \\ y(t) = C_{\sigma(t)}x(t) \end{cases} \quad (33)$$

Theorem 2 *The system (33) is asymptotically stable for any time delay h satisfying $0 \leq h \leq \bar{h}$ if there exist positive definite matrices P_i, Q_i , matrix Y_i , for all $i, j, k \in \psi$, such that*

$$\begin{bmatrix} A_i X_i + X_i A_i^T - B_i Y_i - Y_i^T B_i^T & A_{hi} R_j & X_i \\ * & -R_j & 0 \\ * & * & -R_i \end{bmatrix} < 0 \quad (34)$$

where $Y_i = K_i X_i$.

Proof. Let the Lyapunov function be

$$V_5 = x^T(t) P_{\sigma(t)} x(t) + \int_{t-h}^t x^T(s) Q_{\sigma(s)} x(s) ds \quad (35)$$

Then the derivative of V_5 along the trajectories of the system in (33) is

$$\begin{aligned} \dot{V}_5 &= 2x^T(t) P_{\sigma(t)} (A_{\sigma(t)} - B_{\sigma(t)} K_{\sigma(t)}) x(t) + 2x^T(t) P_{\sigma(t)} A_{h\sigma(t)} x(t-h) \\ &\quad + x^T(t) Q_{\sigma(t)} x(t) - x^T(t-h) Q_{\sigma(t-h)} x(t-h) \end{aligned} \quad (36)$$

Under the particular case $\sigma(t) = i$ and $\sigma(t-h) = j$, and let $P_i = X_i^{-1}$, $Q_i = R_i^{-1}$, one can get that the LMI in (34) means that $\dot{V}_5 < 0$. Therefore, the system (33) is asymptotically stable according to standard Lyapunov stability theory.

Remark 6 *This paper considers the active fault-tolerant case for switched systems with constant delay. Combined the existing time-varying delays results for switched system, the result obtained in this work may be extended to time-varying case, this issue will be one of our future study work.*

5 An illustrative example

Consider the switched system \mathcal{S} consisting of two subsystems described by

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.54 & 1.02 \\ 0.17 & -0.31 \end{bmatrix}, A_2 = \begin{bmatrix} -0.01 & 0.1 \\ 0.01 & 0.04 \end{bmatrix}, A_{h1} = \begin{bmatrix} 0.18 & 0.36 \\ -0.06 & -0.12 \end{bmatrix}, A_{h2} = \begin{bmatrix} 0.11 & 0.18 \\ -0.03 & -0.04 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, C_1 = [0.1 \ 0.2], C_2 = [0.2 \ 0.3], \bar{h} = 6. \end{aligned}$$

By utilizing Algorithm 1 to solve the conditions in Theory 1, we can get a set of solutions as follows:

$$\begin{aligned} P_1 &= \begin{bmatrix} 0.0139 & 0.0460 \\ 0.0460 & 0.1538 \end{bmatrix}, P_2 = \begin{bmatrix} 0.0018 & 0.0015 \\ 0.0015 & 0.0030 \end{bmatrix}, X_1 = 10^3 \times \begin{bmatrix} 8.0608 & -2.4113 \\ -2.4113 & 0.7278 \end{bmatrix}, \\ X_2 &= \begin{bmatrix} 929.2677 & -465.9440 \\ -465.9440 & 562.6869 \end{bmatrix}, Q_1 = 10^{-3} \times \begin{bmatrix} -0.0259 & -0.0660 \\ -0.0660 & -0.1293 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
Q_2 &= 10^{-4} \times \begin{bmatrix} 0.6589 & 0.7236 \\ 0.7236 & 0.7870 \end{bmatrix}, \quad Y_1 = 10^{-3} \times \begin{bmatrix} -0.0926 & -0.1936 \\ -0.1936 & -0.4894 \end{bmatrix}, \\
Y_2 &= 10^{-3} \times \begin{bmatrix} -0.1237 & -0.2492 \\ -0.2488 & -0.5011 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.0064 \\ 0.0271 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.0028 \\ 0.0029 \end{bmatrix}, \\
Z_1 &= 10^{-3} \times \begin{bmatrix} 0.0926 & 0.1936 \\ 0.1936 & 0.4894 \end{bmatrix}, \quad Z_2 = 10^{-3} \times \begin{bmatrix} 0.1251 & 0.2506 \\ 0.2505 & 0.5036 \end{bmatrix}, \\
F_1 &= 0.0830, \quad F_2 = 0.0239, \quad G = 8.9436, \quad \eta = 0.0721.
\end{aligned}$$

