Fault estimation and accommodation for switched systems with time-varying delay

Fault Estimation and Accommodation for Switched Systems with Time-varying Delay

Dongsheng Du, Bin Jiang, and Peng Shi

Abstract: This paper considers the problem of fault estimation and accommodation for a class of switched systems with time-varying delay. An adaptive fault estimation algorithm is proposed to estimate the fault, moreover, constant or time-varying fault can be estimated. Meanwhile, a delay-dependent criteria is obtained with the purpose of reducing the conservatism of the fault estimation algorithm design. On the basis of fault estimation, an observer-based fault tolerant controller is designed to guarantee the stability of the closed-loop system. Additionally, simulation results are presented to illustrate the efficiency of the proposed results.

Keywords: Accommodation, fault estimation, switched systems, time-varying delay.

1. INTRODUCTION

In order to increase the reliability and safety of dynamics systems, fault diagnosis (FD) and fault tolerant control (FTC) have been intensively investigated in recent years. Generally speaking, FTC includes two main approaches: passive and active. In passive FTC systems, which use feedback control laws that are robust with respect to some fixed faults [1, 2]. However, it may be failed for other accidental faults. Under this case, we must resort to active FTC technique, which relies on a fault detection and isolation (FDI) process to identify the fault-induced changes [3]. The control law is reconfigured online in response to the FDI decisions and hence has better fault-tolerance capability. It can be seen that FDI process is the first step in active FTC to monitor the system and determine the location of the fault. Then, fault estimation is utilized to on-line generate magnitude of the fault. Therefore, FD is a very meaningful and challenging topic, which has attracted many researches to devote themselves into this study domain. During the past decades, various effective methods, such as sliding mode observer approach using equivalent output injection signal [4], adaptive technique [5, 6], and learning method based on neural network [7, 8] and so on, have been developed for the fault estimation problem. Among them, adaptive fault diagnosis observer has been proved to be an effective approach. The advantage of the adaptive fault diagnosis observer is that the state vector estimation and actuator fault estimation can be obtained simultaneously.

On the other hand, relatively few FD work was done for switched system compared with the plentiful FD achievements for general dynamic systems. FD for switched systems was considered in [9, 10], passive and active FTC for nonlinear switched systems with actuator faults were separately investigated in [11] and [12]. Sensor fault case for switched systems was studied in [13]. Switched system belongs to hybrid system, which consists a finite number of subsystems and an associated switching signal governing the switching among them. Many physical or man-made systems can be modeled as switched systems, including chemical processes, computer controlled systems, switched circuits and so on. During the past three decades, lots of stability theoretic results have been reported for switched systems [14]- [16]. On the other hand, time delay is the inherent features of many physical process and the big sources of instability and poor performances. Switched systems with time delay have strong engineering background, such as in network control systems [17] and power systems [18]. Existing stability criteria of delay systems can be classified into two types: delay-dependent and delay-independent methods. In recent years, much attention has been drawn to the development of delay-dependent conditions aimed at reducing the conservatism, and many theoretical studies have been conducted for switched systems with time de-

This work was supported in part by the National Natural Science Foundation of P.R. China (Nos. 60874051, 60804011, 60904001), Funding for Outstanding Doctoral Dissertation in Nanjing University of Aeronautics and Astronautics (BCXJ10-03), and the Engineering and Physical Sciences Research Council, UK (EP/F029195).

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lay [19]- [20]. However, to the best of our knowledge, the problem of fault estimation and accommodation for switched systems with time-varying delay has not been considered yet, which motivates us to investigate this meaningful issue.

In this work, based on adaptive fault diagnosis observer method and average dwell-time technique, the problem of fault estimation and accommodation for switched systems with time-varying delay is solved. A novel adaptive estimate law is proposed by solving some constrained linear matrix inequalities. An efficient algorithm is developed to solve these constrained LMIs and simulation results prove the effectiveness of the proposed method. The contributions of this work can be summarized as the following two aspects:

1:) An adaptive estimate law is developed for switched system with time-varying delay. By utilizing piecewise Lyapunov function and average dwell-time technique, a novel adaptive fault estimation algorithm is designed. Moreover, the obtained result is delay-dependent, which makes further efforts to degrade the conservativeness. On the other hand, many fault estimate law is confined to estimate constant fault. The advantage of such proposed estimation algorithm can not only estimate the fault fast and exactly, but also is adaptive to estimate two type of fault: constant fault case and time-varying fault case.

2:) To review the development of FD for switched systems, the fault estimation and accommodation for switched system with time-varying delay is not considered yet. Additionally, the time-delay is time-varying and constrained in an interval set. This condition is more practical than constant time delay case. This paper investigates this issue and efficient results are given. Another point should be pointed out that average dwell-time technique is utilized to stabilize the switched system, which is more general and flexible than dwell time switching and arbitrary switching signal.

The paper is organized as follows. Section 2 gives the model description. Section 3 presents the fault estimation algorithm, and an observer-based fault tolerant controller is designed. An example is illustrated in Section 4 to show the usefulness and applicability of the proposed approaches, and the paper is concluded in Section 5.

2. MODEL DESCRIPTION

The switched system is described as follows :

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t)+A_{d\sigma(t)}x(t-d(t)) \\ &\quad +B_{\sigma(t)}(u(t)+f(t)) \qquad (1) \\ y(t) &= C_{\sigma(t)}x(t) \qquad (2) \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^m$ is the output vector, $u(t) \in \mathbb{R}^p$ is the control input, $f(t) \in \mathbb{R}^l$ is the actuator fault. $\sigma(t) : [0,+\infty) \to \psi = \{1,\cdots,N\}$ is the switching signal, $N > 1$ is the number of subsystems. At an arbitrary continuous time t, $\sigma(t)$, denoted by σ for simplicity, is dependent on t or $x(t)$, or both, or other switching rules. $A_{\sigma(t)}$, $A_{d\sigma(t)}$, $B_{\sigma(t)}$, and $C_{\sigma(t)}$ are constant matrices with appropriate dimensions for all $\sigma(t) \in \psi$. When the i-subsystem is activated, we denote the matrices associated with $\sigma(t) = i$ by $A_{\sigma(t)} = A_i$, $A_{d\sigma(t)} = A_{di}$, $B_{\sigma(t)} = B_i$, and $C_{\sigma(t)} = C_i$. $d(t)$ is the time delay and assumed to satisfy the following condition:

C1: $d(t)$ is differentiable and bounded with a constant delay-derivative bound:

$$h_1 \le d(t) \le h_2, \quad \dot{d}(t) \le d < 1 \qquad (3)$$

f(t) can be thought of as an additional signal expressed as $f(t) = \nu(t-t_0)f_0(t)$, the function $\nu(t-t_0)$ is given by

$$\nu(t-t_0) = \begin{cases} 0, & t \le t_0 \\ 1, & t > t_0 \end{cases} \qquad (4)$$

where t_0 is the time of fault occurring. That is, $f(t)$ is zero prior to the failure time $t \le t_0$ and is $f_0(t)$ after the failure occurs $t > t_0$. It is assumed that the derivation $f_0(t)$ with respect to time is norm bounded, i.e. $\|f_0(t)\| \le f_1$, $\|\dot{f}_0(t)\| \le f_2$, where $0 \le f_1 < \infty$, $0 \le f_2 < \infty$.