Taking the learning law $\Gamma = 200$ and the sampling period $T = 0.01$, the time delay is chosen as $h = 3$, and the control input $u(t)$ is a unit step function. In this example, two cases of faults are considered. When the fault is a constant described as

$$f_1(t) = \begin{cases} 0, & 0 \leq t \leq 5 \\ 0.2(t - 5), & 5 < t \leq 15 \\ 2, & 15 < t \leq 30 \end{cases}$$

In this case, the simulation result is shown in Figure 2. When the fault is a time-varying function described as

$$f_2(t) = \begin{cases} 0, & 0 \leq t \leq 5 \\ 0.3 \sin 2t + 0.5, & 5 < t < 30 \end{cases}$$

The simulation result is shown in Figure 3. From the above simulation results, we can conclude that whether the fault is a constant or a time-varying function, the estimate algorithm proposed here can estimate them quickly and exactly.

By solving the LMI in Theorem 2, one obtains

$$\begin{aligned}
X_1 &= \begin{bmatrix} 177.1841 & -136.2999 \\ -136.2999 & 99.7892 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 80.4373 & -69.2977 \\ -69.2977 & 57.5229 \end{bmatrix}, \\
R_1 &= \begin{bmatrix} 564.7326 & -274.1576 \\ -274.1576 & 301.1453 \end{bmatrix}, \quad R_2 = 10^3 \times \begin{bmatrix} 1.5693 & -1.1476 \\ -1.1476 & 1.0134 \end{bmatrix}, \\
K_1 &= \begin{bmatrix} -338.6052 & -446.6806 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -452.3254 & -530.7370 \end{bmatrix}.
\end{aligned}$$

Take the learning law $\Gamma = 2000$ and the sampling period $T = 0.01$, the time delay is assumed as $h = 3$, and the initial condition is selected as $x(0) = [0.3 \quad -0.2]^T$. If there is no fault, the state response of the closed-loop system is given in Figure 4. If a fault occurs and is supposed as follows:

$$f_3(t) = \begin{cases} 0, & 0 \leq t \leq 20 \\ 6, & 20 < t \leq 100 \end{cases}$$

the state response of the closed-loop system is given in Figure 5. It can be seen from the figure that the closed-loop system is asymptotically stable.

6 Conclusion

In this paper, the problem of active fault tolerant control against actuators failure in switched system with time delay has been addressed. Firstly, an adaptive fault estimation algorithm is proposed, which can exactly and fast estimate the fault. Based on the fault estimation information, observer-based state feedback fault tolerant controller is designed such that the closed-loop system is asymptotically stable. An example is given to illustrate the effectiveness of the proposed method.

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References

- [1] B. Jiang, M. Staroswiecki, and V. Cocquempot, "Fault accommodation for a class of nonlinear dynamic systems," *IEEE Trans. Automat. Contr.*, vol. 51, no. 9, pp. 1578-1583, 2006.
- [2] B. Jiang, S. Marcel, and C. Vincent, "Fault identification for a class of time-delay Systems," *Proceedings of the American Control Conference Anchorage, AK* May 8-10,2002.
- [3] Y. Wang, D. H. Zhou, and F. Gao, "Robust fault-tolerant control of a class of non-minimum phase nonlinear processes," *Journal of Process Control*, vol. 17, no. 6, pp. 523-537, July 2007.
- [4] M. Mahmoud, "Sufficient conditions for the stabilization of feedback delayed discrete time fault tolerant control systems," *Int. J. Innovative Computing, Information and Control*, vol.5, no.5, pp.1137-1146, 2009.
- [5] X. Tang, G. Tao, S. M. Joshi, "Adaptive output feedback actuator failure compensation for a class of non-linear systems," *Int. J. Adaptive Control and Signal Processing*, vol. 19, no. 6, pp. 419-444, 2005.

- [6] P. Shi, E. K. Boukas, S. K. Nguang, and X. Guo, "Robust disturbance attenuation for discrete-time active fault tolerant control systems with uncertainties," *Optimal Control Applications and Methods*, vol.24, pp. 85-101, 2003.
- [7] D. U. Campos-Delgado and K. Zhou, "Reconfigurable fault-tolerant control using GIMC structure," *IEEE Trans. Automat. Contr.*, vol. 48, no. 5, pp. 832-838, 2003.
- [8] S. K. Nguang and P. Shi, "Delay dependent fault estimation for uncertain time delay nonlinear systems: an LMI approach," *Int. J. Robust and Nonlinear Control*, vol. 16, no. 18, pp. 913-933, 2006.
- [9] M. Mahmoud, "An H_∞ stabilizing controller for discrete time fault tolerant control systems with state delays," *ICIC Express Letters*, vol.3, no.1, pp.1-6, 2009.
- [10] M. Mahmoud, "Stabilizing controllers for a class of discrete time fault tolerant control systems," *ICIC Express Letters*, vol.2, no.3, pp.213-218, 2008.
- [11] N. E. Wu, Y. Zhang, K. Zhou, "Detection, estimation, and accommodation of loss of control effectiveness," *Int. J. Adaptive Control and Signal Processing*, vol. 14, no. 7, pp. 775C795, 2000.
- [12] H. Yang, B. Jiang, and V. Cocquempot, "Observer-based fault tolerant control for constraint switched systems," *Int. J. Control, Automation and Systems*, vol. 5, no. 6, pp. 707-711, 2007.
- [13] R. J. Patton, P. Frank, and R. Clark, *Fault Diagnosis in Dynamic Systems: Theory and Application*. Englewood Cliffs, NJ: Prentice-Hall, 1989.
- [14] J. Chen and R. J. Patton, *Robust Model-Based Fault Diagnosis for Dynamic Systems*. Boston, MA: Kluwer, 1999.
- [15] M. Mahmoud, J. Jiang, and Y. Zhang, "Stabilization of active fault tolerant control systems with imperfect fault detection and diagnosis," *Stochastic Analysis and Applications*, vol. 21, no. 3, pp. 673-701, 2003.
- [16] N. E. Wu, "Robust feedback design with optimized diagnostic performance," *IEEE Trans. Automat. Contr.*, vol. 42, no. 9, pp. 1264-1268, 1997.
- [17] M. Maki, J. Jiang, and K. Hagino, "A stability guaranteed active fault-tolerant control system against actuator failures," *Int. J. Robust and Nonlinear Control*, vol. 14, no. 12, pp. 1061-1077, 2004.