The following lemmas are added for the convenes of later proof.

Lemma 1: ([21]) Given matrices $\mathscr{W}$, $\mathscr{X}$ and $\mathscr{Y}$ of appropriate dimensions and with $\mathscr{W}$ symmetrical, then

$$\mathscr{W} + \mathscr{X}F(t)\mathscr{Y} + \mathscr{Y}^T F^T(t)\mathscr{X}^T < 0$$

holds for all $F(t)$ satisfying $F^T(t)F(t) \le I$ if and only if for some $\varepsilon > 0$,

$$\mathscr{W} + \varepsilon\mathscr{X}\mathscr{X}^T + \varepsilon^{-1}\mathscr{Y}^T\mathscr{Y} < 0$$

Lemma 2: ([22]) For any real vectors a, b and matrix $G > 0$ of compatible dimensions, the following inequality holds:

$$a^T b + b^T a \le a^T G a + b^T G^{-1} b, \quad a,b \in R^n$$

Lemma 3: ([23]) Consider the switched system $\dot{x}(t) = f_\sigma(x(t))$, $\sigma \in \psi$ and let $\alpha > 0$, $\mu > 1$ be given constants. Suppose that there exist functions $V_{\sigma(t)} : \mathbb{R}^n \to \mathbb{R}$, and two class functions β_1 and β_2 such that $\beta_1(|x(t)|) \le V_i(x(t)) \le \beta_2(|x(t)|)$, $\dot{V}_i(x(t)) \le -\alpha V_i(x(t)), \forall i \in \psi$, and $\dot{V}_i(x(t)) \le \mu V_j(x(t)), \forall (i,j) \in \psi \times \psi$ then the system is globally uniformly asymptotically stable for any switching signal with average dwell time

$$\tau_a > \tau_a^* = \ln\mu/\alpha$$

3. MAIN RESULTS

3.1. Fault diagnosis observer design

To diagnose the actuator fault, the following fault diagnosis observer is designed:

$$\dot{\hat{x}}(t) = A_{\sigma(t)}\hat{x}(t) + A_{d\sigma(t)}\hat{x}(t-d(t)) + B_{\sigma(t)}(u(t)+\hat{f}(t)) - L_{\sigma(t)}(\hat{y}(t)-y(t)) \quad (5)$$
$$\hat{y}(t) = C_{\sigma(t)}\hat{x}(t) \quad (6)$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state vector of the observer, $\hat{y}(t) \in \mathbb{R}^m$ is the output vector of the observer, $\hat{f}(t)$ is an estimate of $f(t)$, $L_{\sigma(t)}$ is the observer gain.

Denote

$$e(t) = \hat{x}(t) - x(t); \quad r(t) = \hat{y}(t) - y(t); \quad \tilde{f}(t) = \hat{f}(t) - f(t); \quad (7)$$

then the error dynamics is described by

$$\dot{e}(t) = \overline{A}_{\sigma(t)}e(t) + A_{d\sigma(t)}e(t-d(t)) + B_{\sigma(t)}\tilde{f}(t) \quad (8)$$
$$r(t) = C_{\sigma(t)}e(t) \quad (9)$$

where $\overline{A}_{\sigma(t)} = A_{\sigma(t)} - L_{\sigma(t)}C_{\sigma(t)}$. As a result, the purpose of fault estimation is to find a diagnostic algorithm for $\hat{f}(t)$ such that

$$\lim_{t\to\infty} e(t) = 0; \quad \lim_{t\to\infty} \hat{f}(t) = f(t) \quad (10)$$

3.2. Fault estimation algorithm design

In the sequel, a convergent adaptive fault diagnostic algorithm to estimate the fault $f(t)$ is given, which is obtained from the residual $r(t)$. Consequently, $\dot{f}(t) \neq 0$ implies that the residual $r(t)$ with respect to time is

$$\dot{r}(t) = \dot{\hat{f}}(t) - \dot{f}(t) \quad (11)$$

Theorem 1: For the time delay $d(t)$ satisfying the condition C1 and a given scalar $\alpha > 0$, if there exist positive definite matrices P_i, X_i, R_i, Y_i, U_i, V_i, Q_{i1}, Q_{i2}, Q_{i3}, Z_{i1}, Z_{i2}, G, matrices W_i, F_i, N_{i1}, N_{i2}, M_{i1}, M_{i2}, S_{i1}, S_{i2}, and a scalar $\tau > 0$, such that

$$\begin{bmatrix} P_i & I \\ I & X_i \end{bmatrix} \geq 0, \; P_iX_i = I, \quad i = 1,2,\cdots,N. \quad (12)$$

$$\begin{bmatrix} R_i & X_i \\ X_i & V_i \end{bmatrix} \geq 0, Y_iR_i = I, U_iV_i = I, \; i = 1,2,\cdots,N. \quad (13)$$

$$\Psi_i = \begin{bmatrix} \Pi_i & \Gamma_i \\ \Gamma_i^T & \Omega_i \end{bmatrix} < 0 \quad (14)$$

where

$$U_i = h_2Z_{i1} + \bar{h}Z_{i2}, \; \bar{h} = h_2 - h_1,$$
$$\Omega_i = diag\{-Y_i, -h_2e^{-\alpha h_2}Z_{i1}, -h_2e^{-\alpha h_2}Z_{i2}, -\bar{h}e^{-\alpha h_2}(Z_{i1}+Z_{i2}), -\tau I, -\tau I\},$$

$$\Pi_i = \begin{bmatrix} \varphi_{11} & \varphi_{12} & S_{i1} & -M_{i1} & \varphi_{15} \\ * & \varphi_{22} & S_{i2} & -M_{i2} & A_{di}^TC_i^TF_i^T \\ * & * & -e^{-\alpha h_1}Q_{i1} & 0 & 0 \\ * & * & * & -e^{-\alpha h_2}Q_{i2} & 0 \\ * & * & * & * & -2F_iC_iB_i + G \end{bmatrix},$$

$$\Gamma_i = \begin{bmatrix} A_i^TP_i - \tilde{h}C_i^TW_i^T & h_2N_{i1} & \bar{h}S_{i1} & \bar{h}M_{i1} & C_i^TW_i^T & 0 \\ A_{di}^TP_i & h_2N_{i2} & \bar{h}S_{i2} & \bar{h}M_{i2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & F_iC_i \end{bmatrix},$$