- [18] X. Hong, H. Xu, and H. Sun, "Fault-tolerant control of discrete-time switched systems", *IEEE International Conference on Control Automation*, pp. 3082-3086, 2007.
- [19] R. Wang, G. Jin, and J. Zhao, "Robust fault-tolerant control for a class of switched nonlinear systems in lower triangular," *Asian Journal of Control*, vol. 9, no. 1, pp. 68-72, 2007.
- [20] M. Rodrigues, D. Theilliol, and D. Sauter, "Fault-tolerant control design for switched systems," *IFAC Conference on Analysis and Design of Hybrid Systems*, pp. 223-228, 2006.
- [21] L. Zhang and P. Shi, "Stability, l_2 -gain and asynchronous H-infinity control of discrete-time switched systems with average dwell time," *IEEE Trans. on Automatic Control*, vol. 54, no. 9, pp.2193-2200, 2009.
- [22] D. Du, B. Jiang, and S. Zhou, "Delay-dependent robust stabilization of uncertain discrete-time switched systems with time-varying state delays," *Int. J. Systems Science*, vol. 39, no. 3, pp. 305-313, 2008.
- [23] L. Zhang and P. Shi, " $L_2 - L_\infty$ model reduction for switched LPV systems with average dwell time," *IEEE Trans on Automatic Control*, vol. 53, no. 10, pp. 2443-2448, 2008.
- [24] L. Zhang, P. Shi, El K. Boukas, and C. Wang, " H_∞ model reduction for switched linear discrete-time systems with polytopic uncertainties," *Automatica*, vol. 44, no. 11, pp. 2944-2949, 2008.
- [25] A. Benzaouia, O. Benmessaouda, and Y. Shi, "Output feedback stabilization of uncertain saturated discrete-time switching systems," *Int. J. Innovative Computing, Information and Control*, vol.5, no.6, pp.1735-1746, 2009.
- [26] I. R. Pertersen, "A stabilization algorithm for a class of uncertain linear systems," *Systems and Control Letters*, vol. 8, pp. 351-357, 1987.
- [27] Y. Niu, J. Lam, X. Wang, and D. W. C. Ho, "Observer-based sliding mode control for nonlinear state-delayed systems," *Int. J. Systems Science*, vol. 35, no. 2, pp. 139-150, February 2004.
- [28] L. El Ghaoui, F. Oustry, and M. Ait Rami, "A cone complementarity linearization algorithm for static output-feedback and related problems," *IEEE Trans. Autom. Contr.*, vol. 42, no. 8, pp. 1171-1176, 1997.
- [29] D. K. Kim, P. G. Park, and J. W. Ko, "Output-feedback H_∞ control of systems over communication networks using a deterministic switching system approach," *Automatica*, vol. 40, pp. 1205-1212, 2004.

- [30] C. Meyer, S. Schroder and R. W. De Doncker, "Solid-state circuits breakers and current limiters for medium-voltage systems having distributed power systems," *IEEE Trans. Power Electronics*, vol. 19, pp. 1333-1340, 2004.
- [31] X. Sun, J. Zhao, and D. J. Hill, "Stability and L_2 -gain analysis for switched delay systems: A delay-dependent method," *Automatica*, vol. 42, pp. 1769-1774, 2006.
- [32] D. Wang, W. Wang and P. Shi, "Robust fault detection for switched linear systems with state delays," *IEEE Trans. on Systems, Man and Cybernetics, Part B: Cybernetics*, vol. 39, no. 3, pp. 800-805, 2009.
- [33] D. Du, B. Jiang, P. Shi, and S. Zhou, " H_∞ filtering of discrete-time switched systems with state delays via switched Lyapunov function approach," *IEEE Trans. Autom. Control*, vol. 52, no. 8, pp. 1520-1525, 2007.
- [34] H. Yang, B. Jiang, and V. Cocquempot, "A fault tolerant control framework for periodic switched nonlinear systems," *Int. J. Contr.*, vol. 82, no. 1, pp. 117-129, 2009.
- [35] P. Y. Richard, H. Cormerais and J. Buisso, "A generic design methodology for sliding mode control of switched systems," *Nonlinear Analysis*, vol. 65, pp. 1751-177, 2006.
- [36] J. Daafouz, P. Riedinger, and C. Iung, "Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach," *IEEE Trans. Automat. Control*, vol. 47, no. 11, pp. 1883-1887, 2002.