$$\varphi_{11} = P_iA_i + A_i^TP_i - W_iC_i - C_i^TW_i^T + \alpha P_i + N_{i1} + N_{i1}^T + \sum_{j=1}^{3} Q_i,$$
$$\varphi_{12} = P_iA_{di} - N_{i1} + M_{i1} - S_{i1} + N_{i2}^T,$$
$$\varphi_{15} = P_iB_i - C_i^TF_i^T - A_i^TC_i^TF_i^T,$$
$$\varphi_{22} = (d-1)e^{-\alpha h_2}Q_{i3} - N_{i2} - N_{i2}^T + M_{i2} + M_{i2}^T - S_{i2} - S_{i2}^T.$$

then, for the switching signal $\sigma(t)$ with average dwell time satisfying

$$\tau_a \geq \tau_a^* = \frac{\ln\mu}{\alpha} \quad (15)$$

where

$$\mu \geq 1 \; with \; P_i \leq \mu P_j, Q_{i1} \leq \mu Q_{j1}, Q_{i2} \leq \mu Q_{j2}, Q_{i3} \leq \mu Q_{j3}, Z_{i1} \leq \mu Z_{j1}, Z_{i2} \leq \mu Z_{j2}. \quad (16)$$

the following diagnosis algorithm

$$\dot{\hat{f}}(t) = -\Gamma F_i(\dot{r}(t) + r(t)) \quad (17)$$

can realize

$$\lim_{t\to\infty} e(t) = 0; \quad \lim_{t\to\infty} \hat{f}(t) = f(t)$$

where $W_i = P_iL_i$, $*$ denotes the symmetric elements in a symmetric matrix, and $\Gamma = \Gamma^T > 0$ is a given weighting matrix.

Proof. Construct the following piecewise Lyapunov function candidate as

$$V_{\sigma(t)}(t) = e^T(t)P_{\sigma(t)}e(t) + \int_{t-d(t)}^{t} e^T(s)Q_{\sigma(t)3}e^{\alpha(s-t)}e(s)ds$$

$$
\begin{aligned}
&+\sum_{j=1}^{2}\int_{t-h_j}^{t} e^T(s)Q_{\sigma(t)j}e^{\alpha(s-t)}e(s)ds \\
&+\int_{-h_2}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)Z_{\sigma(t)1}e^{\alpha(s-t)}\dot{e}(s)dsd\theta \\
&+\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{e}^T(s)Z_{\sigma(t)2}e^{\alpha(s-t)}\dot{e}(s)dsd\theta \\
&+\tilde{f}^T(t)\Gamma^{-1}\tilde{f}(t) \qquad (18)
\end{aligned}
$$

Set $\sigma(t) = i$ and taking the derivative of (18), we obtain

$$
\begin{aligned}
\dot{V}_i(t) \leq\ & 2e^T(t)P_i\dot{e}(t) + e^T(t)Q_{i3}e(t) - (1-d)\times \\
& e^T(t-d(t))Q_{i3}e^{-\alpha h_2}e(t-d(t)) \\
& +\sum_{j=1}^{2}[e^T(t)Q_{ij}e(t) - e^T(t-h_j)Q_{ij}\times \\
& e^{-\alpha h_j}e(t-h_j)] + \dot{e}^T(t)(h_2Z_{i1}+\bar{h}Z_{i2})\dot{e}(t) \\
& -\int_{t-h_2}^{t}\dot{e}^T(s)Z_{i1}e^{-\alpha h_2}\dot{e}(s)ds \\
& -\int_{t-h_2}^{t-h_1}\dot{e}^T(s)Z_{i2}e^{-\alpha h_2}\dot{e}(s)ds \\
& +\alpha e^T(t)P_ie(t) - \alpha V_i(t) - 2\tilde{f}^T(t)F_iC_i\dot{e}(t) \\
& -2\tilde{f}^T(t)F_iC_ie(t) - 2\tilde{f}^T(t)\Gamma^{-1}\dot{f}(t) \qquad (19)
\end{aligned}
$$

From the Newton-Leibniz formula, the following equations are true for any matrices N_{i1}, N_{i2}, M_{i1}, M_{i2}, S_{i1}, and S_{i2} with appropriate dimensions.

$$
\begin{aligned}
&2[e^T(t)N_{i1} + e^T(t-d(t))N_{i2}]\times \\
&\left[e(t) - e(t-d(t)) - \int_{t-d(t)}^{t}\dot{e}(s)ds\right] = 0 \qquad (20) \\
&2[e^T(t)M_{i1} + e^T(t-d(t))M_{i2}]\,[e(t-d(t)) \\
&-e(t-h_2) - \int_{t-h_2}^{t-d(t)}\dot{e}(s)ds\Big] = 0 \qquad (21) \\
&2[e^T(t)S_{i1} + e^T(t-d(t))S_{i2}]\,[e(t-h_1) \\
&-e(t-d(t)) - \int_{t-d(t)}^{t-h_1}\dot{e}(s)ds\Big] = 0 \qquad (22)
\end{aligned}
$$

Moreover, the following equalities hold:

$$
\begin{aligned}
&\int_{t-h_2}^{t}\dot{e}^T(s)Z_{i1}e^{-\alpha h_2}\dot{e}(s)ds \\
=\ &\int_{t-h_2}^{t-d(t)}\dot{e}^T(s)Z_{i1}e^{-\alpha h_2}\dot{e}(s)ds \\
&+\int_{t-d(t)}^{t}\dot{e}^T(s)Z_{i1}e^{-\alpha h_2}\dot{e}(s)ds \\
&\int_{t-h_2}^{t-h_1}\dot{e}^T(s)Z_{i1}e^{-\alpha h_2}\dot{e}(s)ds \\
=\ &\int_{t-h_2}^{t-d(t)}\dot{e}^T(s)Z_{i2}e^{-\alpha h_2}\dot{e}(s)ds \\
&+\int_{t-d(t)}^{t-h_1}\dot{e}^T(s)Z_{i2}e^{-\alpha h_2}\dot{e}(s)ds \qquad (23)
\end{aligned}
$$

Thus, from (20) and (23), we can obtain that

$$
\begin{aligned}
&-\int_{t-d(t)}^{t}\dot{e}^T(s)Z_{i1}e^{-\alpha h_2}\dot{e}(s)ds - 2\xi^T(t)N_i\times \\
&\int_{t-d(t)}^{t}\dot{e}(s)ds \leq -\int_{t-d(t)}^{t}[\xi^T(t)N_i + \dot{e}(s)Z_{i1}e^{-\alpha h_2}] \\
&\times(Z_{i1}e^{-\alpha h_2})^{-1}[\xi^T(t)N_i + \dot{e}(s)Z_{i1}e^{-\alpha h_2}]ds \\
&+h_2\xi^T(t)N_i(Z_{i1}e^{-\alpha h_2})^{-1}N_i\xi(t) \qquad (24)
\end{aligned}
$$

where

$$
\xi(t) = \begin{bmatrix} e(t) \\ e(t-h) \\ e(t-h_1) \\ e(t-h_2) \\ \tilde{f}(t) \end{bmatrix}, \quad N_i = \begin{bmatrix} N_{i1} \\ N_{i2} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (25)
$$

By using the same methods as (24), from (21)-(23), we obtain

$$
\begin{aligned}
&-\int_{t-h_2}^{t-d(t)}\dot{e}^T(s)Z_{i1}e^{-\alpha h_2}\dot{e}(s)ds \\
&-\int_{t-h_2}^{t-d(t)}\dot{e}^T(s)Z_{i2}e^{-\alpha h_2}\dot{e}(s)ds \\
&-2\xi^T(t)M_i\int_{t-h_2}^{t-d(t)}\dot{e}(s)ds \\
\leq\ &-\int_{t-h_2}^{t-d(t)}[\xi^T(t)M_i + \dot{e}(s)(Z_{i1}+Z_{i2})e^{-\alpha h_2}][(Z_{i1} \\
&+Z_{i2})e^{-\alpha h_2}]^{-1}[\xi^T(t)M_i + \dot{e}(s)(Z_{i1}+Z_{i2})e^{-\alpha h_2}]ds \\
&+\bar{h}\xi^T(t)M_i[(Z_{i1}+Z_{i2})e^{-\alpha h_2}]^{-1}M_i\xi(t) \qquad (26)
\end{aligned}
$$

and

$$
\begin{aligned}
&-\int_{t-d(t)}^{t-h_1}\dot{e}^T(s)Z_{i2}e^{-\alpha h_2}\dot{e}(s)ds \\
&-2\xi^T(t)S_i\int_{t-d(t)}^{t-h_1}\dot{e}(s)ds \\
\leq\ &-\int_{t-d(t)}^{t}[\xi^T(t)S_i + \dot{e}(s)Z_{i2}e^{-\alpha h_2}](Z_{i2}e^{-\alpha h_2})^{-1} \\
&\times[\xi^T(t)S_i + \dot{e}(s)Z_{i2}e^{-\alpha h_2}]ds \\
&+\bar{h}\xi^T(t)S_i(Z_{i2}e^{-\alpha h_2})^{-1}S_i\xi(t) \qquad (27)
\end{aligned}
$$

where

$$
\begin{aligned}
M_i &= \begin{bmatrix} M_{i1}^T & M_{i2}^T & 0 & 0 & 0 \end{bmatrix}^T, \\
S_i &= \begin{bmatrix} S_{i1}^T & S_{i2}^T & 0 & 0 & 0 \end{bmatrix}^T. \qquad (28)
\end{aligned}
$$

By Lemma 2, one can get that

$$
\begin{aligned}
-2\tilde{f}^T(t)\Gamma^{-1}\dot{f}(t) \leq\ & \tilde{f}^T(t)G\tilde{f}(t) \\
+\ & f_2^2\lambda_{max}(\Gamma^{-1}G^{-1}\Gamma^{-1}) \quad (29)
\end{aligned}
$$

From (19), (24), (26), (27), and (29), one can get that

$$\begin{aligned}&\dot{V}_i(t)+\alpha V_i(t)\\ \leq\ &\xi^T(t)\{\Xi_i+\widetilde{A}_i^T(h_2Z_{i1}+\bar{h}Z_{i2})\widetilde{A}_i\\ &+h_2N_i(Z_{i1}e^{-\alpha h_2})^{-1}N_i^T\\ &+\bar{h}M_i[(Z_{i1}+Z_{i2})e^{-\alpha h_2}]^{-1}M_i^T\\ &+\bar{h}S_i(Z_{i2}e^{-\alpha h_2})^{-1}S_i^T\}\xi(t)+\delta \qquad (30)\end{aligned}$$

where

$$\Xi_i = \begin{bmatrix} \varphi_{11} & \varphi_{12} & S_{i1} & -M_{i1} & \bar{\varphi}_{15} \\ * & \varphi_{22} & S_{i2} & -M_{i2} & A_{di}^TC_i^TF_i^T \\ * & * & -e^{-\alpha h_1}Q_{i1} & 0 & 0 \\ * & * & * & -e^{-\alpha h_2}Q_{i2} & 0 \\ * & * & * & * & -2F_iC_iB_i+G \end{bmatrix},$$

$$\begin{aligned}\widetilde{A} &= \begin{bmatrix} \bar{A}_i & A_{di} & B_i & 0 & 0 \end{bmatrix},\\ \bar{\varphi}_{15} &= P_iB_i-C_i^TF_i^T-\bar{A}_i^TC_i^TF_i^T,\\ \delta &= f_2^2\lambda_{max}(\Gamma^{-1}G^{-1}\Gamma^{-1}).\end{aligned}$$

Set $W_i = P_iL_i$ and use Schur complement lemma, one can get that matrix Ξ_i is equivalent to the following formula

$$\Pi_i+\begin{bmatrix}0\\0\\0\\0\\F_iC_i\end{bmatrix}P_i^{-1}\begin{bmatrix}W_iC_i & 0 & 0 & 0 & 0\end{bmatrix} + \begin{bmatrix}C_i^TW_i^T\\0\\0\\0\\0\end{bmatrix}P_i^{-1}\begin{bmatrix}0 & 0 & 0 & 0 & C_i^TF_i^T\end{bmatrix} \qquad (31)$$

Set $\tau = \min\{\varepsilon, \varepsilon^{-1}\}$ and use Lemma 1, if the matrix P_i satisfies (12), one can see that (14) implies (31) < 0.

By using (13), (14), and (30), one obtains that

$$\dot{V}_i(t)+\alpha V_i(t) \leq -\varepsilon\|\xi(t)\|^2+\delta \qquad (32)$$

where ε is the minimum eigenvalue of $-\Psi_i$. It follows that

$$\dot{V}_i(t)+\alpha V_i(t) < 0 \ \ for\ \ \varepsilon\|\xi(t)\|^2 > \delta \qquad (33)$$

witch means that $\xi(t)$ converges to a small set according to Lyapunov stability theory.