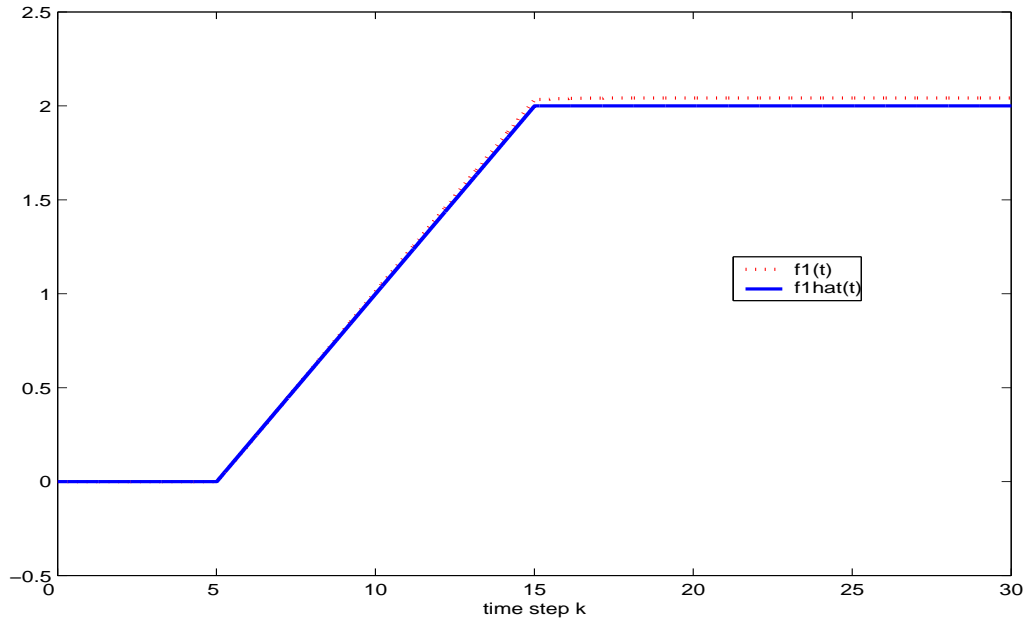


Figure 2. Fault $f_1(t)$ (solid line) and its estimate $\hat{f}_1(t)$ (dotted line)

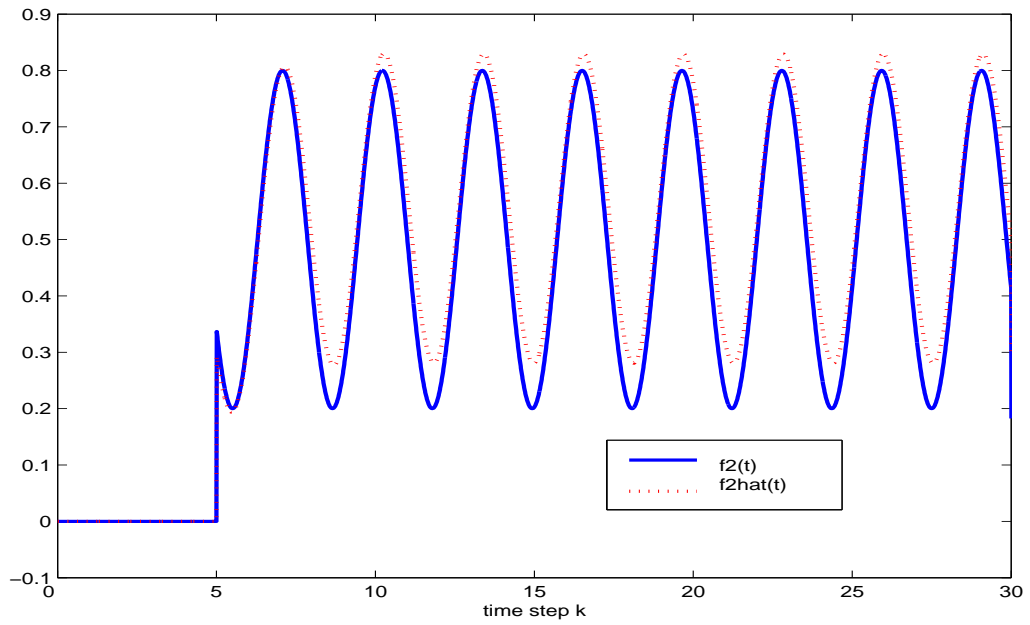


Figure 3. Fault $f_2(t)$ (solid line) and its estimate $\hat{f}_2(t)$ (dotted line)