Integrating the inequalities (33) gives that $V_i(t) \leq e^{-\alpha(t-t_k)}V_i(t_k)$ for any given $t\in[t_k,t_{k+1})$. From (16) and (18), at the switching instant t_k, it follows that

$$\dot{V}_{\sigma(t_k)}(t_k) \leq \mu V_{\sigma(t_k^-)}(t_k^-),\ \ k=0,1,2,\cdots \qquad (34)$$

Thus, using (34) and $N_\sigma(t_0,t) \leq \frac{t-t_0}{\tau_a}$, one can get

$$\begin{aligned}V_i(t) &\leq e^{-\alpha(t-t_k)}\mu V_{\sigma(t_k^-)}(t_k^-) \leq \cdots\\ &\leq e^{-\alpha(t-t_0)}\mu^{N_\sigma(t_0,t)}V_{\sigma(t_0)}(t_0)\\ &= e^{-\alpha(t-t_0)}e^{N_\sigma(t_0,t)\ln\mu}V_{\sigma(t_0)}(t_0)\\ &\leq e^{-\alpha(t-t_0)}e^{\frac{t-t_0}{\tau_a}\ln\mu}V_{\sigma(t_0)}(t_0)\\ &= e^{-(\alpha-\frac{\ln\mu}{\tau_a})(t-t_0)}V_{\sigma(t_0)}(t_0)\\ &= e^{-\lambda(t-t_0)}V_{\sigma(t_0)}(t_0) \qquad (35)\end{aligned}$$

where $\lambda = \frac{1}{2}(\alpha-\frac{\ln\mu}{\tau_a})$.

Set

$$\begin{aligned}a &= \min\lambda_{min}(P_i),\\ b &= \max\lambda_{max}(P_i)+h_1\max\lambda_{max}(Q_{i1})\\ &\quad+h_2\max\lambda_{max}(Q_{i2})+h_2\max\lambda_{max}(Q_{i3})\\ &\quad+\frac{h_2^2}{2}\max\lambda_{max}(Z_{i1})+\frac{h_2^2-h_1^2}{2}\max\lambda_{max}(Z_{i2}) \qquad (36)\end{aligned}$$

It follows (35) and (36) that

$$a^2\|e(t)\|^2 \leq b^2e^{-2\lambda(t-t_0)}\|e_{t_0}\|_c^2 \qquad (37)$$

That is

$$\|e(t)\| \leq \frac{b}{a}e^{-\lambda(t-t_0)}\|e_{t_0}\|_c \qquad (38)$$

witch means that the state error $e(t)\to 0$ and the fault error $\tilde{f}(t)\to 0$. Then the proof is completed.

Remark 1: From the adaptive fault estimation algorithm (17), one can get that it contains the derivative of $r(t)$ and $\dot{r}(t)$. It is feasible when $\dot{r}(t)$ can be obtained. But if the signal $\dot{r}(t)$ can not be easily obtained from certain systems, we should resort to other alternative methods. In order to deal with this problem, $\dot{r}_f(t)$ is introduced to be a substitute for $\dot{r}(t)$ *[24]*. The relationship is defined as follows:

$$\dot{r}_f(t) = -\frac{1}{\varepsilon}(r_f(t)-r(t)) \qquad (39)$$

From (39), one can get that under zero initial condition, using Laplace transform yields

$$\dot{r}_f(t) = \frac{1}{\varepsilon s+1}\dot{r}(t) \qquad (40)$$

Therefore, it it easy to show that the substitute $\dot{r}_f(t)$ can approximate to $\dot{r}(t)$ with any desired accuracy as $\varepsilon\to 0$. Meanwhile, when $s\to 0$, that is $t\to\infty$, $\dot{r}_f(t)$ asymptotically converges to $\dot{r}(t)$.

Remark 2: In the proving process, we set $\dot{\tilde{f}}(t) = \dot{\hat{f}}(t)-\dot{f}(t)$. If the fault is a constant, which means that $\dot{f}(t)=0$. Under this case, the fault estimate algorithm will be changed into $\dot{\hat{f}}(t) = -\Gamma F_i \dot{r}(t)$, which is only adaptive to estimate constant faults. From the above discussion, one can get that the estimate algorithm in (17) is not only adaptive to estimate constant faults, but also adaptive to estimate time-varying faults.

Remark 3: It can be seen that the conditions (12) and (13) are not strict LMI formation due to the equation $P_iX_i = I$, $R_iY_i = I$, and $U_iV_i = I$, which can not be solved directly by Matlab linear matrix inequality Control Toolbox. However, we can solve this nonconvex feasibility problem by formulating it into a special sequential optimization problem subject to LMI constraints. In the following, a specific algorithm is given by utilizing the result in [25].

Algorithm 1:

Step 1: Find a feasible set $\{P_i^{(0)}, X_i^{(0)}, R_i^{(0)}, Y_i^{(0)}, U_i^{(0)}, V_i^{(0)}, Q_{i1}^{(0)}, Q_{i2}^{(0)}, Q_{i3}^{(0)}, Z_{i1}^{(0)}, Z_{i2}^{(0)}, W_i^{(0)}, F_i^{(0)}, G^{(0)}, N_{i1}^{(0)}, N_{i2}^{(0)}, M_{i1}^{(0)}, M_{i2}^{(0)}, S_{i1}^{(0)}, S_{i2}^{(0)}, \tau^{(0)},\}$ satisfying (12), (13) and (14). Set $k = 0$.

Step 2: Solve the following LMI problem

$$\min tr\left(\sum_{i=1}^{N}\left(P_iX_i^{(k)} + P_i^{(k)}X_i + R_iY_i^{(k)} + R_i^{(k)}Y_i + U_iV_i^{(k)} + U_i^{(k)}V_i\right)\right)$$

subject to (12), (13) and (14).

Step 3: Substitute the obtained matrix variables $\{P_i, X_i, R_i, Y_i, U_i, V_i, Q_{i1}, Q_{i2}, Q_{i3}, Z_{i1}, Z_{i2}, W_i, F_i, G, N_{i1}, N_{i2}, M_{i1}, M_{i2}, S_{i1}, S_{i2}, \tau,\}$ into (12), (13) and (14). If the condition (12) is satisfied with

$$|tr(\sum_{i=1}^{N}(P_iX_i + R_iY_i + U_iV_i)) - 3(N+1)n| < \rho$$

for some sufficient small scalar $\rho > 0$, then output the feasible solution $\{P_i, X_i, R_i, Y_i, U_i, V_i, Q_{i1}, Q_{i2}, Q_{i3}, Z_{i1}, Z_{i2}, W_i, F_i, G, N_{i1}, N_{i2}, M_{i1}, M_{i2}, S_{i1}, S_{i2}, \tau,\}$, **EXIT**.

Step 4: If $k > N$ where N is the maximum number of iterations allowed, **EXIT**.

Step 5: Set $k = k+1$, $\{P_i^{(k)}, X_i^{(k)}, R_i^{(k)}, Y_i^{(k)}, U_i^{(k)}, V_i^{(k)}, Q_{i1}^{(k)}, Q_{i2}^{(k)}, Q_{i3}^{(k)}, Z_{i1}^{(k)}, Z_{i2}^{(k)}, W_i^{(k)}, F_i^{(k)}, G^{(k)}, N_{i1}^{(k)}, N_{i2}^{(k)}, M_{i1}^{(k)}, M_{i2}^{(k)}, S_{i1}^{(k)}, S_{i2}^{(k)}, \tau^{(k)},\} = \{P_i, X_i, R_i, Y_i, U_i, V_i, Q_{i1}, Q_{i2}, Q_{i3}, Z_{i1}, Z_{i2}, W_i, F_i, G, N_{i1}, N_{i21}, M_{i1}, M_{i21}, S_{i1}, S_{i2}, \tau,\}$, and go to *Step 2*.

3.3. Fault accommodation

Since the state $x(t)$ is unavailable, the estimation value $\hat{x}(t)$ is substituted for $x(t)$. Therefore, the observer-based normal controller is given

$$u_r(t) = -K_{\sigma(t)}\hat{x}(t) + d(t) \tag{41}$$

where $K_{\sigma(t)}$ is the feedback gain matrix and $d(t)$ is the reference input.

Once a fault occurs, based on the accurate and rapid estimation of the fault , the following observer-based fault-tolerant controller is activated to compensate for the fault.

$$u(t) = u_r(t) - \hat{f}(t) \tag{42}$$

Assuming $d(t) = 0$ and substituting (42) into (1), one obtains

$$\begin{cases} \dot{x}(t) = (A_{\sigma(t)} - B_{\sigma(t)}K_{\sigma(t)})x(t) \\ \quad +A_{d\sigma(t)}x(t-d(t)) + \rho(t) \\ y(t) = C_{\sigma(t)}x(t) \end{cases} \tag{43}$$

where $\rho(t) = -B_{\sigma(t)}K_{\sigma(t)}e(t) - B_{\sigma(t)}\tilde{f}(t)$.

From the result of Theorem 1, one can get that $e(t) \to 0$ and $\tilde{f}(t) \to 0$ when $t \to \infty$. The signal $\rho(t)$ can be treated as a disturbance of the system (43). So, if only the feedback gain K_i can ensure that the following system is asymptotically stable.

$$\begin{cases} \dot{x}(t) = (A_{\sigma(t)} - B_{\sigma(t)}K_{\sigma(t)})x(t) \\ \quad +A_{d\sigma(t)}x(t-d(t)) \\ y(t) = C_{\sigma(t)}x(t) \end{cases} \tag{44}$$

Construct the corresponding piecewise Lyapunov function as

$$\begin{aligned} V_{\sigma(t)}(t) &= x^T(t)P_{\sigma(t)}x(t) \\ &\quad + \int_{t-d(t)}^{t} x^T(s)Q_{\sigma(s)}e^{\alpha(s-t)}x(s)ds \end{aligned} \tag{45}$$

where $P_{\sigma(t)}$ and $Q_{\sigma(t)}$ are positive definite matrices. Thus, we have the following theorem.

Theorem 2: For the time delay $d(t)$ satisfying the condition C1 and a given scalar $\alpha > 0$, if there exist positive definite matrices X_i, T_i, matrix J_i, such that

$$\begin{bmatrix} (1,1) & A_{hi}T_i & X_i \\ * & -(1-d)e^{-\alpha h_2}T_i & 0 \\ * & * & -T_i \end{bmatrix} < 0 \tag{46}$$

where $(1,1) = A_iX_i + X_iA_i^T - B_iJ_i - J_i^TB_i^T + \alpha X_i$, $J_i = K_iX_i$. Then, for the switching signal $\sigma(t)$ with average dwell time satisfying

$$\tau_a \geq \tau_a^* = \frac{\ln\mu}{\alpha} \tag{47}$$

where

$$\mu \geq 1 \; with \; P_i \leq \mu P_j, \; Q_i \leq \mu Q_j. \tag{48}$$

such that the system (44) is asymptotically stable.

Proof. Taking the derivative of $V_{\sigma(t)}$ along the trajectories of the system in (44) is

$$\begin{aligned} \dot{V}_i(t) &\leq 2x^T(t)P_i(A_i - B_iK_i)x(t) + x^T(t)Q_ix(t) \\ &\quad +2x^T(t)P_iA_{di}x(t-d(t)) + \alpha x^T(t)P_ix(t) - \alpha V_i \\ &\quad -(1-d)x^T(t-d(t))Q_ie^{-\alpha h_2}x(t-d(t)) \\ &= \eta^T(t)\Upsilon_i\eta(t) - \alpha V_i \end{aligned} \tag{49}$$

where

$$\eta(t) = \begin{bmatrix} x^T(t) \\ x^T(t-d(t)) \end{bmatrix}^T,$$
$$\Upsilon_i = \begin{bmatrix} (1,1) & P_iA_{di} \\ * & -(1-d)Q_ie^{-\alpha h_2} \end{bmatrix},$$
$$(1,1) = P_i(A_i - B_iK_i) + (A_i - B_iK_i)^T P_i + \alpha P_i + Q_i.$$

Let $P_i = X_i^{-1}$, $Q_i = T_i^{-1}$, one can get that the LMI in (46) means that $\dot{V}_i(t) + \alpha V_i(t) < 0$.

Integrating the inequalities gives that $V_i(t) \leq e^{-\alpha(t-t_k)}V_i(t_k)$ for any given $t \in [t_k, t_{k+1})$. From (46) and (48), at the switching instant t_k, it follows that

$$\dot{V}_{\sigma(t_k)}(t_k) \leq \mu V_{\sigma(t_k^-)}(t_k^-), \ \ k = 0,1,2,\cdots \quad (50)$$

Thus, using (50) and $N_\sigma(t_0,t) \leq \frac{t-t_0}{\tau_a}$, one can get

$$\begin{aligned} V_i(t) &\leq e^{-\alpha(t-t_k)}\mu V_{\sigma(t_k^-)}(t_k^-) \leq \cdots \\ &\leq e^{-\alpha(t-t_0)}\mu^{N_\sigma(t_0,t)}V_{\sigma(t_0)}(t_0) \\ &= e^{-\alpha(t-t_0)}e^{N_\sigma(t_0,t)\ln\mu}V_{\sigma(t_0)}(t_0) \\ &\leq e^{-\alpha(t-t_0)}e^{\frac{t-t_0}{\tau_a}\ln\mu}V_{\sigma(t_0)}(t_0) \\ &= e^{-(\alpha-\frac{\ln\mu}{\tau_a})(t-t_0)}V_{\sigma(t_0)}(t_0) \\ &= e^{-\lambda(t-t_0)}V_{\sigma(t_0)}(t_0) \end{aligned} \quad (51)$$

where $\lambda = \frac{1}{2}(\alpha - \frac{\ln\mu}{\tau_a})$.