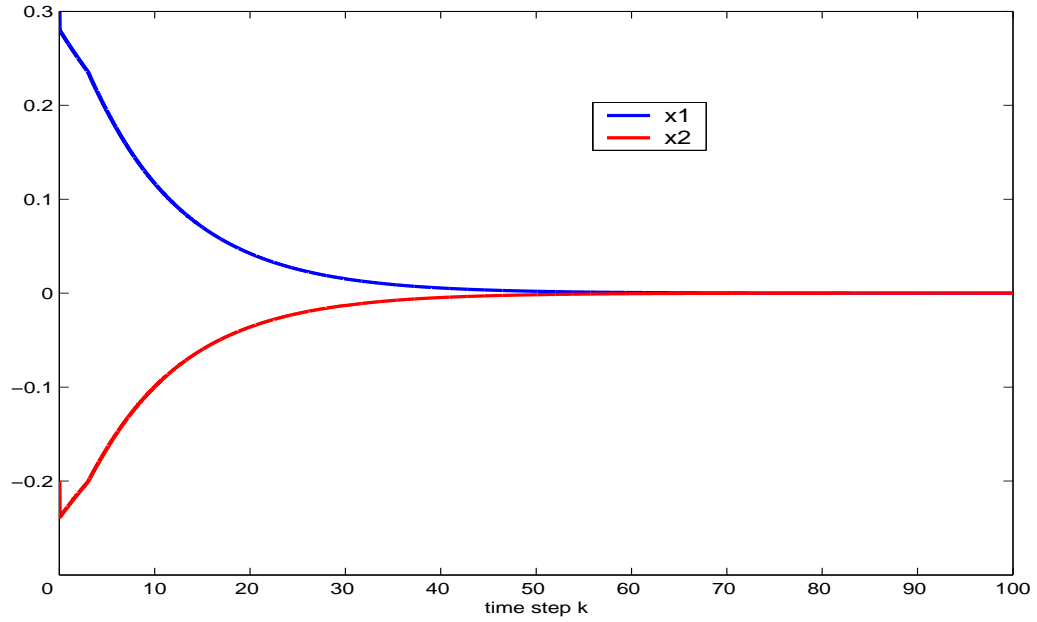


Figure 4. Time response of the state viable $x_1(t)$ and $x_2(t)$ with no fault

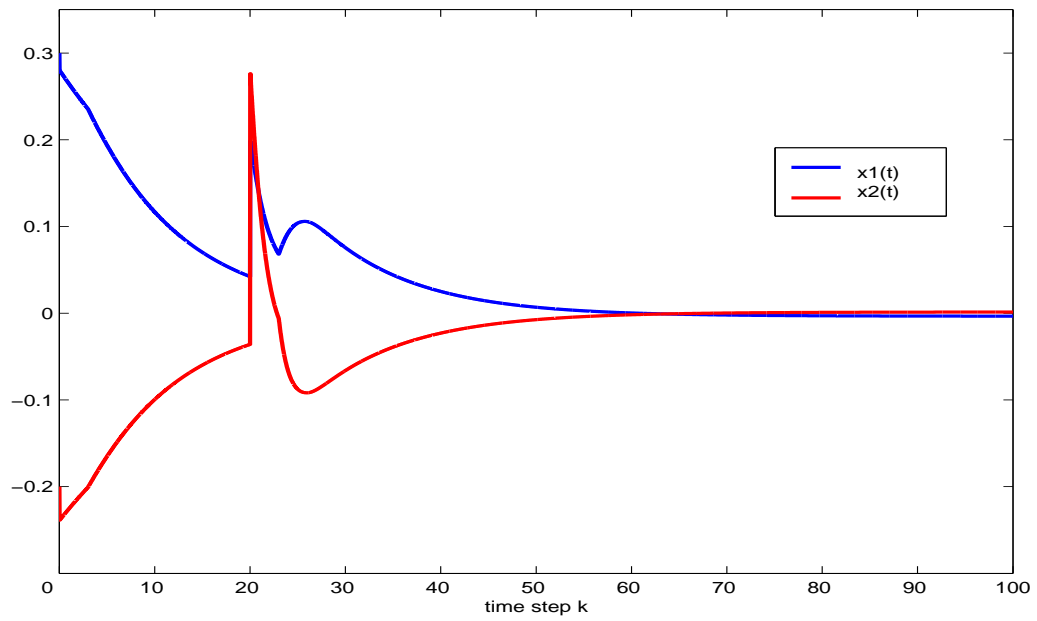


Figure 5. Time response of the state viable $x_1(t)$ and $x_2(t)$ with fault $f_3(t)$