Set

$$\begin{aligned} a &= \min \lambda_{min}(P_i), \\ b &= \max \lambda_{max}(P_i) + h_2 \max \lambda_{max}(Q_i). \end{aligned} \quad (52)$$

It follows (51) and (52) that

$$a^2\|x(t)\|^2 \leq b^2e^{-2\lambda(t-t_0)}\|x_{t_0}\|_c^2 \quad (53)$$

That is

$$\|x(t)\| \leq \frac{b}{a}e^{-\lambda(t-t_0)}\|x_{t_0}\|_c \quad (54)$$

Therefore, the system (44) is asymptotically stable according to standard Lyapunov stability theory.

4. AN ILLUSTRATIVE EXAMPLE

Consider a switched electrical circuit model borrowed from [26], which is shown in Figure 1. This circuit has two modes, that is, $N = 2, \sigma(t) : [0,\infty) \to \{1,2\}$. In mode 1, called the 'on' time, Sw1 is closed and Sw2 is open. In mode 2, called the 'off' time, Sw1 is open and Sw2 is closed. In this system, Sw1 is often a bipolar transistor and Sw2 is a diode. V_c is used to denote the capacitor voltage equal to the output volt age delivered

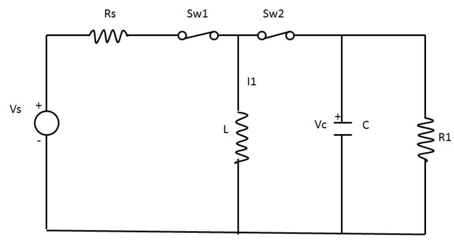

Figure 1. A switched electrical circuit

to the load R_1, and I_1 denotes the inductor current. During the 'on' time, the inductor current is also equal to the input source current. During the 'off' time, the input source current is zero. On the other hand, time delay can enter into this system due to the existence of inductor and/or the transmission channel. Thus, it is reasonable to describe this electrical circuit into a switched time-delay system as system (1). The parameter matrices are chosen as follows:

$$A_1 = \begin{bmatrix} -0.54 & 1.02 \\ 0.17 & -0.31 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} 0.18 & 0.36 \\ -0.06 & -0.12 \end{bmatrix}, C_1 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.01 & 0.1 \\ 0.01 & 0.04 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} 0.11 & 0.18 \\ -0.03 & -0.04 \end{bmatrix}, C_2 = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}.$$

Choose time-varying delay signal as $d(t) = 0.5 + 0.2\sin(t)$, we can get the upper bound of $h_1 = 0.3$, $h_2 = 0.7$, and $d = 0.2$, respectively.

For given $\alpha = 1$ and $\mu = 2.5$, we have $\tau_a^* = 0.9163$. Taking $\tau_a = 2 \geq \tau_a^*$ and the learning law $\Gamma = 200$, the sampling period is chosen as $T = 0.1$, and the control input $u(t)$ is a unit step function. In this example, two cases of faults are considered. When the fault is a constant described as

$$f_1(t) = \begin{cases} 0, & 0 \leq t \leq 5 \\ 4, & 5 < t \leq 30 \end{cases}$$

In this case, the simulation result is shown in Figure 2. When the fault is a time-varying function described as

$$f_2(t) = \begin{cases} 0, & 0 \leq t \leq 5 \\ 0.3\sin(2t) + 0.5, & 5 < t < 30 \end{cases}$$

The simulation result is shown in Figure 3. From the above simulation results, we can conclude that whether the fault is a constant or a time-varying function, the estimate algorithm proposed here can estimate them quickly and exactly.

By solving the LMI in Theorem 2, one obtains

$$K_1 = \begin{bmatrix} -38.6732 & -26.6396 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 52.3254 & -30.7370 \end{bmatrix}.$$

Take the learning law $\Gamma = 100$, the time delay is assumed as $d(t) = 0.5 + 0.2\sin(t)$, and the initial condition is selected as $x(0) = \left[\begin{array}{cc} 0.3 & -0.2 \end{array}\right]^T$. If there is no fault, the state response of the closed-loop system is given in Figure 4. If a fault occurs and is supposed as follows:

$$f_3(t) = \begin{cases} 0, & 0 \le t \le 20 \\ 6, & 20 < t \le 100 \end{cases}$$

the state response of the closed-loop system is given in Figure 5. It can be seen from the figure that the closed-loop system is asymptotically stable.

5. CONCLUSION

In this paper, the problem of fault estimation and accommodation against actuators failure in switched system with time-varying delay has been addressed. Firstly, an adaptive fault estimation algorithm is proposed, which can exactly and fast estimate the fault. Based on the fault estimation information, observer-based state feedback fault tolerant controller is designed such that the closed-loop system is asymptotically stable. An example is given to illustrate the effectiveness of the proposed method.

REFERENCES

[1] G. H. Yang and D. Ye. "Adaptive reliable H_∞ filtering against sensor failures". *IEEE Trans. Signal Processing*, vol. 55, no. 7, pp. 3161-3171, 2007.

[2] S. L. Dai and J. Zhao. "Reliable H_∞ controller design for a class of uncertain linear systems with actuator failures". *Int. J. of Control Automation and Systems*, vol. 6, no. 6, pp. 954-959, 2008.

[3] Y. Q. Wang, D. H. Zhou, S. Qin, and H. Wang. "Active fault-tolerant control for a class of nonlinear systems with sensor faults". *Int. J. of Control Automation and Systems*, vol. 6, no. 3, pp. 339-350, 2008.

[4] C. Edwards, S. K. Spurgeon, R. J. Patton. "Sliding mode observers for fault detection and isolation". *Automatica*, vol. 36, no. 4, pp. 541-553, 2000.

[5] Z. Gao, B. Jiang, P. Shi, and Y. Xu. "Fault accommodation for near space vehicle attitude dynamics via T-S fuzzy models". *Int. J. of Innovative Computing, Information and Control*, vol.6, no.11, pp. 4843-4856, 2010.

[6] D. Ye and G. H. Yang. "Adaptive fault-tolerant dynamic output feedback control for a class of linear time-delay systems". *Int. J. of Control Automation and Systems*, vol. 6, no. 2, pp. 149-159, 2008.

[7] M. M. Polycarpou. "Fault accommodation of a class of multivariable nonlinear dynamical systems using a learning approach". *IEEE Trans. Autom. Control*, vol. 46, no. 5, pp. 736-742, 2001.

[8] A. Fekih, H. Xu, F. N. Chowdhury. "Neural networks based system identification techniques for model based fault detection of nonlinear systems". *Int. J. of Innovative Computing, Information and Control*, vol. 3, no. 5, pp. 1073-1085, 2007.

[9] D. Du, B. Jiang, and Shi Peng. "Fault diagnosis for switched systems with time delay". *Proceedings of the 8th IEEE International Conference on Control and Automation*, Xiamen, China, June 9-11, pp. 2162-2165, 2010.

[10] A. L. Bako, E. Duviella, K. M. Pekpe, and S. Lecoeuche. "Fault diagnosis for switching system using observer Kalman filter identification". *Proceedings of the 17th World Congress The International Federation of Automatic Control*, Seoul, Korea, July 6-11, pp. 10142-10147, 2008.

[11] Z. Xiang, R. Wang, and Q. Chen. "Fault tolerant control of switched nonlinear systems with time dealy under asynchronous switching". *Int. J. Appl. Math. Comput. Sci.*, vol. 20, no. 3, pp. 497-506, 2010.

[12] H. Yang, B. Jiang, and C. Vincent. "Observer-based fault tolerant control for constrained switched systems". *Int. J. of Control, Automation, and Systems*, vol. 5, no. 6, pp. 707-711, December 2007.

[13] S. Shi, Z. Yuan, and Q. Zhang. "Fault-tolerant H_∞ filter design of a class of switched systems with sensor faults". *Int. J. of Innovative Computing, Information and Control*, vol. 5, no. 11, pp. 3827-3838, 2009.

[14] D. Du, B. Jiang, and S. Zhou. "Delay-dependent robust stabilization of uncertain discrete-time switched systems with time-varying state delays". *Int. J. of Systems Science*, vol. 39, no. 3, pp. 305-313, 2008.

[15] X. Lin, S. Li, Y. Zou and Y. Qian. "Generation of stable limit circles for linear systems via switching linear controllers". *ICIC Express Letters*, vol.3, no.3 (B), pp. 663-668, 2009.

[16] G. Zhai, Y. Sun, X. Chen and A. N. Michel. "Stability and L_2 gain analysis for switched symmetric systems with time delay". *Proc. 2003 American Control Conference*, Denver, June, 2003, pp.4-6.

[17] D. K. Kim, P. G. Park, and J. W. Ko. "Output-feedback H_∞ control of systems over communication networks using a deterministic switching system approach". *Automatica*, vol. 40, pp. 1205-1212, 2004.

[18] C. Meyer, S. Schroder and R. W. De Doncker. "Solid-state circuits breakers and current limiters for medium-voltage systems having distributed-power systems, *IEEE Trans. Power Electronics.* vol. 19, pp. 1333-1340, 2004.

[19] X. Sun, J. Zhao, and D. J. Hill,. "Stability and L_2-gain analysis for switched delay systems: a selay-dependent method". *Automatica*, vol. 42, pp. 1769-1774, 2006.

[20] D. Du, B. Jiang, P. Shi, and S. Zhou. "H_∞ filtering of discrete-time switched systems with state delays via switched Lyapunov function approach". *IEEE Trans. Autom. Contr.*, vol. 52, no. 8, pp. 1520-1525, 2007.

[21] I. R. Pertersen. "A stabilization algorithm for a class of uncertain linear systems". *Systems and Control Letters*, vol. 8, pp. 351-357, 1987.

[22] Y. Niu, J. Lam, X. Wang, and D. W. C. HO. "Observer-based sliding mode control for nonlinear state-delayed systems". *Int. J. of Systems Science*, vol. 35, no. 2, pp. 139-150, February 2004.

[23] J. P. Hespanha, and A. S. Morse. "Stability of switched systems with average dwell time". *In Proc. 38th Conf. Decision Control*, Phoenix, AZ, pp. 2655-2660, 1999.

[24] M. Mahmoud. "An H_∞ stabilizing controller for discrete time fault tolerant control systems with state delays". *ICIC Express Letters*, vol.3, no.1, pp.1-6, 2009.

[25] L. El Ghaoui, F. Oustry, and M. Ait Rami. "A cone complementarity linearization algorithm for static output-feedback and related problems". *IEEE Trans. Autom. Contr.*, vol. 42, no. 8, pp. 1171-1176, 1997.

[26] A. Allison, D. Abbott. "Some benefits of random variables in switched control systems". *Microelectronics Journal*, vol. 31, no. 7, pp. 515-522, 2000.

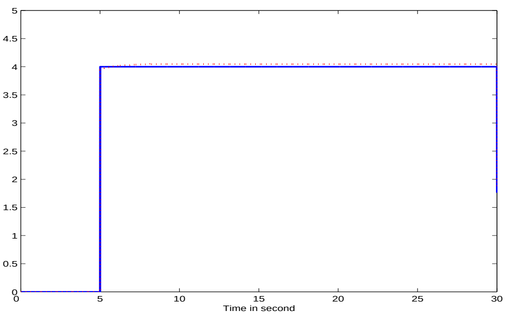

Figure 2. Fault $f_1(t)$ (dotted line) and its estimate $\hat{f}_1(t)$ (solid line)

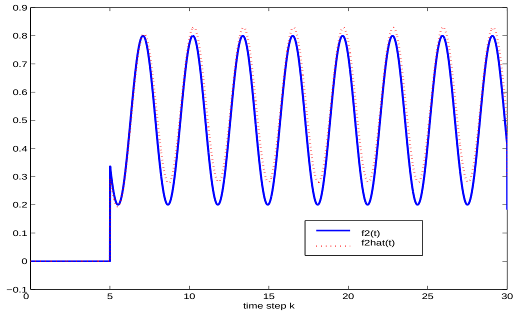

Figure 3. Fault $f_2(t)$ (dotted line) and its estimate $\hat{f}_2(t)$ (solid line)

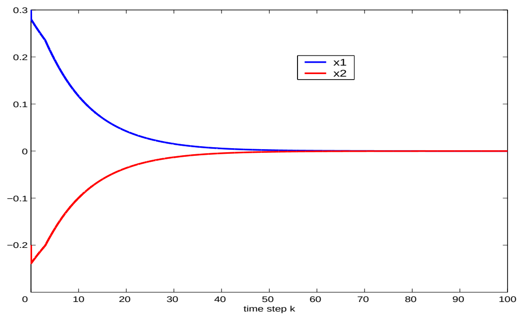

Figure 4. Time response of the state viable $x_1(t)$ and $x_2(t)$ with no fault

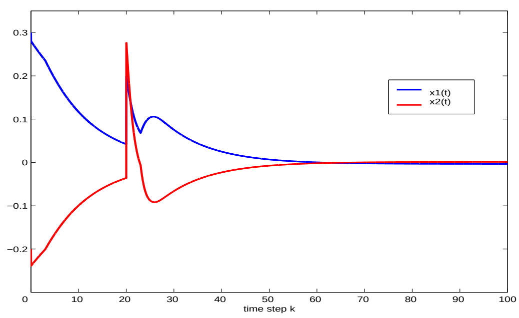

Figure 5. Time response of the state viable $x_1(t)$ and $x_2(t)$ with fault $f_3(t)$

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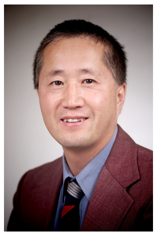

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